Cosmology 2: Inflation

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Initial Condition problems of Cosmology

1. Flatness Problem

$$\frac{3 M_{Pl}^2}{8\pi}\left(H^2 + \frac{k}{a^2}\right) = \rho \qquad \rho_c \equiv \frac{3 M_{Pl}^2}{8\pi} H^2$$

Divide through $\Longrightarrow 1 + \frac{k}{a^2 H^2} = \Omega$

Friedmann: $$\frac{k}{H^2 a^2} = \Omega - 1$$ ← time dependent in the way it is defined here

observation → today: $$\boxed{\frac{k}{H_0^2 a_0^2} = \Omega_0 - 1 < 10^{-2}}$$

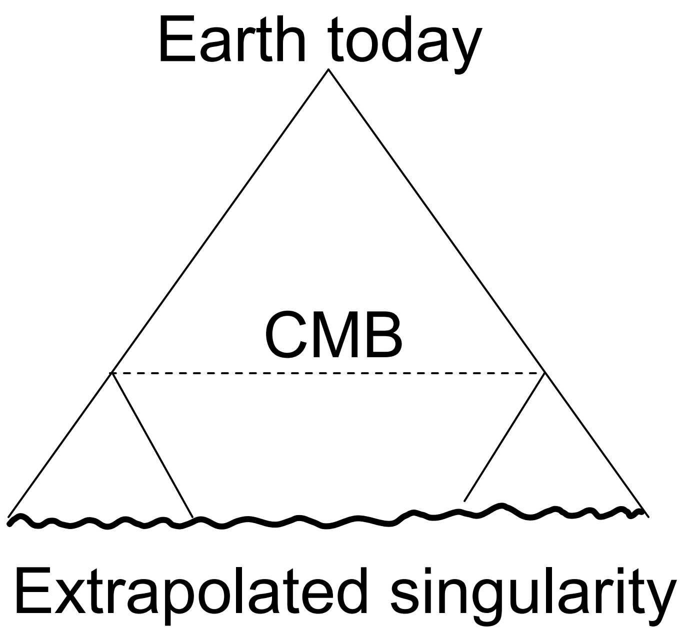
early universe: $$\frac{k}{H_e^2 a_e^2} = \frac{k}{H_0^2 a_0^2}\left(\frac{a_r}{a_0}\right)\left(\frac{a_e}{a_r}\right)^2 < (10^{-2})(10^{-4})(10^{-6})^2 = 10^{-18}$$

at nucleosynthesis

Why small?

Initial spatial curvature had to be finely tuned for universe to be this old and flat. Why?

2. Horizon/causality problem: Why homogeneous and isotropic on "acausal" scales?"



$$d_{LSS} \equiv a(t_0) \int_{t_{LSS}}^{t_0} \frac{dt'}{a(t')}$$

$$d_H(t_{LSS}) \equiv a(t_{LSS}) \int_{t_s}^{t_{LSS}} \frac{dt'}{a(t')}$$

Why don't we see the singularity imprinted in the CMB?

Somewhat ill posed: Maybe singularity is imprinted on the CMB.

3. Unobserved relic problem

e.g. Suppose the SM is embedded in a larger theory with gauge group G_1

$$G_1 \to G_2 \to \ldots \to SU(3)_c \times SU(2)_L \times U(1)_Y$$

Monopoles arise whenever $\Pi_2(G_i / G_j) \neq I$

$$n_M \sim H^3 \sim T_c^6 / M_{pl}^3$$

$$n_M / s \sim \frac{T_c^3}{M_{pl}^3} \sim \left(\frac{10^{14}}{10^{19}}\right)^3 \sim 10^{-15}$$

