Lecture 2: Weakly-coupled Higgs bosons

- Problems with the SM Higgs boson.
- Two-Higgs-doublet models.
- Minimal supersymmetric standard model Higgs sector
- The next-to-minimal supersymmetric standard model Higgs bosons.

Problems with the SM Higgs boson

- The electroweak symmetry breaking was put in by hand

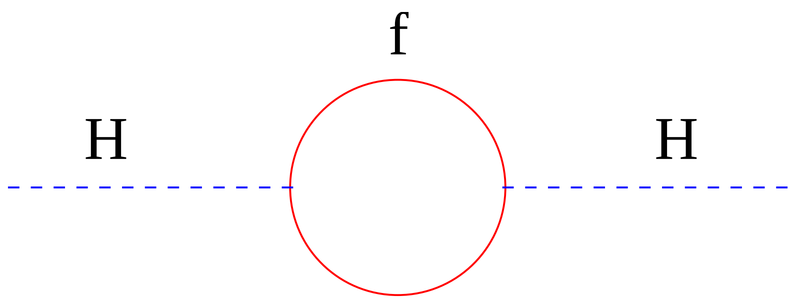
$$V_{\text{Higgs}} = \mu^2|\phi|^2 + \lambda|\phi|^4$$

  By some unknown dynamics that the SM did not address the parameter $\mu^2 < 0$.

- Large Hierarchy between M_{planck} and M_{weak}.

Gauge Hierarchy Problem

Scalar boson mass has no symmetry protection.



$$\Delta M_H^2 = \frac{|\lambda_f|^2}{16\pi^2}\left[-2\Lambda_{UV}^2 + 6m_f^2 \ln\left(\frac{\Lambda_{UV}}{m_f}\right) + ...\right]$$

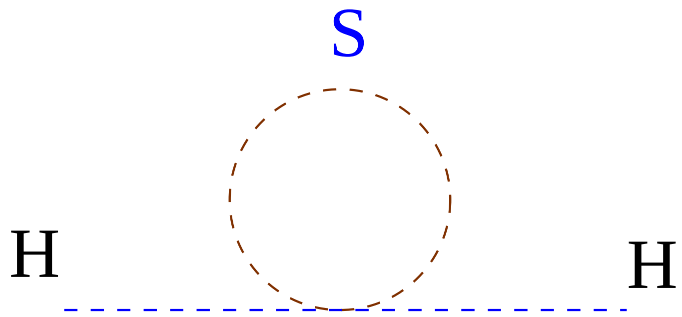
The physical Higgs boson is then

$$(M_H^2)_{\rm phys} = (M_H^2)_{\rm bare} + \Delta M_H^2 \simeq (100\ {\rm GeV})^2$$

We need a huge finely tuned cancellation in order to achieve a physical $(100\ \text{GeV})^2$ Higgs boson.

In literature, there are two classes of models to solve the hierarchy problem.

- Weakly-coupled models, e.g., supersymmetry. It predicts new scalars such that they systematically cancel the quadratic divergences



$$\Delta M_H^2 = \frac{\lambda_S}{16\pi^2}\left[\Lambda_{UV}^2 - 2m_S^2 \ln\left(\frac{\Lambda_{UV}}{m_S}\right) + \ldots\right]$$

The leading term in Λ_{UV} will cancel if

$\lambda_S = |\lambda_f|^2$ and if there are 2 such scalars

- Λ_{UV} is of order TeV. The SM would be replaced by a new theory at the TeV scale. Just like the 4-fermi interaction was replaced by the W-boson propagator. Examples include some new dynamics at TeV scale, the technicolor type models, topcolor models, little Higgs models.

Extensions to the Standard Model Higgs sector

Weakly-coupled models usually contain more than one Higgs doublets, may be two or more, triplets, or singlets. The MSSM contains two Higgs doublets. The NMSSM contains two doublets and one singlet.

Basic Constraints for adding extra Higgs fields:

1. The first constraint is the experimental value of

$$\rho \equiv \frac{m_W^2}{m_Z^2 \cos^2\theta_{\rm w}} \simeq 1$$

   very close to 1. The structure of the Higgs sector will affect the ρ parameter. Doublets and singlets will satisfy $\rho = 1$ automatically. But it is not true for an arbitrary Higgs representation. The general formula for arbitrary representations is

$$\rho = \frac{\sum_{T,Y} [4T(T+1) - Y^2] |V_{T,Y}|^2 c_{T,Y}}{\sum_{T,Y} 2Y^2 |V_{T,Y}|^2}$$

where $V_{T,Y} = \langle \phi_{T,Y} \rangle$, T is the total $SU(2)_L$ isospin and Y is the hypercharge. The constant $c_{T,Y}$ is

$$c_{T,Y} = \begin{cases} 1, & (T,Y) \in \text{complex representation} \\ \frac{1}{2}, & (T,Y) \in \text{real representation} \end{cases}$$

It is easy to see that for arbitrary $V_{T,Y}$ the condition

$$4T(T+1) - Y^2 = 2Y^2 \qquad \Leftrightarrow \qquad (2T+1)^2 - 3Y^2 = 1$$

can make sure $\rho = 1$.

Consider an example of Higgs triplet of $T = 1, Y = 0$ OR $T = 1, Y = 2$

$$\begin{pmatrix} \phi^+ \\ \phi^0 \\ \phi^- \end{pmatrix}, \qquad \begin{pmatrix} \phi^{++} \\ \phi^+ \\ \phi^0 \end{pmatrix}$$

Obviously, the triplets do not satisfy $(2T+1)^2 - 3Y^2 = 1$ condition. One can satisfy the $\rho = 1$ within experimental uncertainty by restricting the VEV of the triplet (use the current value from PDG):

$$1.0002\,^{+0.0007}_{-0.0004} = \frac{8|V_{1,0}|^2 + 2|V_{1/2,1}|^2}{2|V_{1/2,1}|^2}$$

which gives

$$\frac{|V_{1,0}|}{|V_{1/2,1}|} \le 0.03$$

2. The second constraint is the flavor-changing neutral current:

$$s \leftrightarrow d, \qquad c \leftrightarrow u$$

A theorem due to Glashow and Weinberg stated that tree-level FCNC mediated by Higgs bosons will be absent if all fermions of a given electric charge couple to no more than one Higgs doublet. There are two natural choices:

- Model I: of 2HDM is that one of the Higgs doublets do not couple to fermions at all;
- Model II: of 2HDM is that the $Y = 1$ doublet couples to the up-type fermions while the $Y = -1$ doublet couples to the down-type fermions and the charged leptons. This is also the basis for the MSSM.

Two Higgs Doublet Models

There are two complex $Y = 1$ doublets, ϕ_1 and ϕ_2 with the following Higgs potential

$$\begin{aligned} V(\phi_1, \phi_2) &= \lambda_1(\phi_1^\dagger\phi_1 - v_1^2)^2 + \lambda_2(\phi_2^\dagger\phi_2 - v_2^2)^2 + \lambda_3 \left[(\phi_1^\dagger\phi_1 - v_1^2) + (\phi_2^\dagger\phi_2 - v_2^2)\right]^2 \\ &+ \lambda_4 \left[(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_2) - (\phi_1^\dagger\phi_2)(\phi_2^\dagger\phi_1)\right]^2 \\ &+ \lambda_5 \left[\Re e(\phi_1^\dagger\phi_2) - v_1 v_2 \cos\xi\right]^2 + \lambda_6 \left[\Im m(\phi_1^\dagger\phi_2) - v_1 v_2 \sin\xi\right]^2 \end{aligned}$$

Some comments are in order here.

