Cosmology 1: Basic Cosmology

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General Introductory References

Texts Edward Kolb and Michael Turner, THE EARLY UNIVERSE.

Scott Dodelson, MODERN COSMOLOGY

Michel Le Bellac, THERMAL FIELD THEORY

CMB Hu and Sugiyama, astro-ph/9411008.

W. Hu, U. Seljak, M. J. White and M. Zaldarriaga, Phys. Rev. D **57**, 3290 (1998)

Inflation Mukhanov, Feldman, Brandenberger, Phys. Rept. 215 (1992).

Lidsey, Liddle, Kolb, Copeland Barreiro, and Abney Rev. Mod. Phys 69, 373 (1997).

Lyth and Riotto, hep-ph/9807278.

EW Baryogenesis P. Huet and A. E. Nelson, Phys. Rev. D **53**, 4578 (1996)

hep-ph/9901312, hep-ph/9807454, hep-ph/0609145

Cosmology related to MSSM

Chung, Everett, Kane, King, Lykken, and Wang hep-ph/0312378.

What is cosmology?

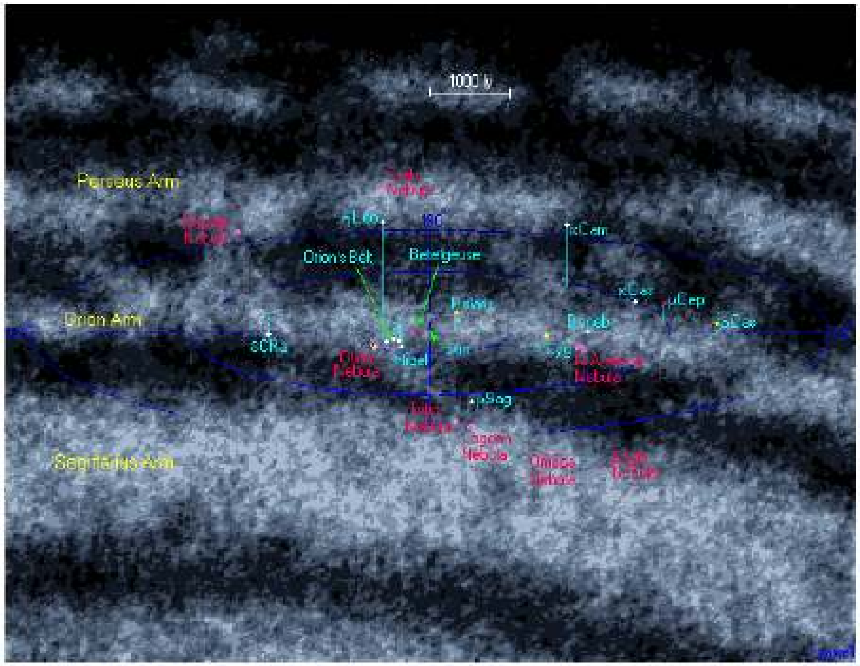
- Study of the origin and large scale structure of the universe
  - Large scale > 10 kpc (= 30,000 lyr ; galaxy size).
  - Largest scale observed (around 10,000 Mpc).
- Traditionally: gravitational and thermal history
  - Far away galaxies seem to receding away from us with a velocity proportional to its distance. (universe is not static or stationary -- history)
  - There is a thermal background radiation at 2.7 degrees Kelvin. (thermal history)

$$\hbar \neq 0 \qquad T \neq 0 \qquad G_N \neq 0 \qquad \frac{1}{c} \neq 0$$

Started with stars

With stars, homogeneity and isotropy is unclear.

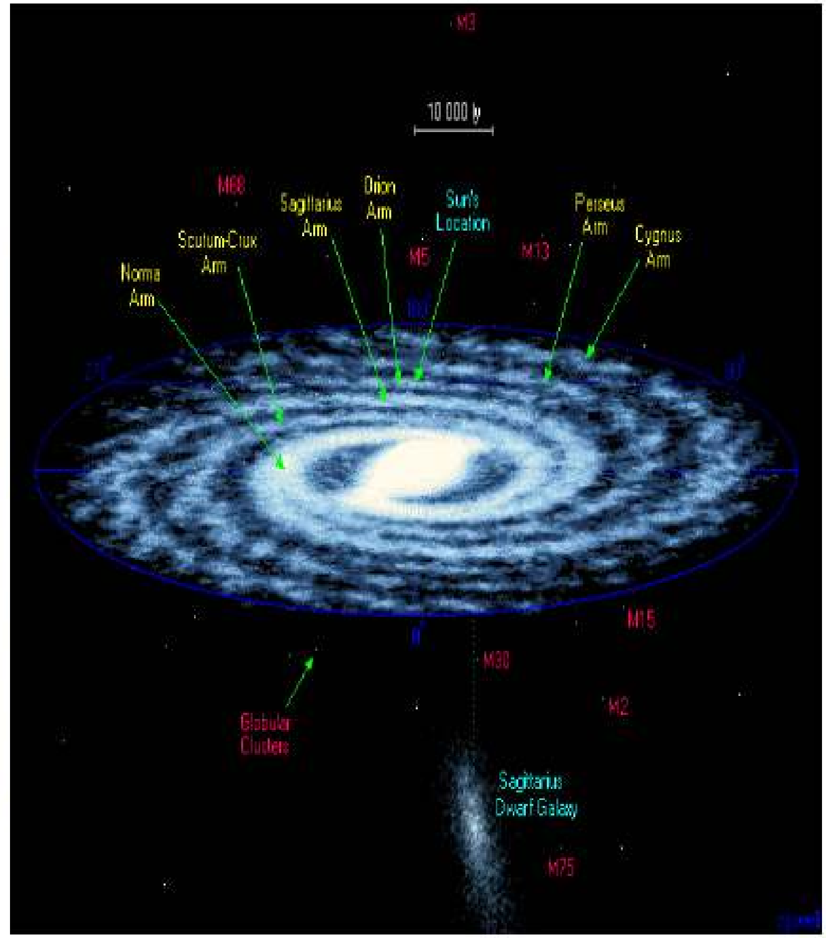
The Orion Arm



Bright giant and supergiant stars shown in the Orion arm.

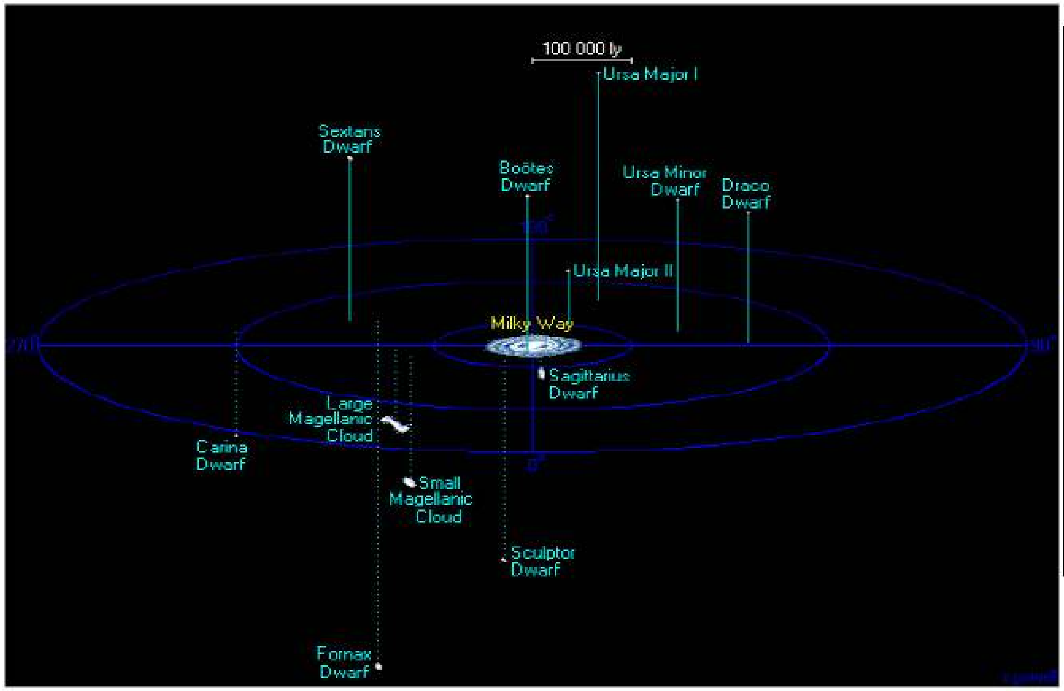
ρ Cassiopeia is 10^3 more luminous than the sun (dim to naked eye)

Milky Way ($\sim 10^{11}$ stars)



- Sagittarius Galaxy is being engulfed by our galaxy
- globular cluster outside of galactic plane (only around 10^6 stars)

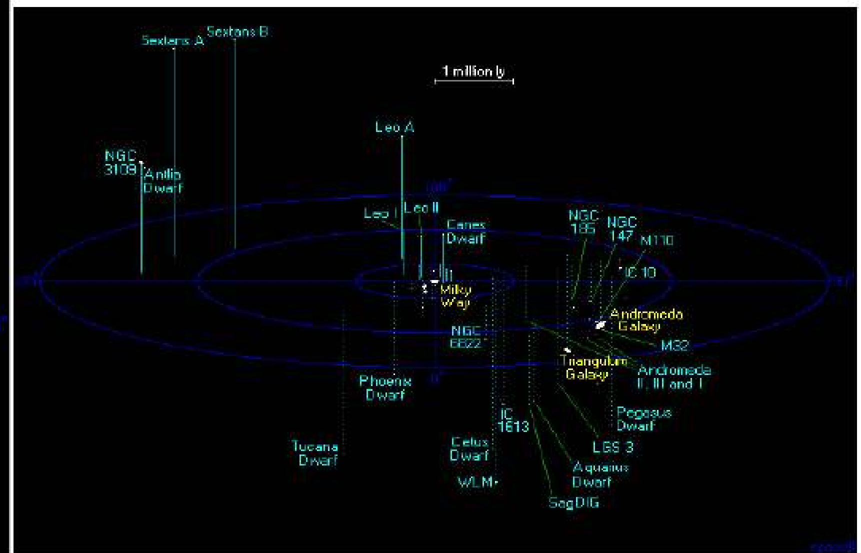
Sattellite Galaxies



Around 12 dwarf galaxies are in this range

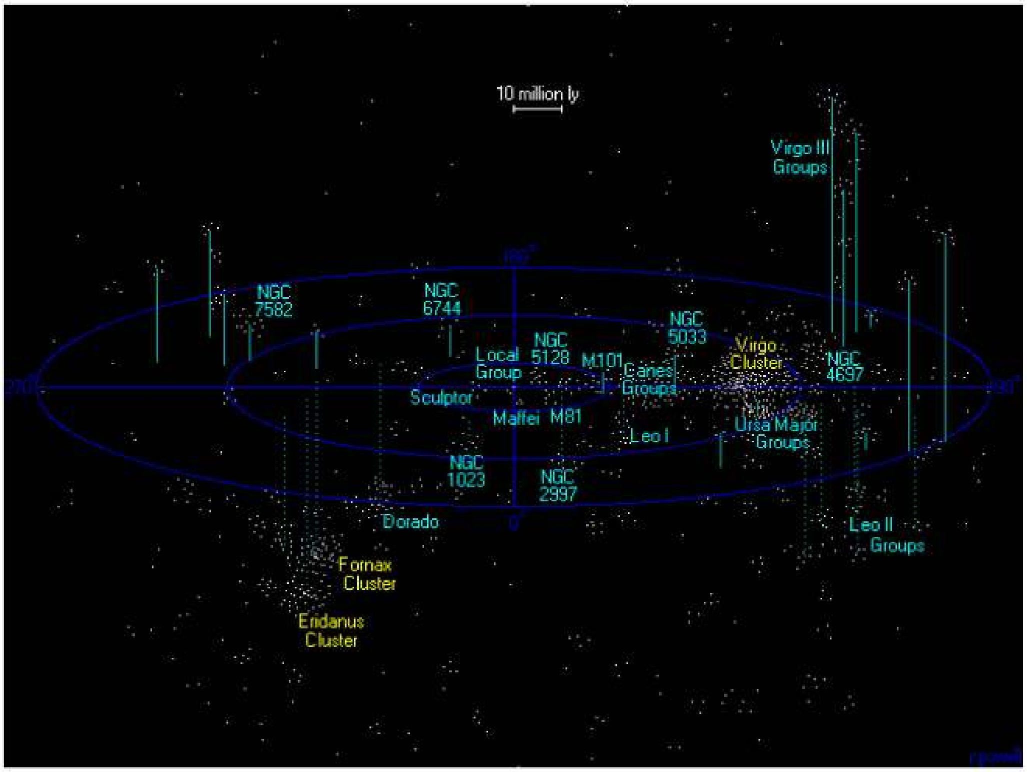
(typically $O(10^7 \sim 10^8)$ stars per dwarf)

Local Group of Galaxies



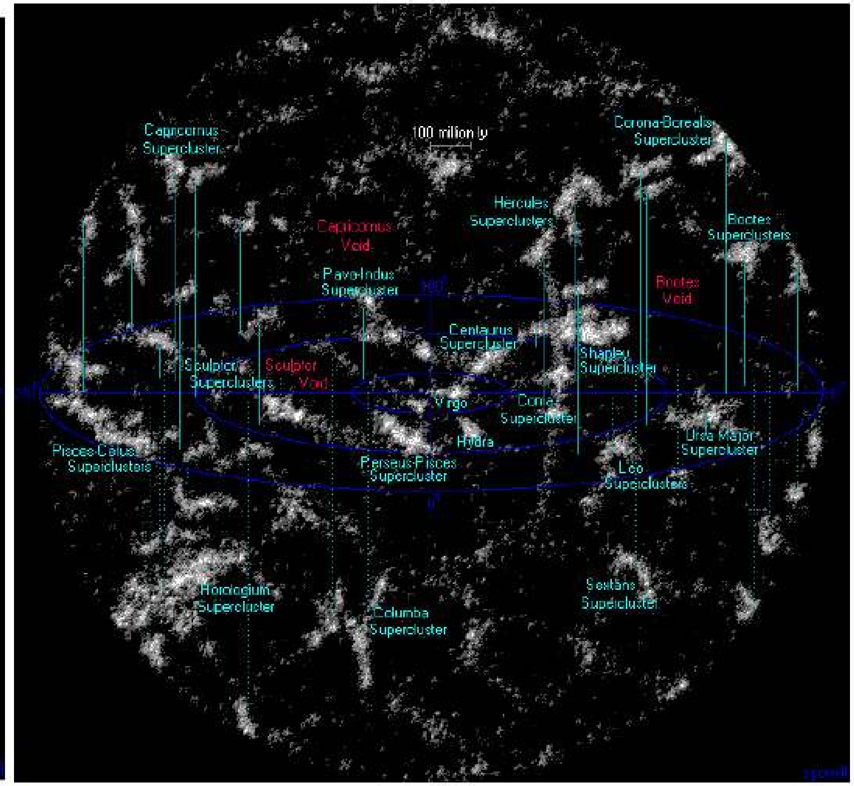
- Within about 5 million ly, there are about 46 dwarf galaxies known. Since faint, more discovery is likely.
- Number of large galaxies is 3 in this range

Virgo Supercluster



- Our local group is falling into the Virgo cluster ($\Theta(10^2)$ galaxies)
- All groups here are collectively called a supercluster
  $\sim \Theta(10^2)$ groups $\sim \Theta(10^3)$ large galaxies
  $\sim \Theta(10^4)$ stars

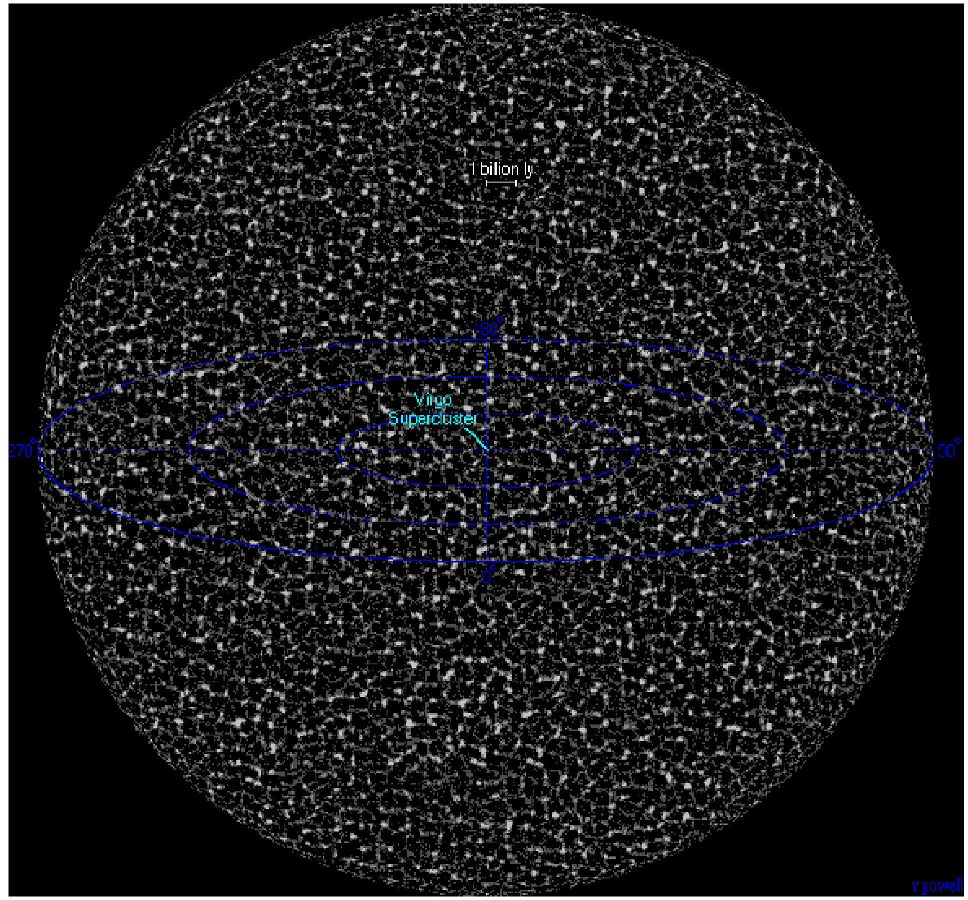
Neighboring Superclusters



- Galaxies and clusters form sheets and walls leaving large voids ($\Theta(10^8)$ ly)
- 100 superclusters in 10^9 ly.
- homogeneity scale is larger than 10^9 ly

The visible universe ($\sim O(10^{10})$ ly)

Only on largest scales, homogeneity and isotropy seems like a plausible assumption if consider just stars.