- All λs are real. This potential is the most general with respect to gauge invariance.
- For a large range of parameters correct pattern of EWSB is guaranteed. The minimum of the potential occurs at

$$\langle\phi_1\rangle = \begin{pmatrix} 0 \\ v_1 \end{pmatrix}, \qquad \langle\phi_2\rangle = \begin{pmatrix} 0 \\ v_2 e^{i\xi} \end{pmatrix},$$

which breaks the $SU(2)_L \times U(1)_Y \to U(1)_{\rm em}$.

- If $\sin\xi \neq 0$ then CP is violated in the Higgs sector. But if $\lambda_5 = \lambda_6$ the last two terms can be combined into a single one $|\phi_1^\dagger \phi_2 - v_1 v_2 e^{i\xi}|^2$ and the phase can be removed by a redefinition of one of the fields, e.g.,

$$\phi_2 \longrightarrow \phi_2 e^{i\xi}$$

  which does not change any other terms in the potential.

- We set $\xi = 0$, there will be no CP violation in the Higgs sector.

- Define the ratio of the VEVs

$$\tan\beta = \frac{v_2}{v_1}$$

Spectrum

There are 8 d.o.f. in two complex doublets. 3 of which will be eaten to become the longitudinal components of the gauge bosons. We substitute

$$\phi_1 = \begin{pmatrix} \phi_1^+ \\ \phi_1^0 \end{pmatrix}, \qquad \phi_2 = \begin{pmatrix} \phi_2^+ \\ \phi_2^0 \end{pmatrix}$$

into the potential.

- Charged Higgs: The mass terms of the charged fields are

$$\lambda_4 (\phi_1^- \;\; \phi_2^-) \begin{pmatrix} v_2^2 & -v_1 v_2 \\ -v_1 v_2 & v_1^2 \end{pmatrix} \begin{pmatrix} \phi_1^+ \\ \phi_2^+ \end{pmatrix}$$

It can be diagonalized by

$$\begin{pmatrix} G^\pm \\ H^\pm \end{pmatrix} = \begin{pmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{pmatrix} \begin{pmatrix} \phi_1^\pm \\ \phi_2^\pm \end{pmatrix}$$

After subsituting we obtain

$$\lambda_4 (G^- \;\; H^-) \begin{pmatrix} 0 & 0 \\ 0 & v_1^2 + v_2^2 \end{pmatrix} \begin{pmatrix} G^+ \\ H^+ \end{pmatrix}$$

The charged Higgs mass is

$$m^2_{H+} = \lambda_4(v_1^2 + v_2^2)$$

- Pseudoscalar: Again look for the mass terms for $\Im m\phi_1^0$ and $\Im m\phi_2^0$:

$$\lambda_6(\phi_1^{0,i} \quad \phi_2^{0,i}) \begin{pmatrix} v_2^2 & -v_1v_2 \\ -v_1v_2 & v_1^2 \end{pmatrix} \begin{pmatrix} \phi_1^{0,i} \\ \phi_2^{0,i} \end{pmatrix}$$

We rotate them by the same angle as the charged fields:

$$\begin{pmatrix} G^0 \\ A^0 \end{pmatrix} = \sqrt{2} \begin{pmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{pmatrix} \begin{pmatrix} \phi_1^{0,i} \\ \phi_2^{0,i} \end{pmatrix}$$

Then the mass term becomes

$$\frac{\lambda_6}{2}(G^0 \quad A^0) \begin{pmatrix} 0 & 0 \\ 0 & v_1^2 + v_2^2 \end{pmatrix} \begin{pmatrix} G^0 \\ A^0 \end{pmatrix}$$

The G^0 is the goldstone boson. The pseudoscalar mass is

$$m^2_A = \lambda_6(v_1^2 + v_2^2)$$

- Neutral Higgs bosons: We rotate the real part of ϕ_1^0 and ϕ_2^0 as

$$\begin{pmatrix} H^0 \\ h^0 \end{pmatrix} = \sqrt{2} \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} \phi_1^{0,r} - v_1 \\ \phi_2^{0,r} - v_2 \end{pmatrix}$$

where it is assumed $m_{H^0} > m_{h^0}$. The mass matrix was

$$(\phi_1^{0,r} - v_1 \;\; \phi_2^{0,r} - v_2) \begin{pmatrix} 4v_1^2(\lambda_1 + \lambda_3) + v_2^2\lambda_5 & (4\lambda_3 + \lambda_5)v_1 v_2 \\ (4\lambda_3 + \lambda_5)v_1 v_2 & 4v_2^2(\lambda_2 + \lambda_3) + v_1^2\lambda_5 \end{pmatrix} \begin{pmatrix} \phi_1^{0,r} - v_1 \\ \phi_2^{0,r} - v_2 \end{pmatrix}$$

The masses can be obtained as

$$m^2_{H^0,h^0} = \frac{1}{2}\left[M_{11} + M_{22} \pm \sqrt{(M_{11} - M_{22})^2 + 4M_{12}^2} \right]$$

and the mixing angle is

$$\sin 2\alpha = \frac{2M_{12}}{\sqrt{(M_{11} - M_{22})^2 + 4M_{12}^2}}, \qquad \cos 2\alpha = \frac{M_{11} - M_{22}}{\sqrt{(M_{11} - M_{22})^2 + 4M_{12}^2}},$$

- So totally, we have 5 physical Higgs bosons: 2 charged, 2 CP even, and 1 CP odd.

MSSM Higgs Sector (Model II)

In model II, up-type fermions couple to ϕ_1 while down-type fermions couple to ϕ_2:

$$\mathcal{L} = -y_u \overline{Q}_L u_R \tilde{\phi}_2 - y_d \overline{Q}_L d_R \phi_1 + h.c.$$

We obtain the Yukawa interactions

$$\begin{aligned}
\mathcal{L} \quad = \quad & -\frac{g m_u}{2 m_w s_\beta} \bar{u} u (\sin\alpha H^0 + \cos\alpha h^0) + \frac{g m_u \cot\beta}{2 m_w} \bar{u} i \gamma^5 u A^0 \\
& - \frac{g m_d}{2 m_w c_\beta} \bar{d} d (\cos\alpha H^0 - \sin\alpha h^0) + \frac{g m_d \tan\beta}{2 m_w} \bar{d} i \gamma^5 d A^0 \\
& + \frac{g}{\sqrt{2} m_W} \Big[\bar{d} (m_u \cot\beta P_R + m_d \tan\beta P_L) u \; H^- \\
& \quad + \bar{u} (m_u \cot\beta P_L + m_d \tan\beta P_R) d \; H^+ \Big]
\end{aligned}$$

MSSM Higgs potential

The Higgs fields of the model consist of the two Higgs doublets

$$H_u = \begin{pmatrix} H_u^+ \\ H_u^0 \end{pmatrix}, \quad H_d = \begin{pmatrix} H_d^0 \\ H_d^- \end{pmatrix}$$

The Higgs potential receives contributions from F terms, D terms, and the soft terms

$$\begin{aligned} W &= \epsilon^{ab} y^u Q^a H_u^b U^c - \epsilon^{ab} y^d Q^a H_d^b D^c + \mu \epsilon^{ab} H_u^a H_d^b \\ V_F &\equiv \left| \frac{\partial W}{\partial \phi_i} \right|^2 = |\mu|^2 (|H_u|^2 + |H_d|^2) \\ V_D &\equiv \frac{1}{2}(D^a D^a + D' D') = \frac{1}{8}(g^2 + g'^2)(|H_d|^2 - |H_u|^2)^2 + \frac{1}{2} g^2 |H_u^\dagger H_d|^2 \\ V_{\text{soft}} &= m_{H_u}^2 |H_u|^2 + m_{H_d}^2 |H_d|^2 + (B\epsilon^{ab} H_u^a H_d^b + h.c.) \end{aligned}$$

where $D^a = g\,\phi_i^\dagger \frac{\tau^a}{2} \phi_i$, $D' = g' \phi_i^\dagger \frac{Y}{2} \phi_i$. Putting all terms together the Higgs potential is

$$\begin{aligned} V_H &= (m_{H_u}^2 + |\mu|^2)|H_u|^2 + (m_{H_d}^2 + |\mu|^2)|H_d|^2 + (B\epsilon^{ab} H_u^a H_d^b + h.c.) \\ &+ \frac{1}{8}(g^2 + g'^2)(|H_d|^2 - |H_u|^2)^2 + \frac{1}{2} g^2 |H_u^\dagger H_d|^2 \end{aligned}$$

We can make a comparison with the Higgs potential of the general 2HDM and we should relate the coefficients λ_{1-6} to the present parameters

$$
\begin{aligned}
\lambda_2 &= \lambda_1 \\
\lambda_3 &= \frac{1}{8}(g^2 + g'^2) - \lambda_1 \\
\lambda_4 &= \lambda_1 - \frac{1}{2}(g^2 + g'^2) \\
\lambda_5 &= 2\lambda_1 - \frac{1}{2}g'^2 = \lambda_6 \\
m^2_{H_u} + |\mu|^2 &= 2\lambda_1 v_2^2 - \frac{1}{2}m_Z^2 \\
m^2_{H_d} + |\mu|^2 &= 2\lambda_1 v_1^2 - \frac{1}{2}m_Z^2 \\
B &= -v_1 v_2 \lambda_5 = -\frac{1}{2}(4\lambda_1 - g'^2)
\end{aligned}
$$

Therefore, instead of 6 free parameter in the general 2HDM we have only TWO independent parameters in this Higgs sector. We can therefore pick two of them, usually one takes

$$\tan\beta, \qquad m_{A^0}$$

All the other Higgs masses and the mixing angle can be expressed in terms of $\tan\beta$ and m_A.

$$
\begin{aligned}
m^2_{H+} &= m^2_A + m^2_W \\
m^2_{H^0,h^0} &= \frac{1}{2}\left[m^2_A + m^2_Z \pm \sqrt{(m^2_A + m^2_Z)^2 - 4m^2_Z m^2_A \cos^2 2\beta}\right] \\
\cos 2\alpha &= -\cos 2\beta \left(\frac{m^2_A - m^2_Z}{m^2_{H^0} - m^2_{h^0}}\right) \\
\sin 2\alpha &= -\sin 2\beta \left(\frac{m^2_{H^0} + m^2_{h^0}}{m^2_{H^0} - m^2_{h^0}}\right)
\end{aligned}
$$

where $0 \leq \beta \leq \pi/2$, which implies that $-\pi/2 \leq \alpha \leq 0$. These mass relations

$$
\begin{aligned}
m_w &\leq m_{H+} \\
m_Z &\leq m_{H^0} \\
m_{h^0} &\leq m_A \\
m_{h^0} &\leq m_Z
\end{aligned}
$$

The last relation guarantees a light Higgs boson.

Higgs mass bound

On tree-level, the lightest CP-even Higgs boson has to be lighter than the Z boson. Searches at LEP1 and LEP2 have put a bound of 114.4 GeV on m_H. If the tree-level mass relations always hold, then the SUSY would be ruled out. Fortunately, the radiative corrections to the Higgs boson mass is large.

$$
\begin{aligned}
m_h^2 &= m_h^2(\text{tree}) + m_h^2(\text{loop}) \\
m_h^2(\text{tree}) &\approx m_Z^2 - \frac{4m_Z^2 m_A^2}{m_A^2 - m_Z^2} \cot\beta \\
m_h^2(\text{loop}) &= \frac{3m_t^4}{4\pi^2 v^2} \left[\ln\left(\frac{m_{\tilde{t}_1} m_{\tilde{t}_2}}{m_t^2}\right) + \frac{|X_t|^2}{m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2} \ln\left(\frac{m_{\tilde{t}_1}^2}{m_{\tilde{t}_2}}\right) \right. \\
&\left. + \frac{1}{2}\left(\frac{|X_t|^2}{m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2}\right)^2 \left(2 - \frac{m_{\tilde{t}_1}^2 + m_{\tilde{t}_2}^2}{m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2} \ln\left(\frac{m_{\tilde{t}_1}^2}{m_{\tilde{t}_2}^2}\right)\right)\right]
\end{aligned}
$$

where $|X_t| = A_t - \mu^* \cot\beta$. Here $v = 174$ GeV.

The radiation correction is dominated by the stop loop. If the mixing is small, the correction is mainly due to the first term:

$$m_h^2(\text{loop}) \approx 4400 \ln(m_{\tilde{t}}/m_t)$$

It implies

$$m_{\tilde{t}_1} \approx m_t \; \exp\left(\frac{m_h^2 - m_Z^2}{4400\,\text{GeV}^2}\right)$$

The minimum of $m_{\tilde{t}}$ is about 510 GeV in order to obtain $m_h > 115$ GeV.

If $|X_t|$ is large, then $m_{\tilde{t}_1}$ will be much smaller than $m_{\tilde{t}_2}$. This is a very interesting scenario for the baryogenesis and searches at the LHC.

Phenomenology of the MSSM or Model II Higgs bosons

- $b \to s\gamma$
  The major contribution comes from the charged-Higgs loop of the 2HDM The effective Hamiltonian at a scale of order $O(m_b)$ is

$$\mathcal{H}_{\rm eff} = -\frac{G_F}{\sqrt{2}} V_{ts}^* V_{tb} \left[\sum_{i=1}^{6} C_i(\mu) Q_i(\mu) + C_{7\gamma}(\mu) Q_{7\gamma}(\mu) + C_{8G}(\mu) Q_{8G}(\mu) \right] .$$

The decay rate of $B \to X_s\gamma$ normalized to the experimental semileptonic decay rate is

$$\frac{\Gamma(B \to X_s\gamma)}{\Gamma(B \to X_c e \bar\nu_e)} = \frac{|V_{ts}^* V_{tb}|^2}{|V_{cb}|^2} \frac{6\,\alpha_{\rm em}}{\pi f(m_c/m_b)} |C_{7\gamma}(m_b)|^2 ,$$

where $f(z) = 1 - 8z^2 + 8z^6 - z^8 - 24z^4 \ln z$. The Wilson coefficient $C_{7\gamma}(m_b)$ is

$$C_{7\gamma}(\mu) = \eta^{\frac{16}{23}} C_{7\gamma}(M_W) + \tfrac{8}{3}\left(\eta^{\frac{14}{23}} - \eta^{\frac{16}{23}}\right) C_{8G}(M_W) + C_2(M_W) \sum_{i=1}^{8} h_i \eta^{a_i} ,$$

where $\eta = \alpha_s(M_W)/\alpha_s(\mu)$. The coefficients $C_i(M_W)$ at the leading order in 2HDM II are

$$\begin{aligned} C_j(M_W) &= 0 \qquad\qquad (j = 1,3,4,5,6)\ , \\ C_2(M_W) &= 1\ , \\ C_{7\gamma}(M_W) &= -\frac{A(x_t)}{2} - \frac{A(y_t)}{6}\cot^2\beta - B(y_t)\ , \\ C_{8G}(M_W) &= -\frac{D(x_t)}{2} - \frac{D(y_t)}{6}\cot^2\beta - E(y_t)\ , \end{aligned}$$

where $x_t = m_t^2/M_W^2$, and $y_t = m_t^2/m_{H^\pm}^2$.