Because gravity is attractive, universe is very clumpy.

Star homogeneity scale:
Larger than about 100 Mpc

For recent speculations of an inhomogeneous universe, see **0711.3459** and **0712.0370**

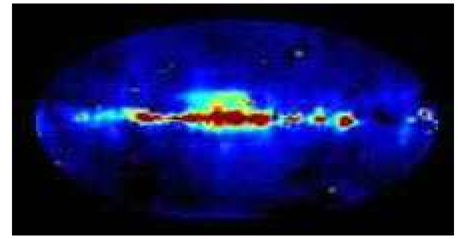
- Number of superclusters in the visible universe = 10 million
- Number of galaxy groups in the visible universe = 25 billion
- Number of large galaxies in the visible universe = 350 billion
- Number of dwarf galaxies in the visible universe = 7 trillion
- Number of stars in the visible universe = 30 billion trillion (3×10^{22})

Multi-wavelengths, Multiple probes

1) Best evidence for isotropy
2) **Farthest and oldest** we can see with a direct probe

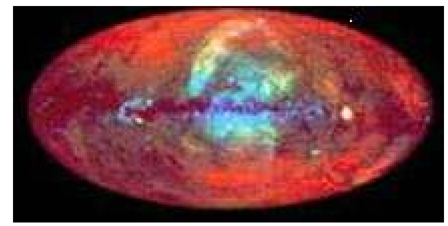
Electromagnetic

Gamma rays

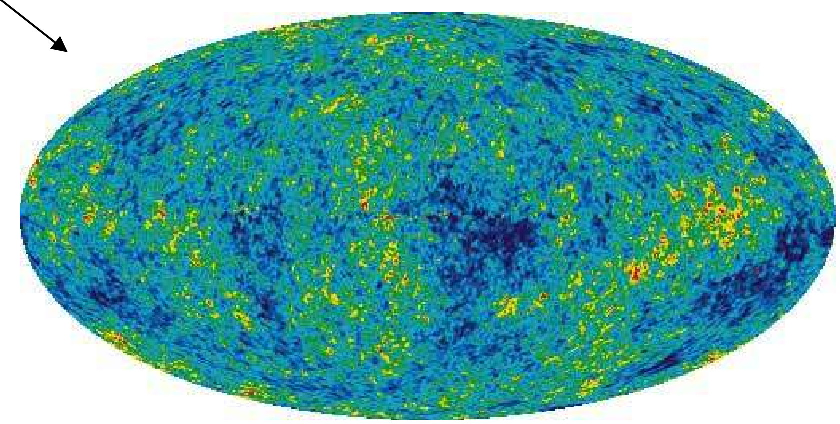


High E

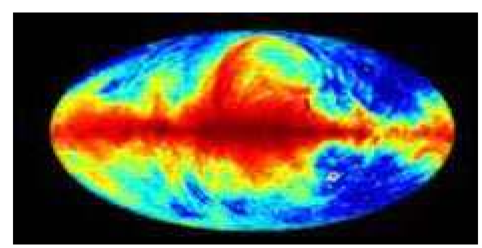
X-rays



CMB

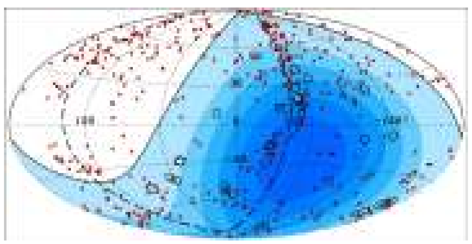


Radio

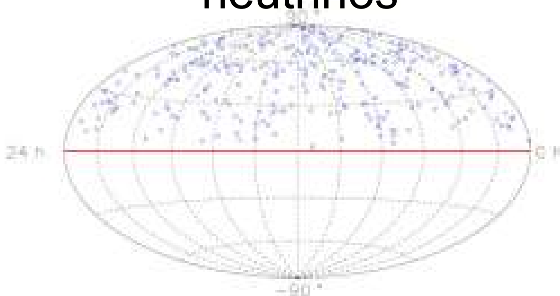


Low E

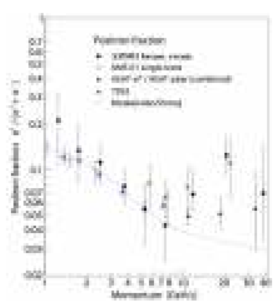
Cosmic rays: Protons + (?)