The experimental data on $b \to s\,\gamma$ rate in 2003 was

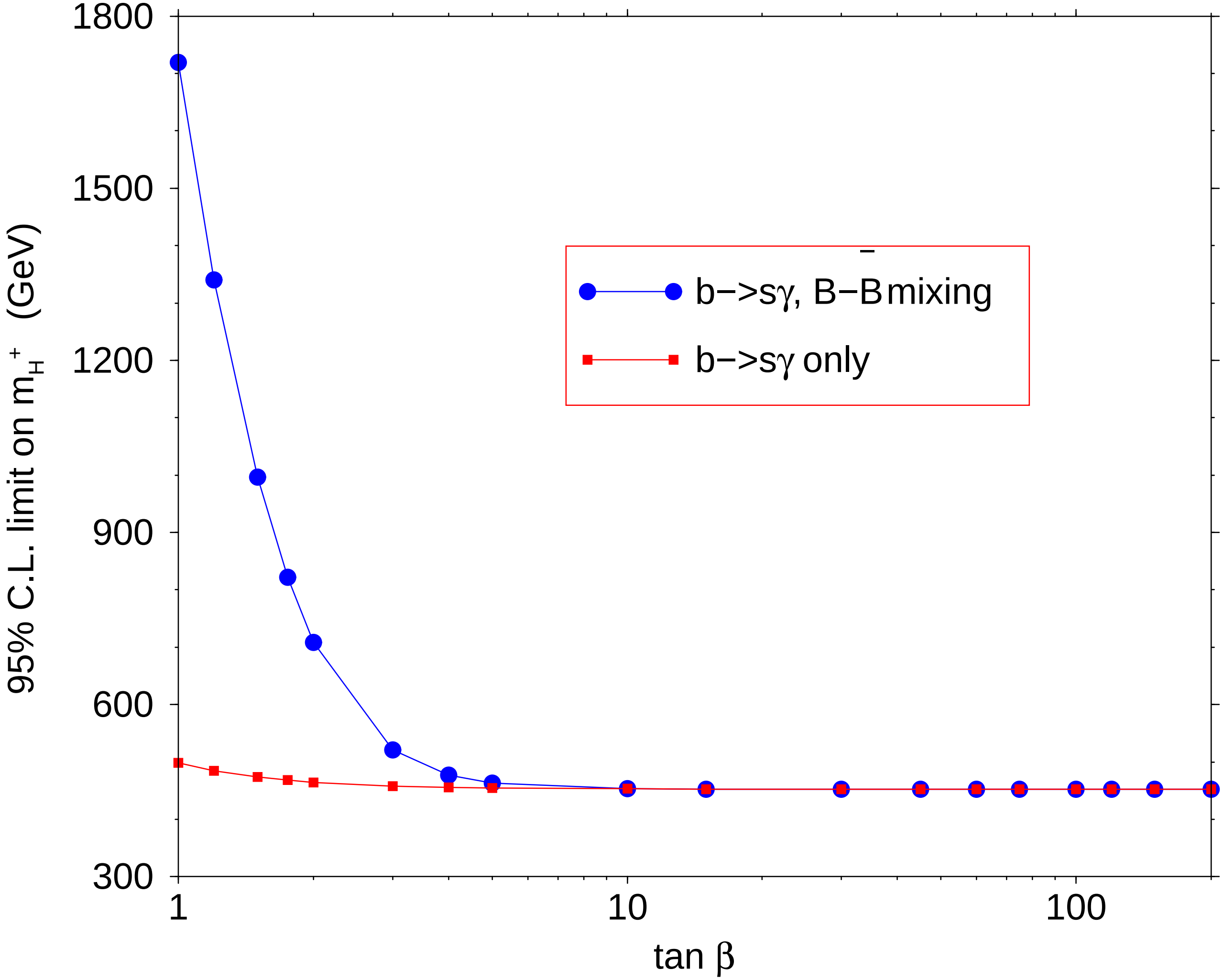
$$B(b \to s\,\gamma)|_{\rm exp} = 3.88 \pm 0.36({\rm stat}) \pm 0.37({\rm sys})^{+0.43}_{-0.28}({\rm theory})\ .$$

The SM prediction is

$$B(b \to s\,\gamma)|_{\rm SM} = (3.64 \pm 0.31) \times 10^{-4}\ ,$$

which agrees very well the data. The constraint on new physics contribution is, explicitly,

$$\Delta B(b \to s\,\gamma) \equiv B(b \to s\,\gamma)|_{\rm exp} - B(b \to s\,\gamma)|_{\rm SM} = (0.24^{+0.67}_{-0.59}) \times 10^{-4}\ ,$$


1800
1500
1200
900
600
300
1
10
100
tan β
95% C.L. limit on m_H+ (GeV)
b−>sγ, B−B̄ mixing
b−>sγ only

KC, Kong 2003

- $B^0 - \overline{B^0}$

The quantity that parameterizes the $B^0 - \overline{B^0}$ mixing is

$$x_d \equiv \frac{\Delta m_B}{\Gamma_B} = \frac{G_F^2}{6\pi^2} |V_{td}^*|^2 |V_{tb}|^2 f_B^2 \, B_B \, m_B \eta_B \tau_B \, M_W^2 \, (I_{WW} + I_{WH} + I_{HH}) \ ,$$

$$I_{WW} = \frac{x}{4} \left[1 + \frac{3-9x}{(x-1)^2} + \frac{6x^2 \log x}{(x-1)^3} \right] ,$$

$$I_{WH} = xy \cot^2\beta \left[\frac{(4z-1)\log y}{2(1-y)^2(1-z)} - \frac{3\log x}{2(1-x)^2(1-z)} + \frac{x-4}{2(1-x)(1-y)} \right] ,$$

$$I_{HH} = \frac{xy\cot^4\beta}{4} \left[\frac{1+y}{(1-y)^2} + \frac{2y\log y}{(1-y)^3} \right] ,$$

with $x = m_t^2/M_W^2$, $y = m_t^2/m_{H^\pm}^2$, $z = M_W^2/m_{H^\pm}^2$.

$$x_d = 0.755 \pm 0.015 \ .$$

We use the following input parameters $|V_{tb}V_{td}^*| = 0.0079 \pm 0.0015$, $f_B^2 B_B = (198 \pm 30 \text{ GeV})^2 (1.30 \pm 0.12)$, $m_B = 5279.3 \pm 0.7$ MeV, $\eta_B = 0.55$, and $\tau_B = 1.542 \pm 0.016$ ps. Note that the value of $|V_{tb}V_{td}^*|$ is in fact determined by the measurement of x_d.