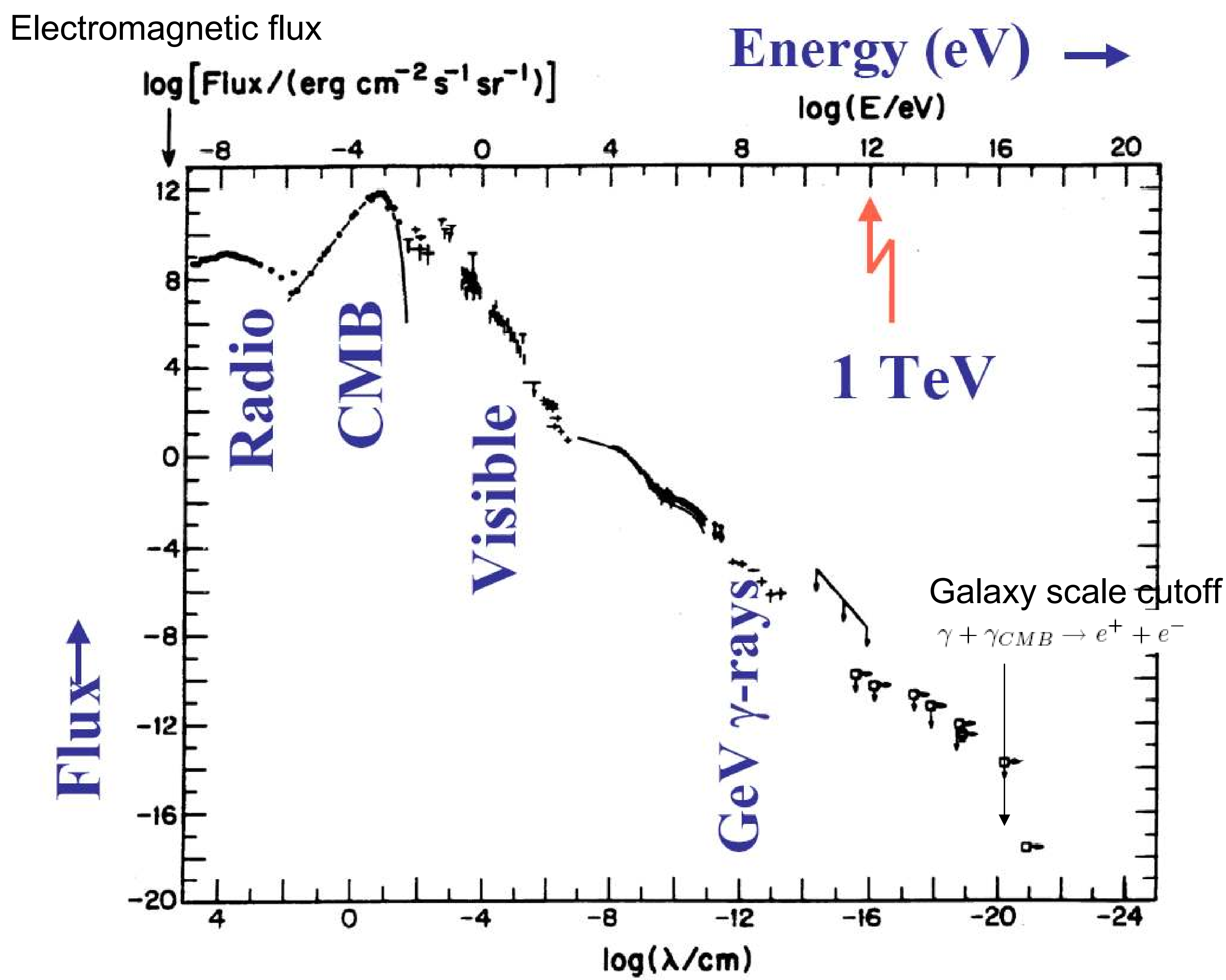


neutrinos

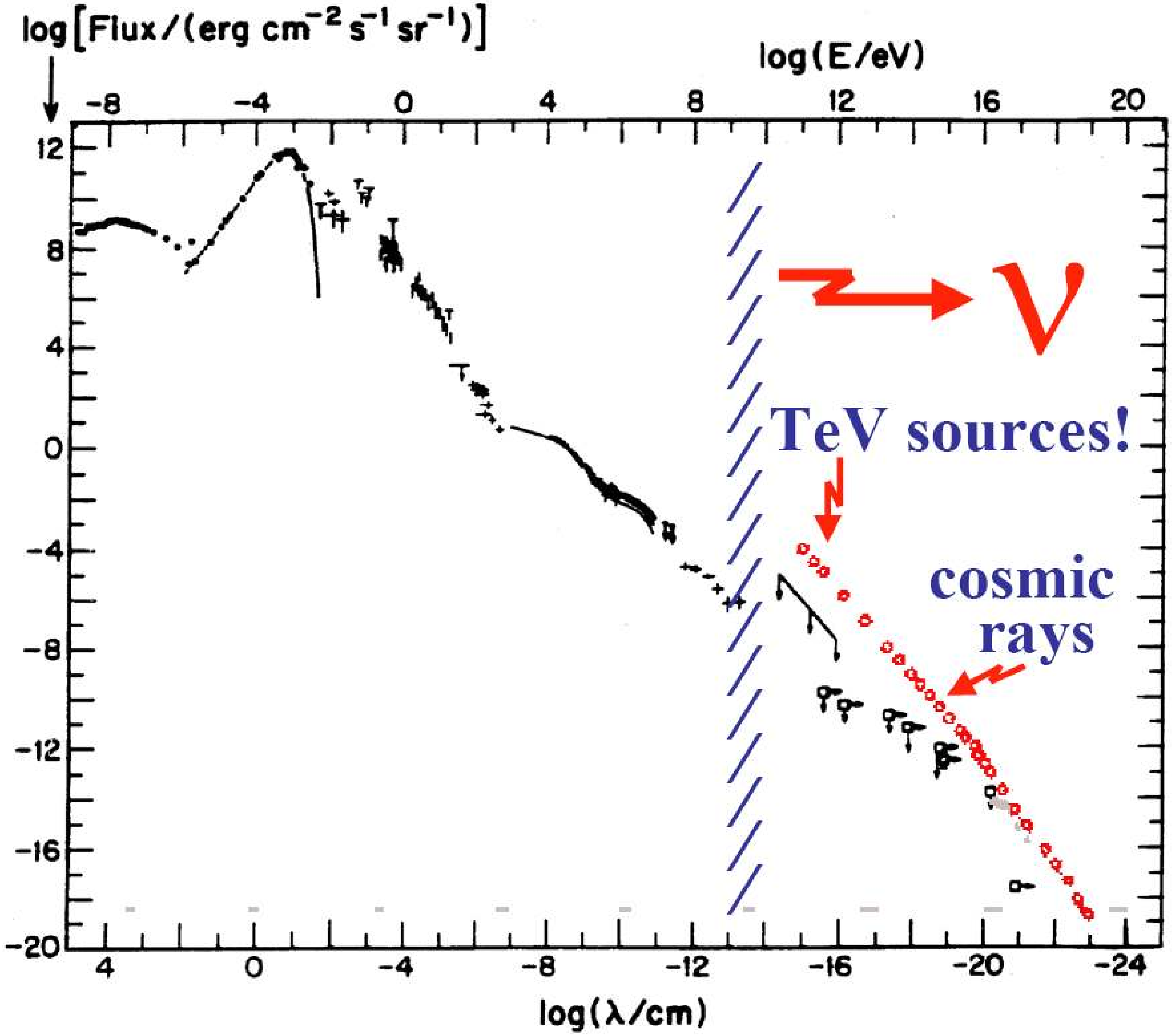


antimatter





Electromagnetic flux
Energy (eV)
log[Flux/(erg cm^{-2} s^{-1} sr^{-1})]
log(E/eV)
-8
-4
0
4
8
12
16
20
12
8
4
0
-4
-8
-12
-16
-20
Radio
CMB
Visible
GeV γ-rays
1 TeV
Galaxy scale cutoff
$\gamma + \gamma_{CMB} \rightarrow e^+ + e^-$
Flux
4
0
-4
-8
-12
-16
-20
-24
log(λ/cm)



[courtesey of F. Halzen]

Theory

1) Homogeneous
2) Perturbations
3) CMB scalar
4) Large scale structure
5) Polarization

General Theoretical Problem

- Einstein equations (Equivalence principle)

$$S=\frac{-1}{16\pi}\int d^4x\sqrt{-g}\,R+\int d^4x\sqrt{-g}\,L_M$$

$$R_{\mu\nu}[g_{\mu\nu}]-\frac{1}{2}g_{\mu\nu}R[g_{\mu\nu}]=8\pi T_{\mu\nu}[g_{\mu\nu}]\longrightarrow$$ Put in known fields (more later . . .)

- Boltzmann Equations

$$\left[p^\alpha\frac{\partial}{\partial X^\alpha}-\Gamma^\alpha_{\beta\gamma}p^\beta p^\gamma\frac{\partial}{\partial p^\alpha}\right]f(X^\alpha,p^\alpha)=\boxed{C[f]}$$

Collision term; Approximation

$$T^\mu{}_\nu=g_X\int\frac{d^3P}{(2\pi)^3}\frac{1}{\sqrt{|g|}}\frac{P^\mu P_\nu}{P^0}f_X(t,\vec{x},\vec{p})$$

$$\vec{p}=g_{ij}P^iP^j\hat{p}$$

- Other relevant field equations (e.g. magnetic field, inflaton, axion, quintessence)

Difficulties/opportunities: 1) nonlinearity 2) large number of degrees of freedom 3) unknown initial conditions

Usual Approach To Resolving Difficulties

- Non-linearity:
  - Perturbation theory when possible (e.g. large scales)
  - Numerical simulations (Smaller than about 10 Mpc)
- Large number of degrees of freedom:
  - Statistical observables
  - Categorize based on dominant interactions
- Unknown initial conditions:
  - Thermal equilibrium
  - Homogeneity and isotropy
  - Inflation
  - Adiabatic vacuum

Perturbation Theory: Zeroth order

Homogeneous and isotropic fluid.

CMB + philosophical prejudice of non-preferred frame

$\longrightarrow$ $\boxed{\text{Homogeneous and isotropic on large length scales}}$

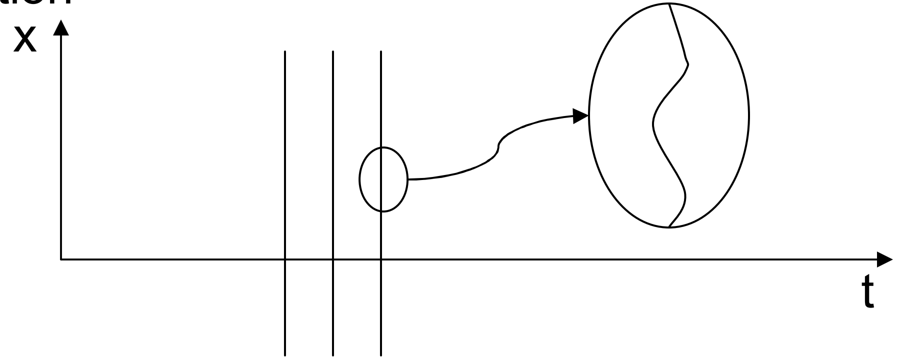
Geometry: $ds^2 = dt^2 - a^2(t)\left[\dfrac{dr^2}{1-kr^2} + r^2 d\Theta^2 + r^2 \sin^2\Theta \, d\varphi^2\right]$

·characterizes the curvature of space at a fixed time

$${}^3R = 6\,k/a^2 < 0.01\,H^2 \sim (10^{-44}\,GeV)^2$$

Energy: $T_{\mu\nu} = (\rho(t) + P(t))\,u_\mu u_\nu + P(t)\,g_{\mu\nu}$ e.g. $T_{00} = \rho$ $T_{11} = a^2 P$

Foliation



SM + BSM physics determines
The properties of these objects

e.g. equation of state

Background

- Einstein

$$H^2 + \frac{k}{a^2} = \frac{\rho}{3} \qquad H \equiv \frac{a'(t)}{a(t)} \qquad \frac{M_{pl}}{\sqrt{8\pi}} = \frac{1}{\sqrt{G_N 8\pi}} = 1$$

$$2H'(t) + 3H^2 + \frac{k}{a^2} = -P$$

expansion rate

combine: $\frac{a''(t)}{a(t)} = \frac{-1}{6}(\rho + 3P)$ ← ordinary matter: decelerate

alternate: $d(\rho a^3) = -P\, d(a^3)$ ← perfect fluid energy conservation and adiabatic flow

- Notation and examples

$$\boxed{w \equiv \frac{P}{\rho} = equation\ of\ state}$$

Short distance physics

Matter dominated $\left\{\rho \propto \frac{1}{a^3}\ ,\ \ w = 0\right\} \rightarrow \ a \propto t^{2/3}$

Radiation dominated $\left\{\rho \propto \frac{1}{a^4}\ ,\ \ w = \frac{1}{3}\right\} \rightarrow \ a \propto t^{1/2}$

Homogeneity and Isotropy

- "on the average" Homogeneity and isotropy

$$ds^2 = dt^2 - a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2 d\Theta^2 + r^2 \sin^2\Theta \, d\varphi^2\right]$$

$\uparrow$

·characterizes the curvature of space at a fixed time

$${}^3R = 6\,k/a^2 < 0.01\,H^2 \sim (10^{-44}\,GeV)^2$$

- Stress Tensor: Perfect fluid

$$T_{\mu\nu} = (\rho(t) + P(t))\,u_\mu u_\nu + P(t)\,g_{\mu\nu} \qquad e.g.\ \ T_{00} = \rho \qquad T_{11} = a^2 P$$

- Open Problem: Is the naïve averaging of the background density correct?