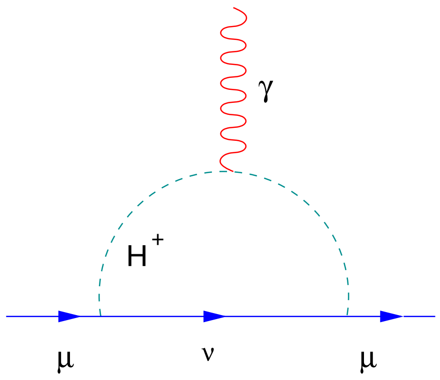
- $g-2$
  The data and the calculations of the SM in 2003 was

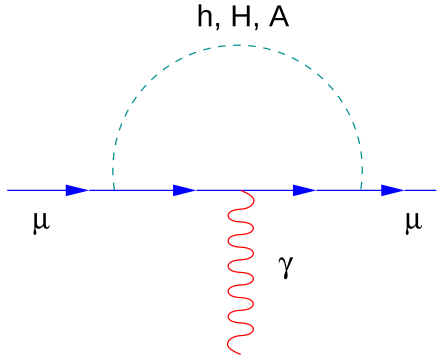
$$\Delta a_\mu \equiv a_\mu^{\rm exp} - a_\mu^{\rm SM} = 426 \pm 165 \times 10^{-11} \qquad (2.6\sigma)$$

At the present moment, the deviation is (Hagiwara et al. 2007)

$$\Delta a_\mu = (276 \pm 81) \times 10^{-11} \qquad (3.3\sigma)$$

For 2HDM: all higgs bosons contribute to a_μ at one-loop level.





$$\Delta a_\mu^h \simeq \frac{m_\mu^2}{8\pi^2 m_h^2} \left(\frac{g m_\mu}{2 m_W} \frac{\sin\alpha}{\cos\beta} \right)^2 \left(-\frac{7}{6} - \ln(m_\mu^2/m_h^2) \right)$$

$$\Delta a_\mu^H \simeq \frac{m_\mu^2}{8\pi^2 m_H^2}\left(\frac{g m_\mu}{2m_W}\frac{\cos\alpha}{\cos\beta}\right)^2 \left(-\frac{7}{6} - \ln(m_\mu^2/m_H^2)\right)$$

$$\Delta a_\mu^A \simeq -\frac{m_\mu^2}{8\pi^2 m_A^2}\left(\frac{g m_\mu}{2m_W}\tan\beta\right)^2 \left(-\frac{11}{6} - \ln(m_\mu^2/m_A^2)\right)$$

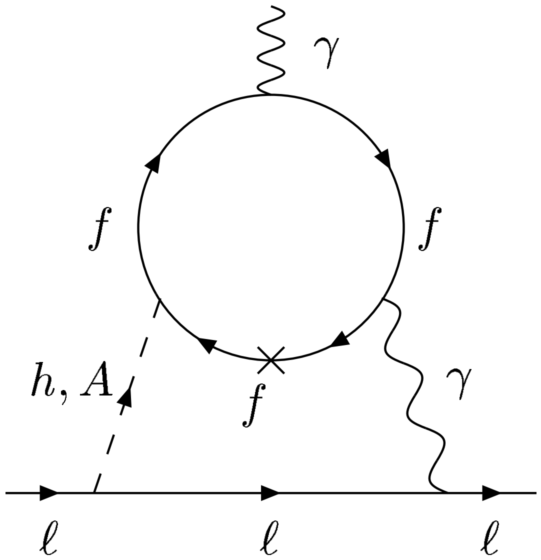
$$\Delta a_\mu^{H^+} \simeq \frac{m_\mu^2}{8\pi^2 m_{H^+}^2}\left(\frac{g m_\mu}{2m_W}\tan\beta\right)^2 \left(-\frac{1}{6} - \frac{1}{12}\frac{m_\mu^2}{m_{H^+}^2}\right)$$

Dominated by small h and A.

$$\Delta a_\mu^h(\mathrm{one-loop})\ \ \text{is positive}$$

$$\Delta a_\mu^A(\mathrm{one-loop})\ \ \text{is negative}$$

Two-loop Barr-Zee diagrams with heavy fermions.



$$\Delta a_\mu^h = -\frac{\alpha^2}{4\pi^2 \sin^2\theta_W} \frac{m_\mu^2 \lambda_\mu}{M_W^2} \sum_{f=t,b,\tau} N_c^f \mathcal{Q}_f^2 \lambda_f f\left(\frac{m_f^2}{m_h^2}\right) ,$$

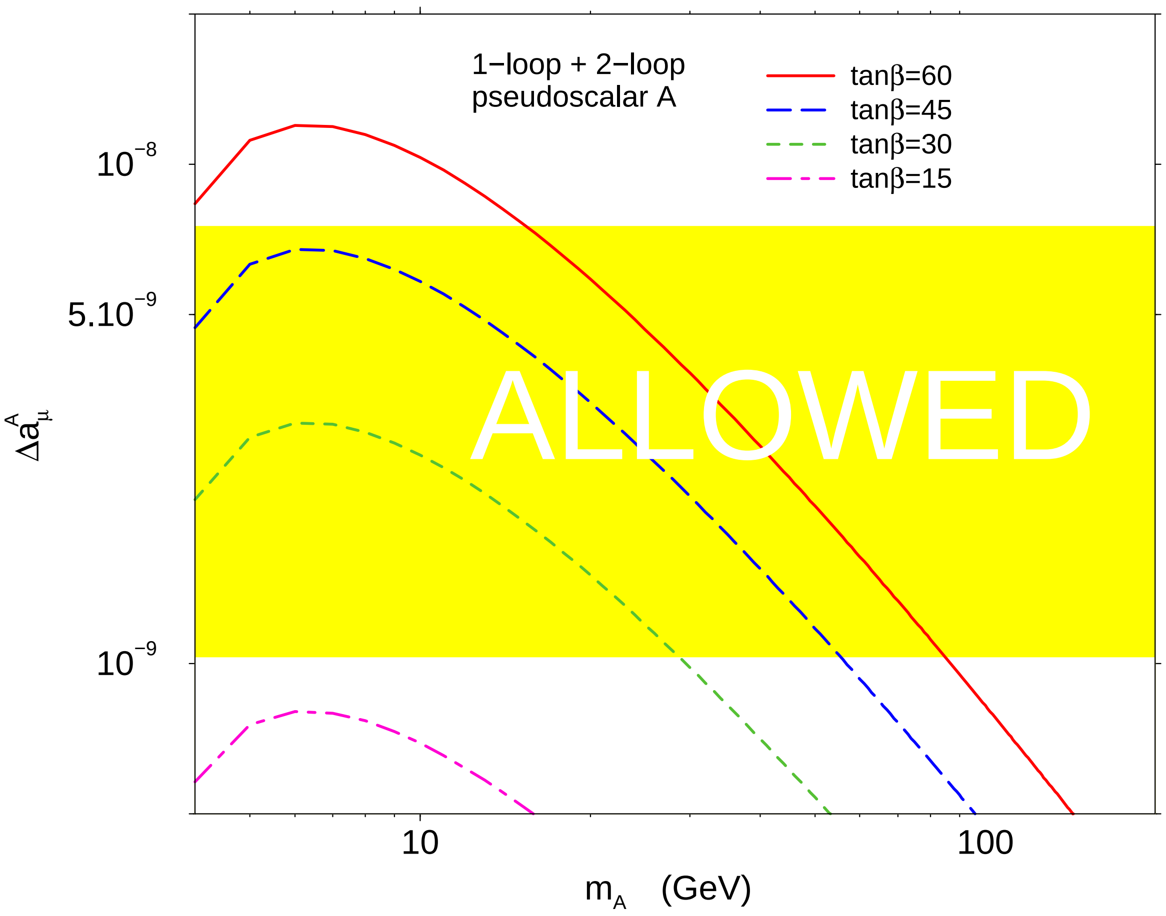
$$\Delta a_\mu^A = \frac{\alpha^2}{4\pi^2 \sin^2\theta_W} \frac{m_\mu^2 A_\mu}{M_W^2} \sum_{f=t,b,\tau} N_c^f \mathcal{Q}_f^2 A_f g\left(\frac{m_f^2}{m_A^2}\right)$$

Dominated by τ and b loops

$$\Delta a_\mu^h(\text{two} - \text{loop}) \text{ is negative}$$

$$\Delta a_\mu^A(\text{two} - \text{loop}) \;\; \text{is} \;\; \text{positive}$$

Since the deviation is positive, we want to make A^0 light and the h^0 heavy such that the overall contribution is positive and large enough.