Explicit Stress-Energy Components

$$(\rho_c \equiv 3H_0^2 \sim 10^{-46}\,GeV^4)$$

$$\Omega_X = \rho_X / \rho_c$$

- Popular Model

energy conservation

$$\rho_\gamma = \rho_\gamma(t_0)\left(\frac{a_0}{a}\right)^4 \qquad P_\gamma = \frac{\rho_\gamma}{3} \qquad \Omega_\gamma \sim 10^{-5}$$

noninteracting particle

$$\rho_{b,c} = \rho_{b,c}(t_0)\left(\frac{a_0}{a}\right)^3 \qquad P_{b,c} = 0 \qquad \Omega_b \approx 0.044 \qquad \Omega_c \approx 0.22$$

$$\rho_\nu = \rho_\nu(t_0)\left(\frac{a_0}{a}\right)^3 \qquad P_\nu = 0 \qquad \Omega_\nu \sim 0.01$$

negative pressure

$$\rho_\Lambda = \rho_\Lambda(t_0) \qquad P_\Lambda \approx -\rho_\Lambda \qquad \Omega_\Lambda \approx 0.73$$

$$\rho_{tot} = \rho_\gamma + \rho_\nu + \rho_b + \rho_c + \rho_\Lambda \qquad \Omega_{tot} \approx 1.0$$

$$P_{tot} = P_\gamma + P_\nu + P_b + P_c + P_\Lambda$$

What about interactions and Momentum distributions?

Momentum Distribution of Things that Once Scattered "Quickly"

- If interactions are fast, thermal equilibrium is established (e.g. motivates initial conds.): Motivated by Boltzmann H-theorem → Interactions lead to **equilibrium phase space distribution**
- CMB has equilib. spectrum today → possibility: CMB once in equilib.
- Anything that strongly interacted with CMB was in equilib.

$$\frac{dn_X}{dt} + 3Hn_X = -\Gamma_{X\to\gamma\gamma}(n_X - n_X^{eq})$$

Out of equilib: $\frac{\langle \Gamma_{X\to\gamma\gamma}\rangle}{3H} \lesssim 1$

More formally:

$$[p^\alpha \frac{\partial}{\partial x^\alpha} - \Gamma^\alpha_{\beta\gamma} p^\beta p^\gamma \frac{\partial}{\partial p^\alpha}] f(x^\alpha, p^\alpha) = C[f]$$

$$\partial_t \int d^3p f + 3H \int d^3p f = \int \frac{d^3p}{E} C[f] \qquad n(t) = \frac{g}{(2\pi)^3} \int d^3p f$$

$$\frac{g_X}{(2\pi)^3} \int \frac{d^3p_X}{E_X} C[f] = -\int d\Pi_X d\Pi_a d\Pi_b d\Pi_c (2\pi)^4 \delta^{(4)}(p_X + p_a - p_b - p_c) \times$$
$$\left[|M|^2_{X+a\to b+c} f_X f_a (1 \pm f_b)(1 \pm f_c) - |M|^2_{b+c\to X+a} f_b f_c (1 \pm f_X)(1 \pm f_a) \right]$$

$$d\Pi_X \equiv \frac{g_X}{(2\pi)^3} \frac{d^3p_X}{2E_X}$$

Equilibrium Distributions:

- Equilibrium Thermodynamics $n = \frac{g}{(2\pi)^3} \int f(E)\, d^3p$

$$\rho = \frac{g}{(2\pi)^3} \int f(E)\, E\, d^3p \qquad E = \sqrt{p^2 + m^2}$$

$$P = \frac{g}{(2\pi)^3} \int \frac{p^2}{3E} f(E)\, d^3p \qquad f(E) = \frac{1}{\exp\frac{(E-\mu)}{T} \pm 1}$$

can fall exponentially with temperature

- Photon Temperature = “temperature of universe”

$$\rho_\gamma = \frac{\pi^2}{30} 2 T^4 \qquad \rho_R \equiv \frac{\pi^2}{30} g_*(T)\, T^4$$

- Entropy (conservation gives T history)

$$s = \frac{(\rho + p)}{T} \equiv \frac{2\pi^2}{45} g_{*s}(T)\, T^3$$

Early Universe (T>1 MeV)

$g_*(T) \approx g_{*S}(T)$

SM only: $g_*(T > 300\, GeV) \approx 107$

today (for massless neutrinos): $T \approx 2.34 \times 10^{-4}\, eV$ $\quad g_{*S} \approx 3.9$ $\quad g_* \approx 3.36$

$g_\gamma = 2 \rightarrow$ photons dominate and can be measured!

Jargon

- Equilibrium conditions: $\Gamma \gg H$
  - Kinetic equilibrium: $X\{Y\} \rightarrow X\{Z\}$ Y and Z in equilib with photon
    - maintains same temperature
    - particle number does not change
  - Chemical equilibrium: $X\{Y\} \rightarrow \{Z\}$ Y and Z in equilib with photon
    - maintains same temperature
    - particle number changes
    - particle number is determined by temperature
- Boltzmann equations govern approach to equilibrium
- Out of equilibrium:
    - Kinetic: decoupled
    - Chemical: freeze out

Universe is not Homogeneous and Isotropic

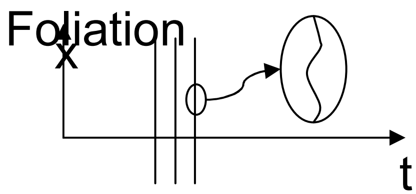
- Now, that you know how to describe homogeneous and isotropic fluid, what about spatial inhomogeneities?

perturbations

- Introduce perturbation variables and use field/Boltzmann equations
- Consider Large scale structure formation and CMB for T< keV

$$g_{00} = -(1+2\Psi(t,\vec{x})) \qquad g_{ij} = \delta_{ij}\,(1+2\Phi(t,\vec{x}))\,a^2(\eta)$$

$$p^\mu = \left(E(1-\Psi),\ p\,\hat{p}^i\,\frac{1-\Phi}{a}\right)$$



$$T^\mu{}_\nu = g_X \int \frac{d^3P}{(2\pi)^3}\frac{1}{\sqrt{|g|}}\frac{P^\mu P_\nu}{P^0} f_X(t,\vec{x},\vec{p})$$

$$\Theta \equiv \frac{\Delta T}{T}(x^\mu, \hat{p})$$

encodes anisotropy

For γ: $f_\gamma = f^{(0)} - p\,\frac{\partial f^{(0)}}{\partial p}\,\Theta$

$T^0{}_0 = \rho_\gamma(1+4\Theta_0)$

For ν: $T^0{}_0 = \rho_\nu(1+4N_0)$

$$\Theta(\hat{p}) = \sum_\ell \Theta_\ell P_\ell(\mu)(2\ell+1)\,i^\ell$$

$\mu \equiv \cos\theta$

Protons or DM: $T^0{}_0 = \rho_X(1+\delta_X)$

$$v_X^i = \frac{1}{n_X}\int \frac{d^3p}{(2\pi)^3} f_X \frac{p\,\hat{p}^i}{E}$$

What are the equations governing $\{\Phi, \Psi, \Theta, \delta, \delta_b, v, v_b \mathcal{N}\}$?

Compton Scattering:

Low temperature:

$$\gamma: \quad \dot\Theta(\eta,\vec k,\mu) + ik\mu\,\Theta(\eta,\vec k,\mu) = -\dot\Phi(\eta,\vec k) - ik\mu\,\Psi(\eta,\vec k) - \dot\tau\left[\Theta_0(\eta,\vec k) - \Theta(\eta,\vec k,\mu) + \mu v_b(\eta,\vec k)\right]$$

$$\text{DM:} \quad \dot\delta(\eta,\vec k) + ikv(\eta,\vec k) = -3\dot\Phi(\eta,\vec k)$$

$$\dot v(\eta,\vec k) + Hv(\eta,\vec k) = -ik\Psi(\eta,\vec k)$$

Angular coupling

$\boxed{\text{almost ODEs.}}$

$$\text{Baryons:} \quad \dot\delta_b(\eta,\vec k) + ikv_b(\eta,\vec k) = -3\dot\Phi(\eta,\vec k)$$

$$\dot v_b(\eta,\vec k) + Hv_b(\eta,\vec k) = -ik\Psi(\eta,\vec k) + \frac{\dot\tau(\eta)}{R(\eta)}\left[v_b(\eta,\vec k) + 3i\,\Theta_1(\eta,\vec k)\right]$$

* We are neglecting Θ_2 and pol. for now

$$\nu: \quad \dot{\mathcal{N}} + ik\mu\mathcal{N} = -\dot\Phi - ik\mu\Psi$$

$$\frac{1}{R(\eta)} \equiv \frac{4}{3}\frac{\rho_\gamma^{(0)}}{\rho_b^{(0)}}$$

$$\Theta_1(\eta,\vec k) \equiv i\int_{-1}^{1}\frac{d\mu}{2}P_1(\mu)\,\Theta(\eta,k,\mu)$$

$$\Theta_0(\eta,\vec k) \equiv \int_{-1}^{1}\frac{d\mu}{2}\,\Theta(\eta,k,\mu)$$

Gravity:

$$k^2\Phi + 3\frac{\dot a}{a}\left(\dot\Phi - \Psi\frac{\dot a}{a}\right) = 4\pi G a^2\left[\rho_{dm}\delta + \rho_b\delta_b + 4\rho_\gamma\Theta_0 + 4\rho_\nu \mathcal{N}_0\right]$$

$$k^2(\Phi + \Psi) = -32\pi G a^2\left[\rho_\gamma\Theta_2 + \rho_\nu \mathcal{N}_2\right]$$

Language: $k\eta \ll 1$ "outside of horizon"

$k\eta \gg 1$ "inside the horizon"

Why? Consider $ds^2 = a^2(\eta)\left(d\eta^2 - |d\vec{x}|^2\right)$

Take 2 points separated by coordinate distance $|\Delta\vec{x}|$.

It's proper distance is $D_p = a(\eta)|\Delta\vec{x}|$.

The relative "speed" is

$$\frac{d}{dt}D_p = \frac{1}{a}\frac{d}{d\eta}\left(a|\Delta\vec{x}|\right) = \frac{\frac{da}{d\eta}}{a^2}\, a|\Delta\vec{x}| = Ha|\Delta\vec{x}|$$

Hence if we identify $|\Delta\vec{x}| \sim \frac{1}{|\vec{k}|}$, the two points will be causally disconnected (because of speed of light being 1) if

$$\frac{Ha}{|\vec{k}|} > 1$$

Now, note if $a \propto \eta^{q}$ w/ $q \sim O(1)$, $Ha = \frac{q}{\eta}$

$$\Rightarrow \frac{Ha}{|\vec{k}|} = \frac{q}{k\eta}$$

Hence if $k\eta \ll O(1)$, then $\frac{Ha}{k} > 1$, and the mode is outside of the horizon and cannot evolve causally.

$$T(\eta,\vec{x},\hat{p}) \equiv T(\eta)\left[1+\Theta(\eta,\vec{x},\hat{p})\right]$$

$$\Theta(\eta,\vec{x},\hat{p}) = \sum_{\ell=1}^{\infty}\sum_{m=-\ell}^{\ell} a_{\ell m}(\eta,\vec{x})\, Y_{\ell m}(\hat{p})$$

$$\langle a_{\ell m}(\eta_0,\vec{x})\, a_{\ell' m'}(\eta_0,\vec{x})\rangle \equiv \delta_{\ell\ell'}\,\delta_{mm'}\, C_\ell$$

$$\boxed{C_\ell = \frac{2}{\pi}\int dk\, k^2\, P(k) \left|\frac{\Theta_\ell(k)}{\delta(k)}\right|^2}$$

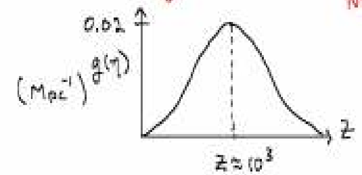
Expectation value definition

Visibility function $g(\eta) \equiv -\dot{\tau}\, e^{-\tau}$ appears in the integral.

$$e^{-\tau(\eta)} = e^{\int_{\eta_0}^{\eta} d\eta\, n_e \sigma_T a}$$

today

Note $\underbrace{\int_0^{\eta_0} d\eta\, g(\eta) = 1}_{\text{Normalized like a probability density}}$



$$\langle \tilde{\delta}(t_0,\vec{k})\, \tilde{\delta}(t,\vec{p})\rangle = (2\pi)^3 P(k)\, \delta^{(3)}(\vec{k}+\vec{p})$$

Note assumed isotropy.

restricts to LSS

Sachs-Wolfe Effect

$$\Theta_\ell(k,\eta_0) = \boxed{\int_0^{\eta_0} d\eta\, g(\eta)\left[\Theta_0(k,\eta) + \Psi(k,\eta)\right] j_\ell[k(\eta_0-\eta)]}$$

$$-\int_0^{\eta_0} d\eta\, g(\eta)\left[\frac{i v_b(k,\eta)}{k}\right]\frac{d}{d\eta} j_\ell[k(\eta_0-\eta)] \quad \text{local dipole}$$

$$\boxed{+\int_0^{\eta_0} d\eta\, e^{-\tau}\left[\dot{\Psi}(k,\eta) - \dot{\Phi}(k,\eta)\right] j_\ell[k(\eta_0-\eta)]}$$

Integrated Sachs-Wolfe Effect

potentials are either constant (MD) or decaying (ΛD)

Makes a significant contribution only after recombination

Sensitive to initial spectrum:

$$\Theta_0(k,\eta_*) \sim \frac{\Phi_p(k)}{2}\cos\left[k\int_0^{\eta_*} d\eta'\, c_s(\eta')\right]\exp\left(\frac{-k^2}{k_D^2}\right)$$

One connection to Inflation

$$k^3 P_\Phi(k) \propto k^0$$

$$\langle \Phi_p(\vec{k})\,\Phi_p^*(\vec{k}')\rangle\big|_{\frac{k}{aH}\to 0} = (2\pi)^3 P_\Phi(k)\,\delta^{(3)}(\vec{k}-\vec{k}')$$

Some intuition:

$$C_l^{SW} = \frac{2}{\pi}\int dk k^2 \frac{P(k)}{|\delta(k)|^2}\left|\int_0^{\eta_0} d\eta g(\eta)\left[\Theta_0(k,\eta)+\Psi(k,\eta)\right] j_l[k(\eta_0-\eta)]\right|^2$$

$$C_l^{SW} \sim \int dk k^2 \left|\frac{\Phi_p(k)}{2}\cos\left[\frac{2c_s(\eta_*)k}{a_* H_*}\right]\exp\left(\frac{-k^2}{k_D^2}\right) j_l\left(\frac{2k}{aH_0}\right)+\Psi(k,\eta_*)\right|^2$$

$$k_D^2 \equiv \frac{1}{-\int \frac{d\eta\left[\frac{R^2}{1+R}+\frac{4}{5}\right]}{6(1+R)\dot{\tau}}}$$

$$c_s^2 \equiv \frac{1}{3(1+R)} \qquad R \equiv \frac{3}{4}\frac{\rho_b}{\rho_\gamma}$$

<0;Dark matter dominated: No oscillation

$$\tau(\eta) \equiv \int_\eta^{\eta_0} d\eta' \, n_e \sigma_T a \sim \frac{n_e \sigma_T}{H} \sim \frac{\Gamma}{H} >> 1$$

$$k \to \frac{l a H_0}{2}$$

Diffusion damping

$$c_s l \frac{a_0}{a_*}\frac{H_0}{H_*} = \pi$$ First peak

$$\frac{H_0}{H_*} \approx \left(\frac{a_*}{a_0}\right)^{3/2}$$ CDM dominates most of H history

$$\boxed{l \approx \frac{\pi}{c_s}\left(\frac{a_0}{a_*}\right)^{1/2} \sim 6.8(10^3)^{1/2} \sim 200}$$

With sufficient attention to details, the equations in previous slides allow one to understand parametric dependences of this observable.

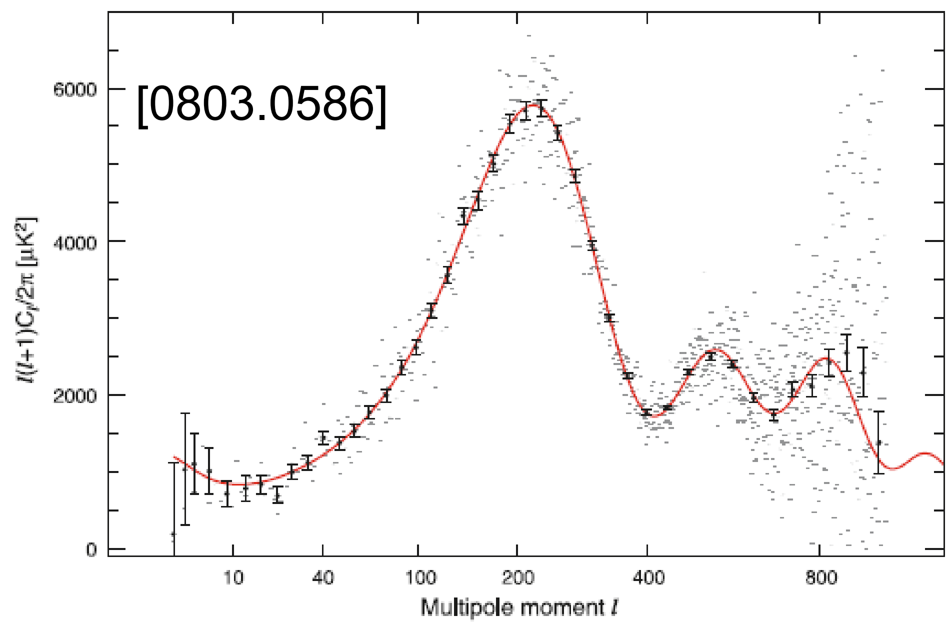
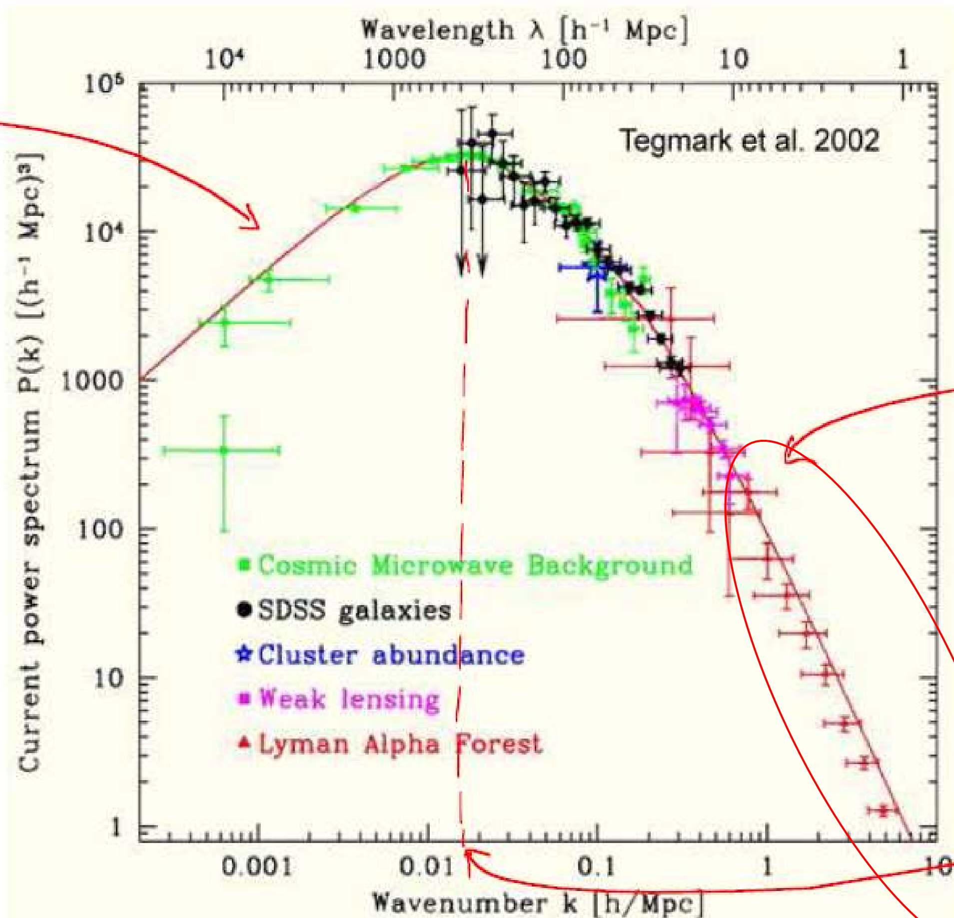