KC, Kong 2003

- The ρ parameter constrains the spectrum of the 2HDM. Essentially, it prefers small mass splitting among the bosons. However, some level of fine-tuning among various contributions are still valid.
- There have been numerous collider searches for Higgs bosons of the 2HDM or the MSSM, in both the LEP2 and Tevatron. We do not list here.

Adding an extra Higgs singlet field

The NMSSM Superpotential

Superpotential:

$$W = \mathbf{h_u}\hat{Q}\,\hat{H}_u\,\hat{U}^c - \mathbf{h_d}\hat{Q}\,\hat{H}_d\,\hat{D}^c - \mathbf{h_e}\hat{L}\,\hat{H}_d\,\hat{E}^c + \lambda\hat{S}\,\hat{H}_u\,\hat{H}_d + \frac{1}{3}\kappa\,\hat{S}^3.$$

When the scalar field S develops a VEV $\langle S\rangle = v_s/\sqrt{2}$, the μ term is generated

$$\mu_{\rm eff} = \lambda\frac{v_s}{\sqrt{2}}$$

It was motivated by the μ problem.

Higgs Sector

Higgs fields:

$$H_u = \begin{pmatrix} H_u^+ \\ H_u^0 \end{pmatrix}, \quad H_d = \begin{pmatrix} H_d^0 \\ H_d^- \end{pmatrix}, \quad S\,.$$

Tree-level Higgs potential: $V = V_F + V_D + V_{\rm soft}$:

$$\begin{aligned} V_F &= |\lambda S|^2(|H_u|^2 + |H_d|^2) + |\lambda H_u H_d + \kappa S^2|^2 \\ V_D &= \frac{1}{8}(g^2 + g'^2)(|H_d|^2 - |H_u|^2)^2 + \frac{1}{2} g^2 |H_u^\dagger H_d|^2 \\ V_{\rm soft} &= m_{H_u}^2 |H_u|^2 + m_{H_d}^2 |H_d|^2 + m_S^2 |S|^2 + [\lambda A_\lambda S H_u H_d + \frac{1}{3}\kappa A_\kappa S^3 + \text{h.c.}] \end{aligned}$$

Minimization of the Higgs potential links $M_{H_u}^2$, $M_{H_d}^2$, M_S^2 with VEV's of H_u, H_d, S.

In the electroweak symmetry, the Higgs fields take on VEV:

$$\langle H_d \rangle = \frac{1}{\sqrt{2}}\binom{v_d}{0}, \qquad \langle H_u \rangle = \frac{1}{\sqrt{2}}\binom{0}{v_u}, \qquad \langle S \rangle = \frac{1}{\sqrt{2}} v_s$$

Then the mass terms for the Higgs fields are:

$$\begin{aligned} V &= \begin{pmatrix} H_d^+ & H_u^+ \end{pmatrix} \mathcal{M}^2_{\text{charged}} \begin{pmatrix} H_d^- \\ H_u^- \end{pmatrix} \\ &+ \frac{1}{2} \begin{pmatrix} \Im m H_d^0 & \Im m H_u^0 & \Im m S \end{pmatrix} \mathcal{M}^2_{\text{pseudo}} \begin{pmatrix} \Im m H_d^0 \\ \Im m H_u^0 \\ \Im m S \end{pmatrix} \\ &+ \frac{1}{2} \begin{pmatrix} \Re e H_d^0 & \Re e H_u^0 & \Re e S \end{pmatrix} \mathcal{M}^2_{\text{scalar}} \begin{pmatrix} \Re e H_d^0 \\ \Re e H_u^0 \\ \Re e S \end{pmatrix} \end{aligned}$$

We rotate the charged fields and the scalar fields by the angle β to project out the Goldstone modes. We are left with

$$V_{\rm mass} = m^2_{H^\pm} H^+ H^- + \frac{1}{2}\,(P_1 \;\; P_2)\mathcal{M}^2_P \begin{pmatrix} P_1 \\ P_2 \end{pmatrix} + \frac{1}{2}\,(S_1 \;\; S_2 \;\; S_3)\mathcal{M}^2_S \begin{pmatrix} S_1 \\ S_2 \\ S_3 \end{pmatrix}$$

where

$$\begin{aligned}
\mathcal{M}^2_{P\,11} &= M^2_A \,, \\
\mathcal{M}^2_{P\,12} &= \mathcal{M}^2_{P\,21} = \frac{1}{2}\cot\beta_s\,\left(M^2_A \sin 2\beta - 3\lambda\kappa v_s^2\right)\,, \\
\mathcal{M}^2_{P\,22} &= \frac{1}{4}\sin 2\beta \cot^2\beta_s\,\left(M^2_A \sin 2\beta + 3\lambda\kappa v_s^2\right) - \frac{3}{\sqrt{2}}\kappa A_\kappa v_s\,,
\end{aligned}$$

with

$$M^2_A = \frac{\lambda v_s}{\sin 2\beta}\left(\sqrt{2}A_\lambda + \kappa v_s\right)\,, \qquad \tan\beta_s = \frac{v_s}{v}$$

$$\begin{aligned}
\mathcal{M}^2_{S\,11} &= M^2_A + \left(M^2_Z - \frac{1}{2}\lambda^2 v^2\right)\sin^2 2\beta\,, \\
\mathcal{M}^2_{S\,12} &= \mathcal{M}^2_{S\,12} = -\frac{1}{2}\sin 4\beta\left(M^2_Z - \frac{1}{2}\lambda^2 v^2\right)\,,
\end{aligned}$$

$$\begin{aligned}
\mathcal{M}^2_{S\,13} &= M^2_{S\,31} = -\frac{1}{2}\cot\beta_s \cos 2\beta \left(M_A^2 \sin 2\beta + \lambda\kappa v_s^2\right) \;, \\
\mathcal{M}^2_{S\,22} &= M_Z^2 \cos^2 2\beta + \frac{1}{2}\lambda^2 v^2 \sin^2 2\beta \;, \\
\mathcal{M}^2_{S\,23} &= M^2_{S\,32} = \frac{1}{2}\left(2\lambda^2 v_s^2 - M_A^2 \sin^2 2\beta - \lambda\kappa v_s^2 \sin 2\beta\right) \cot\beta_s \;, \\
\mathcal{M}^2_{S\,33} &= \frac{1}{4} M_A^2 \sin^2 2\beta \cot^2\beta_s + 2\kappa^2 v_s^2 + \kappa A_\kappa v_s/\sqrt{2} - \frac{1}{4}\lambda\kappa v^2 \sin 2\beta
\end{aligned}$$

The MSSM limit can be recovered by $\lambda \to 0$ and $\cot\beta_s \to 0$.