Fig. 5.— The temperature angular power spectrum corresponding to the *WMAP*-only best-fit ΛCDM mo The grey dots are the unbinned data; the black data points are binned data with 1σ error bars inclu both noise and cosmic variance computed for the best-fit model.

$$\boxed{P(k) \sim \delta^2 \propto k \quad \text{for} \quad k < k_{eq}}$$

$$\boxed{P(k) \sim \delta^2 \propto \frac{1}{k^3}\left(\ln\left(\frac{\sqrt{2}\,k}{k_{eq}}\right)\right)^2 \quad \text{for} \quad k > k_{eq}}$$

$\sim k$



k^{-3}

Non-linear

Bound on $\Omega_m h^2$

$$\boxed{k_{eq} \sim 0.073\,\mathrm{Mpc}^{-1}\,\Omega_m h^2}$$

Some intuition:

After horizon entry and matter domination: $\delta \propto a$

Hence, we know now what the power spectrum looks like: For $k < k_{eq}$,

$$\delta \sim \delta_i \frac{a_0}{a_H}$$ From solution.

$$\frac{k}{a_H} = H_H \sim H_0 \left(\frac{a_0}{a_H}\right)^{3/2}$$

$$\frac{k}{a_0} \sim H_0 \sqrt{\frac{a_0}{a_H}}$$

$$\Rightarrow \delta \sim \delta_i \left(\frac{k}{a_0} \frac{1}{H_0}\right)^2$$

If $\Phi_i \sim \delta_i(k)$ and $\Phi_i^2 \sim \frac{c^2}{k^3}$, ← Something inflation gives

$$\Rightarrow \delta_i \sim \frac{c}{k^{3/2}}$$

$$\Rightarrow \delta \propto \sqrt{k} \Rightarrow \boxed{P(k) \sim \delta^2 \propto k \quad \text{for} \quad k < k_{eq}}$$

Polarization

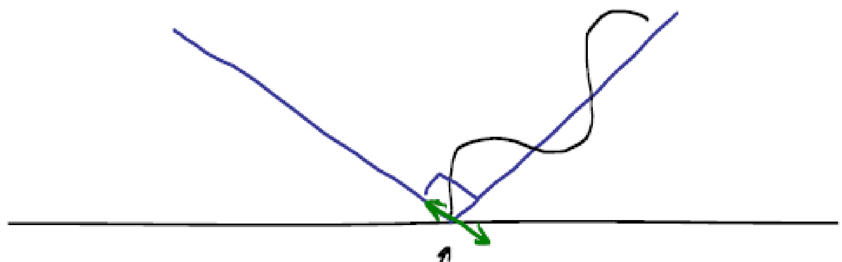
One of the CMB information that we have not measured yet is called B-mode polarization. This can contain information about inflation as well as cosmic strings. PLANCK sattellite which is to be launched this year may be able to measure this.

Hence, we turn to the discussion of CMB polarization.

How is polarized light produced by matter w/ a particular preferred direction (non-isotropic)?

☆ Thomson scattering produces polarization

Example



Since the leading radiation multipole is produced by the dipole radiation, the radiation for this scattering angle is suppressed.

Hence, if the incident light is unpolarized, the reflected light is polarized.

pol. vectors

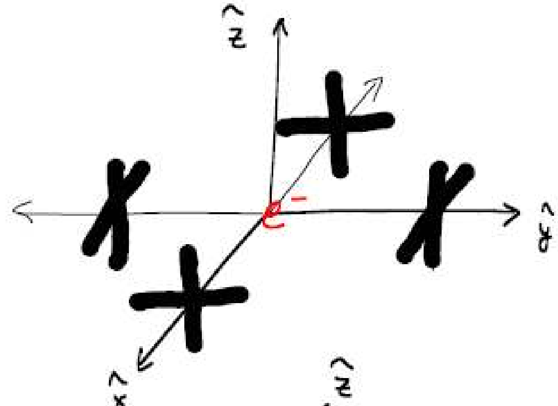
$$\left\langle \sum |M|^2 \right\rangle = (4\pi)^2 \alpha^2 \, |\varepsilon_i \cdot \varepsilon_f|^2 \;)$$

In this way glare of light reflected from the floor is polarized light.

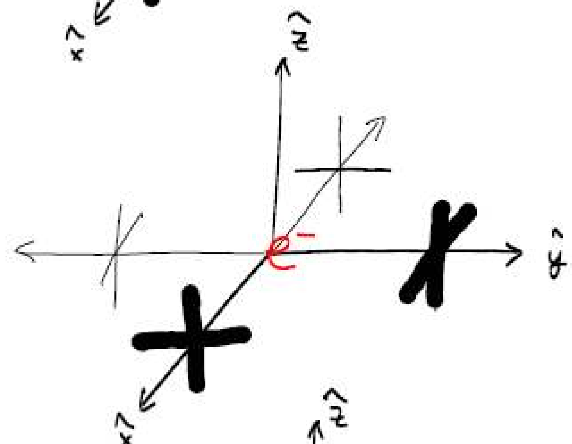
Pictoral physical intuition

e.g. Suppose the outgoing radiation is in the $\boxed{\hat{z}}$ direction

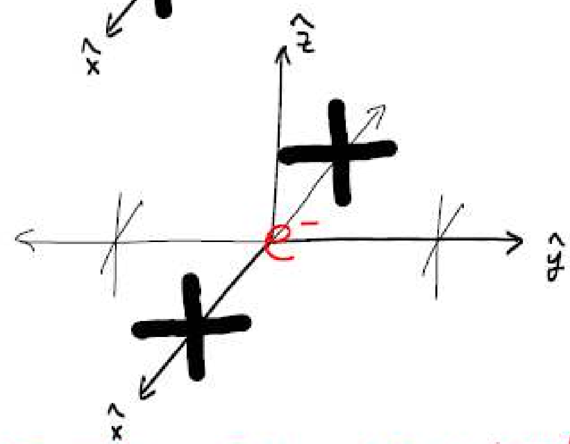
Monopole



Dipole



Quadrupole



Quadrupole anisotropy source is required to produce pol.

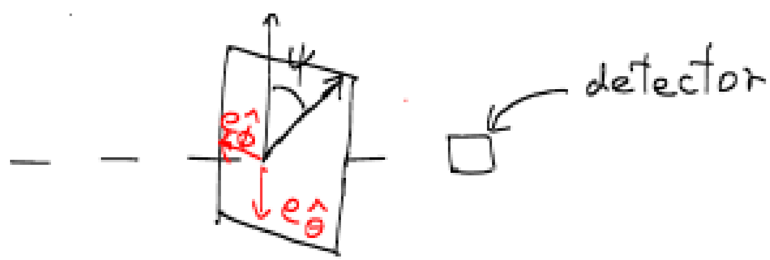
$$A_\mu(x) = \sum_\lambda \int \frac{d^3k}{\sqrt{2k_0(2\pi)^3}} \left[a^{(\lambda)}(k)\, \varepsilon_\mu^{(\lambda)}(k)\, e^{-ik\cdot x} + a^{(\lambda)\dagger}(k)\, \varepsilon_\mu^{(\lambda)*}(k)\, e^{ik\cdot x} \right]$$

Helicity basis: $\hat{\varepsilon}^{(\pm)} = \frac{-1}{\sqrt{2}} \left(e_{\hat\theta} \pm i\, e_{\hat\phi} \right)$

Intensity tensor: $f_{\xi\xi'} \equiv \langle \hat{a}^{(\xi)\dagger} \hat{a}^{(\xi)} \rangle$

$$f_{\xi\xi'} = \begin{pmatrix} f_{++} & f_{+-} \\ f_{-+} & f_{--} \end{pmatrix} \equiv \begin{pmatrix} f_I - f_V & f_Q - i f_U \\ f_Q + i f_U & f_I + f_V \end{pmatrix}$$

Interpretation?