The charged Higgs mass:

$$M^2_{H^\pm} = M^2_A + M^2_W - \frac{1}{2}\lambda^2 v^2$$

The scalar Higgs bosons:

The mass matrix $\mathcal{M}^2_S$ is diagonalized by an orthogonal transformation

$$\begin{pmatrix} H_3 \\ H_2 \\ H_1 \end{pmatrix} = O \begin{pmatrix} S_1 \\ S_2 \\ S_3 \end{pmatrix}$$

In the approximation of large $\tan\beta$ and large M_A, the physical scalar Higgs bosons masses are

$$\begin{aligned} m^2_{H_3} &= M^2_A \left(1 + \frac{1}{4}\cot^2\beta_s \sin^2 2\beta\right) , \\ m^2_{H_{2/1}} &= \frac{1}{2}\Bigg[m^2_Z + \frac{\kappa v_s}{2}(4\kappa v_s + \sqrt{2}A_\kappa) \\ &\quad \pm \sqrt{\left(m^2_Z - \frac{\kappa v_s}{2}(4\kappa v_s + \sqrt{2}A_\kappa)\right)^2 + \cot^2\beta_s \left(2\lambda^2 v^2_s - M^2_A \sin^2 2\beta\right)^2}\, \Bigg] \end{aligned}$$

Pseudoscalar Higgs bosons

The pseudoscalar fields, P_i $(i = 1,\, 2)$, is further rotated to mass basis A_1 and A_2, through a mixing angle:

$$\begin{pmatrix} A_2 \\ A_1 \end{pmatrix} = \begin{pmatrix} \cos\theta_A & \sin\theta_A \\ -\sin\theta_A & \cos\theta_A \end{pmatrix} \begin{pmatrix} P_1 \\ P_2 \end{pmatrix}$$

with

$$\tan\theta_A = \frac{\mathcal{M}^2_{P\,12}}{\mathcal{M}^2_{P\,11} - m^2_{A_1}} = \frac{1}{2}\cot\beta_s\,\frac{M_A^2 \sin 2\beta - 3\lambda\kappa v_s^2}{M_A^2 - m^2_{A_1}}$$

In large $\tan\beta$ and large M_A, the tree-level pseudoscalar masses become

$$\begin{aligned} m^2_{A_2} &\approx M_A^2\,(1 + \frac{1}{4}\cot^2\beta_s \sin^2 2\beta), \\ m^2_{A_1} &\approx -\frac{3}{\sqrt{2}}\kappa v_s A_\kappa \end{aligned}$$

Parameters of NMSSM: NMHDECAY

Additional parameters other than the usual MSSM's

$$\lambda, \quad \kappa, \quad A_\lambda, \quad A_\kappa, \quad \mu_{\text{eff}}$$

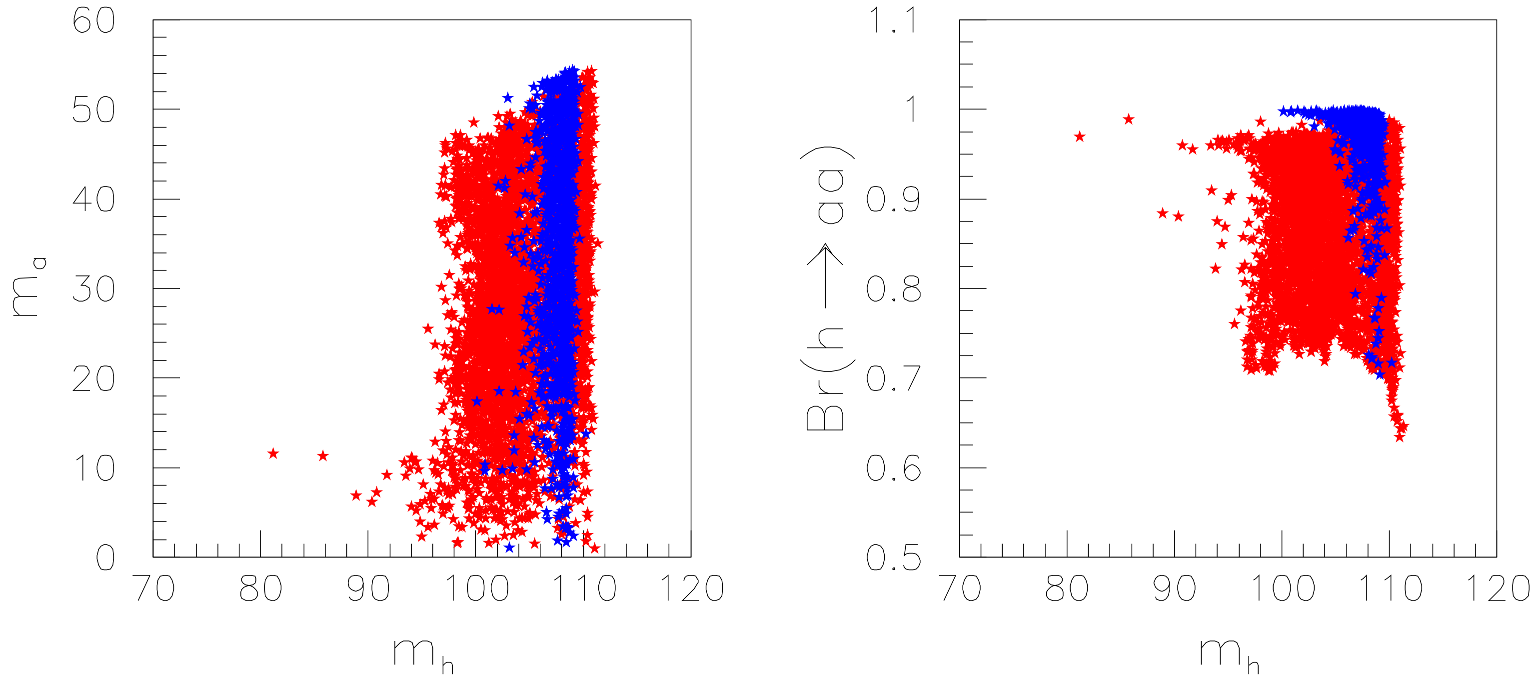
Constraints inside the NMHDECAY (Ellwanger, Gunion, Hugonie):

- One-loop radiative corrections to Higgs potential
- $b \to s\gamma$ constraint
- Dark matter relic density constraint: $[0.095, 0.112]$
- LEP2 bounds

A study of $h \to a_1 a_1 \to 4b$ in NMSSM (KC, Song, Yan RPL 2007)

NMSSM (A)	NMSSM (B)
$\lambda = 0.18,\ \kappa = -0.43$	$\lambda = 0.26,\ \kappa = 0.51$
$\tan\beta = 29$	$\tan\beta = 23$
$A_\lambda = -437$ GeV	$A_\lambda = -222$ GeV
$A_\kappa = -4$ GeV	$A_\kappa = -13$ GeV
$\mu_{\rm eff} = -143$ GeV	$\mu_{\rm eff} = 144$ GeV
$m_{h_1} = 110$ GeV	$m_{h_1} = 109$ GeV
$m_{a_1} = 30$ GeV	$m_{a_1} = 39$ GeV
$B(h_1 \to a_1 a_1) = 0.92$	$B(h_1 \to a_1 a_1) = 0.99$
$B(a_1 \to b\bar{b}) = 0.93$	$B(a_1 \to b\bar{b}) = 0.92$
$g_{VVh_1}/g^{\rm SM}_{VVh} = 0.99$	$g_{VVh_1}/g^{\rm SM}_{VVh} = -0.99$
$g_{tth_1}/g^{\rm SM}_{tth} = 0.99$	$g_{tth_1}/g^{\rm SM}_{tth} = -0.99$
$g_{tta_1}/g^{\rm SM}_{tth} = -2.4 \times 10^{-3}$	$g_{tta_1}/g^{\rm SM}_{tth} = -1.2 \times 10^{-2}$
$C^2_{4b} = 0.80$	$C^2_{4b} = 0.83$

$$C^2_{4b} \equiv \left(\frac{g_{ZZh}}{g^{\rm SM}_{ZZh}} \right)^2 \times B(h \to a_1 a_1) \times B^2(a_1 \to b\bar{b})$$



⋆: bench-mark point A-like

⋆: bench-mark point B-like

All evade the Higgs mass bound

Further decay in $h \to a_1 a_1$

Further decay of a_1 includes

$$h \to a_1 a_1 \to (2\gamma,\ 2\tau,\ 2b,\ 2g)\ (2\gamma,\ 2\tau,\ 2b,\ 2g)$$

- If a_1 is very light and so energetic that the two photons are very collimated. It may be difficult to resolve them. Effectively, like $h \to \gamma\gamma$.
- If the mixing angle is larger than 10^{-3} and a_1 is heavier than a few GeV, it can decay into $\tau^+\tau^-$. Thus, 4τs in the final state (Graham, Pierce, Wacker 2006).
- If a_1 is heavier than $2m_b$, a_1 will dominately decay into $b\bar{b}$.
- The gluon mode suffers from QCD background.