$f(\psi)$ is observed

$$\boxed{\begin{aligned} f_Q &= \tfrac{1}{2}\left[f(0) - f\left(\tfrac{\pi}{2}\right) \right] \\ f_U &= \tfrac{1}{2}\left[f\left(\tfrac{\pi}{4}\right) - f\left(\tfrac{-\pi}{4}\right) \right] \end{aligned}}$$

$$f_I = \tfrac{1}{2}\left[f(0) + f\left(\tfrac{\pi}{2}\right) \right]$$

Polarization information.

To describe polarization anisotropies, we need a basis for describing vector functions on a unit sphere just as we needed $Y_{lm}(\theta, \phi)$ for describing scalar functions on the unit sphere.

A representation $\hat{t}(\theta, \phi) Y_{\ell m}(\theta, \phi)$ is a tensor product of $(\ell = 1) \otimes (\ell) = (\ell+1) \oplus (\ell) \oplus (\ell - 1)$ which is a reducible representation. Hence, it would be nicer to identify an irreducible representation.

Suppose we make a coordinate transformation $x^k \rightarrow x^{k\prime} = R^k{}_\ell x^\ell$ where $R^k{}_\ell$ is a rotation matrix. Then

$$Y_{\ell m}(\hat{x}) \longrightarrow Y_{\ell m}(\hat{x}') = \sum_{m'} Y_{\ell m'}(\hat{x}) D^\ell_{m'm}(R^{-1})$$

Now, note that since R^{-1} can be parameterized by Euler angles (α, β, γ), $D^\ell_{m'm} = D^\ell_{m'm}(\alpha, \beta, \gamma)$

A basis for expanding vector functions on the unit sphere: spin spherical harmonics,

Spin spherical harmonics:

$$\boxed{Y^s_{\ell m}(\theta, \phi) = (-1)^s \sqrt{\frac{2\ell+1}{4\pi}}\, D^\ell_{-s\,m}(\phi, \theta, 0)}$$

E and B modes (partiy eigenstates)

$$f_{\substack{+\\-}}(\hat{p}) = f_Q \pm i f_u = \sum_{\ell=2}^{\infty} \sum_{m=-\ell}^{\ell} a_{\pm 2,\ell m} Y_{\ell m}^{\pm 2}$$

since $s = \pm 2$

(Note that δf_u may be a parity eigenstate, but it is not a spin eigenstate)

$$Y_{\ell m}^{s} \longrightarrow (-1)^{\ell} Y_{\ell m}^{-s}$$

$$\delta f_{E,\ell}^{(m)}(\eta, \vec{k}, p) \pm i \delta f_{B,\ell}^{(m)}(\eta, \vec{k}, p) = i^{\ell} \sqrt{\frac{2\ell+1}{4\pi}} \int d\Omega\, Y_{\ell m}^{\pm 2 *}(\Omega) \left[\delta f_Q(\eta, \vec{k}, p\hat{p}) \pm i \delta f_U(\eta, \vec{k}, p\hat{p}) \right](\Omega)$$

$$\delta f_{E,\ell}^{(m)} \longrightarrow (-1)^{\ell} \delta f_{E,\ell}^{(m)}$$

$$\delta f_{B,\ell}^{(m)} \longrightarrow (-1)^{\ell+1} \delta f_{B,\ell}^{(m)}$$

$$E(\Omega) \equiv \sum_{\ell m} \delta f_{E,\ell}^{(m)} Y_{\ell m}(\Omega)$$

$$B(\Omega) \equiv \sum_{\ell m} \delta f_{B,\ell}^{(m)} Y_{\ell m}(\Omega)$$

Under parity, $E \rightarrow E$

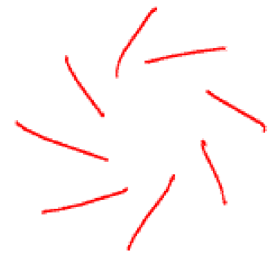
under parity, $B \rightarrow -B$

E and B are parity eigenstate angular functions containing polarization information.

E and B are scalars like temperature.

Polarization modes which are parity even: E-mode

Polarization modes which are parity odd: B-mode



Something needs to break rotational invariance around the axis of propagation. Gravity waves parallel to the direction of propagation breaks rotational invariance due to its spin 2 nature.

∴ Thomson scattering in the presence of gravity waves produces B-mode polarization. Otherwise, thomson scattering by itself cannot produce B-modes.

How does one write Boltzmann for $f_{(\pm)\pm}$?

Scalar + tensor

$$\delta g_{\mu\nu} = a^2(\eta)\begin{pmatrix} -2\Psi & \\ & 2(\Phi\delta_{ij} + E_{ij}) \end{pmatrix}$$

Scalar (Ψ, Φ) tensor (E_{ij})

$E_i{}^i = 0$

$E^i{}_{|i} = 0$

Example: Consider gravity wave (tensor perturbation) propagating along the $\hat{z}$ axis

This can be written as $m = \pm 2$ modes

$$E^T_{ij} = -\sqrt{\frac{3}{8}}\begin{pmatrix} E^{(+2)} + E^{(-2)} & i\left(E^{(+2)} - E^{(-2)}\right) & 0 \\ i\left(E^{(+2)} - E^{(-2)}\right) & -\left(E^{(+2)} + E^{(-2)}\right) & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

This couples to $m = \pm 2$ of ν and γ:

$$\frac{d^2}{d\eta^2}E^{(\pm 2)} + 2\,\frac{\frac{da}{d\eta}}{a}\,\frac{d}{d\eta}E^{(\pm 2)} + k^2 E^{(\pm 2)} = \frac{8\pi}{M_{Pl}^2}\,a^2\,\Pi^{(\pm 2)}$$

$$\Pi^{(\pm 2)} = \frac{8}{15}\rho_\nu \Theta^{(\pm 2)}_{\nu,2} + \frac{8}{15}\rho_\gamma \Theta^{(\pm 2)}_{I,2}$$

Defining temperature to be

$$\Theta^{(m)}_{X,\ell} = \frac{-\,\delta f^{(m)}_{X,\ell}}{\left(\dfrac{\partial f^{(0)}_{I}}{\partial \ln p}\right)}$$

not necessarily I

normalization

$$\delta f^{(m)}_{I,\ell}(\eta,\vec{k},p) = i^{\ell}\sqrt{\frac{2\ell+1}{4\pi}}\int_{4\pi} d\Omega\, Y^{*}_{\ell m}(\Omega)\,\delta f_{I}(\Omega)$$

$$f^{(0)}_{I} \equiv \frac{1}{e^{\frac{p}{T}}-1}\,,$$ we find the following equations.

$$\frac{d}{d\eta}\Theta^{(\pm2)}_{I,\ell} = k\left(\frac{\sqrt{\ell^2-4}}{2\ell-1}\,\Theta^{(\pm2)}_{I,\ell-1} - \frac{\sqrt{(\ell-1)(\ell+3)}}{2\ell+3}\,\Theta^{(\pm2)}_{I,\ell+1}\right) - \dot{E}^{(\pm2)}\delta_{\ell=2} + |\dot{\tau}|\left(-\Theta^{(\pm2)}_{I,\ell} + \delta_{\ell 2}\,\frac{\Theta^{(\pm2)}_{I,2} - \sqrt{6}\,\Theta^{(\pm2)}_{E,2}}{10}\right)$$

$$\frac{d}{d\eta}\Theta^{(\pm2)}_{E,\ell} = k\left(\frac{\ell^2-4}{2(2\ell-1)}\,\Theta^{(\pm2)}_{E,\ell-1} - \frac{(\ell-1)(\ell+3)}{\ell(2\ell+3)}\,\Theta^{(\pm2)}_{E,\ell+1} - \frac{4}{\ell(\ell+1)}\,\Theta^{(\pm2)}_{B,\ell}\right) + |\dot{\tau}|\left(-\Theta^{(\pm2)}_{E,\ell} + \delta_{\ell 2}\,\frac{6\,\Theta^{(\pm2)}_{E,2} - \sqrt{6}\,\Theta^{(\pm2)}_{I,2}}{10}\right)$$

$$\frac{d}{d\eta}\Theta^{(\pm2)}_{B,\ell} = k\left(\frac{\ell^2-4}{\ell(2\ell-1)}\,\Theta^{(\pm2)}_{B,\ell-1} - \frac{(\ell-1)(\ell+3)}{\ell(2\ell+3)}\,\Theta^{(\pm2)}_{B,\ell+1} + \frac{4}{\ell(\ell+1)}\,\Theta^{(\pm2)}_{E,\ell}\right) \overbrace{-|\dot{\tau}|}^{\text{damping}}\,\Theta_{B,\ell}$$

a) The only source of B-mode is $m=\pm2$ component of E

b) $m=\pm2$ component of E-mode is non-vanishing only when $\delta_{\ell 2}$ term $\Theta^{(\pm2)}_{I,2}$ is non-vanishing and $|\dot{\tau}|\neq 0$

c) $\Theta^{(\pm2)}_{I,2}$ is non-vanishing only when $\dot{E}^{(\pm2)}$ is non-vanishing

$\Rightarrow$ $\boxed{\text{Requires tensor perturbations}}$

Summary of Part 1

- Universe is clumpy today (homogeneity and isotropy is not obvious).
- Multiple probes are giving us a better picture.
- Boltzmann equations + Einstein's equations can be used to model and accurately describe most of the cosmological observations such as CMB and large scale structure.
- Inflationary (to be discussed in the next lecture) sensitivity is in the spectral information of the observables.
- B-mode polarization requires Thomson scattering in the presence of nonzero spin background. Gravity waves can provide such background. We are awaiting Planck to see if any B-mode exists.