Higgs Production at the LHC

- Gluon fusion $gg \to h \to \eta\eta \to 4b$ suffers from huge QCD background.
- WW fusion $qq \to qqWW \to qqh \to qq(4b)$ also suffers from QCD background.
- Wh, Zh associated production:

$$Wh \to (\ell\nu) + (4b)\,, \quad Zh \to (\ell\ell) + (4b)$$

The charged lepton removes most QCD background.

- $t\bar{t}h \to (bW)(bW) + (4b)$, combinatorial background.

Require at least one charged lepton and 4 b-tagged jets in the final state.

Production and decay

We used MADGRAPH with the effective vertex g_{vvh} to calculate the signal cross sections. Decay of the W/Z and h:

$$p_T(\ell) > 15\,\mathrm{GeV}, \qquad |\eta(\ell)| < 2.5\,,$$
$$p_T(b) > 15\,\mathrm{GeV}, \qquad |\eta(b)| < 2.5\,, \quad \Delta R(bb, b\ell) > 0.4\,,$$

We employ a B-tagging efficiency of 70% for each B tag, and a probability of 5% for a light-quark jet faking a B tag.

Backgrounds

- It is possible for the photon in $\gamma + nj$ background to fake an electron in the EM calorimeter.
- The backgrounds from $W + nj$ and $Z + nj$ contribute at a very low level and are reducible as we require 4 b-tagged jets in the final state.
- The background from $WZ \to \ell\nu b\bar{b}$ is also reducible by the 4 b-tagging requirement.
- $t\bar{t}$ production with one of the top decay hadronically and the other semi-leptonically. The jet from the W may fake a b jet.
- $t\bar{t}b\bar{b}$ production, irreducible.
- $W/Z + 4b$ production, irreducible.

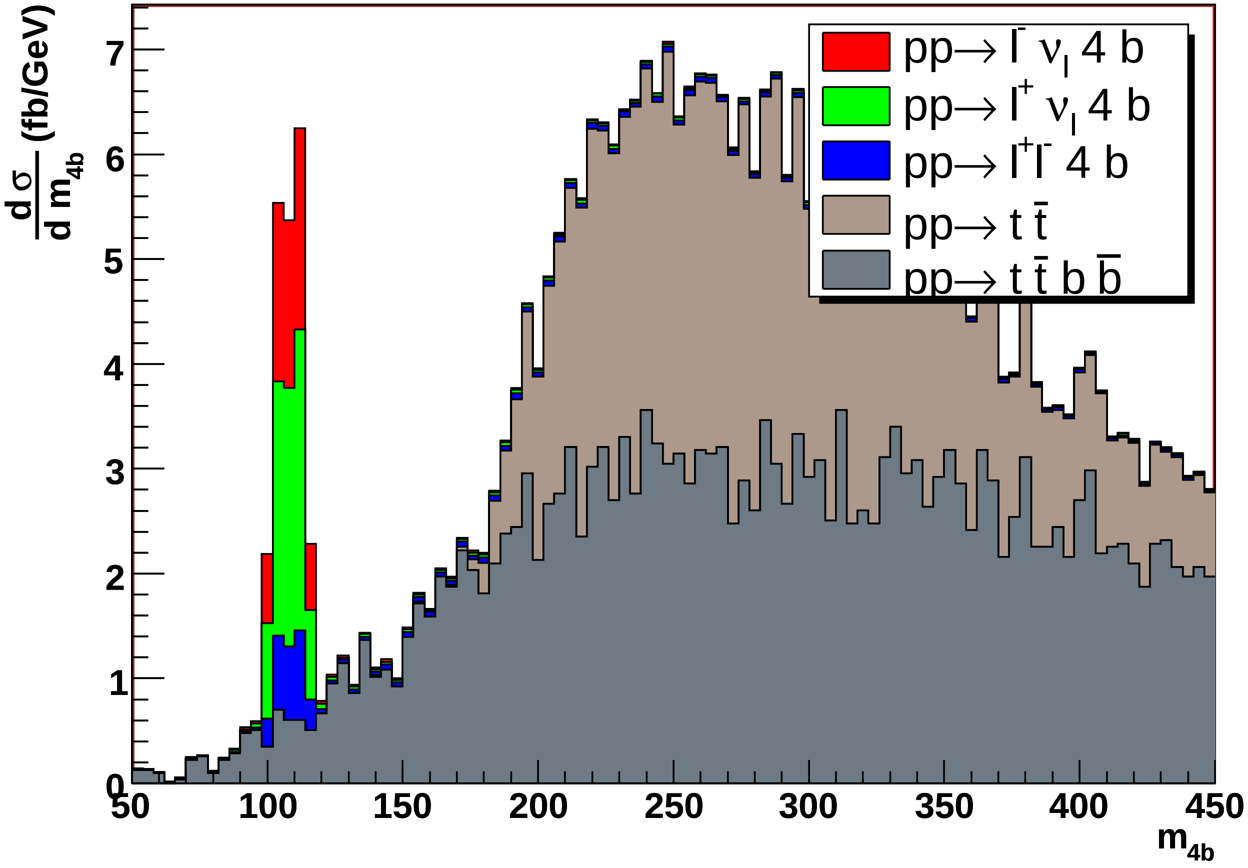
Event rates

Channels	NMSSM (A)	NMSSM (B)	SLHμ (A)	SLHμ (B)
W^+h signal	3.13 fb	9.54 fb	1.27 fb	0.63 fb
W^-h signal	2.35 fb	6.55 fb	0.87 fb	0.44 fb
Zh signal	1.05 fb	2.76 fb	0.36 fb	0.18 fb

Background

Channels	cross sections (fb)
$t\bar{t}$	172 (NMSSM & SLHμ)
$t\bar{t}b\bar{b}$	236 (NMSSM), 284 (SLHμ A), 429 (SLHμ B)
$W+4b$	3.80 (NMSSM), 4.16 (SLHμ A), 4.63 (SLHμ B)
$Z+4b$	3.85 (NMSSM & SLHμ)

$t\bar{t}b\bar{b}$ background is enhanced by $t\bar{t}\eta$ production in SLH model.



Apply the invariant mass cuts:

$$m_h - 15 \text{ GeV} < M_{4b} < m_h + 15 \text{ GeV} \;,$$

Significance of the signal

Total signal and background cross sections under the signal peak:

	NMSSM (A)	NMSSM (B)	SLHμ (A)	SLHμ (B)
signal	6.53 fb	18.85 fb	2.50 fb	1.25 fb
bkgd	4.83 fb	4.77 fb	13.83 fb	22.45 fb
$S/\sqrt{B}$	29.7	86.3	6.7	2.6

$S/\sqrt{B}$ for $L = 100$ fb^{-1}

Impact of the channel $Wh \to Wa_1a_1 \to \ell\nu + 4b$

- The emergence of the Higgs boson decay mode into two pseudoscalar bosons can relieve the so-called little hierarchy problem and reduce the LEP2 Higgs boson mass bound.
- It may affect the golden search modes ($h \to \gamma\gamma, b\bar{b}$) of the Higgs boson significantly.
- With the $h \to a_1a_1 \to 4b$, together with at least a charged lepton from the W or Z boson decay, a significant Higgs boson signal is observable at the LHC.