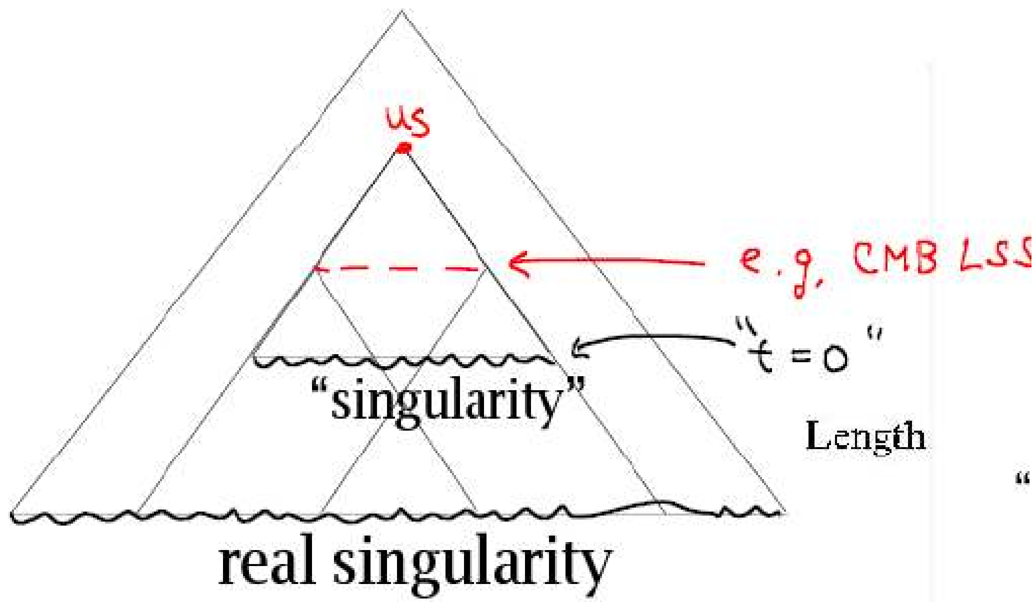
$m_M \sim 10^{16}\, GeV \;\; \to \;\; \Omega_M \sim 10^{11}$ unacceptably large!

[Guth, 80]

<u>Inflationary solution</u>: Blow up small flat universe.

- small flat patch blown up to encompass the entire universe
- Lengthen the time it takes to reach the singularity

$$d_H(t_{LSS}) \equiv a(t_{LSS}) \int_{t_s}^{t_{LSS}} \frac{dt'}{a(t')} \gg d_{LSS} \equiv a(t_0) \int_{t_{LSS}}^{t_0} \frac{dt'}{a(t')}$$



$$n(\text{after}) = n(\text{before})(\frac{a_{\text{before}}}{a_{\text{after}}})^3$$

- Dilute away massive particles.

$$\ln(a_{\text{after}}/a_{\text{before}}) \gg 1$$

How does one achieve this?
Guth proposed to have field theory models with initial conditions such that there is a quasi-dS phase.

Intuition: Einstein: $(\frac{\dot{a}}{a})^2 = \frac{\rho_\Lambda}{3M_p^2} \equiv H_I^2$

Solution: $a(t) = \exp[H_I t]$

$$\therefore t_s = -\infty$$

Horizon at CMB LSS: $d_H(t_{LSS}) = \frac{1}{H_I}(\exp[H_I(t_{LSS} - t_s)] - 1) \to \infty$

Clearly, satisfying

$$d_H(t_{LSS}) \equiv a(t_{LSS}) \int_{t_s}^{t_{LSS}} \frac{dt'}{a(t')} \gg d_{LSS} \equiv a(t_0) \int_{t_{LSS}}^{t_0} \frac{dt'}{a(t')}$$

$$n(\text{after}) = n(\text{before})(\frac{a_{\text{before}}}{a_{\text{after}}})^3$$

$$\ln(a_{\text{after}}/a_{\text{before}}) \gg 1$$

Single Field Slow Roll Inflation

[Linde, Steinhardt, Albrecht]

$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 - V(\phi) + \mathcal{L}_I[\phi, X, \psi_{SM}, ...]$$

How does one construct inflationary models?

- $\frac{d^2 a}{dt^2} > 0$ for about 60 e-folds. $\frac{a(t_f)}{a(t_i)} = e^N$ ← efold
- Inflation must end.
- Spatial inhomogeneities of ϕ must be sufficiently small to be consistent with cosmology (too big = too many black holes, too small = not enough structure).
- After inflation ends, the universe must reheat to $T > 10\,MeV$.
- After inflation ends, unwanted relics must not be created (e.g. low enough temperature).

Quantitatively:

$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 - V(\phi)$$

How to choose the potential and initial conditions?

- Negative Pressure and 60 e-folding

$$\epsilon \equiv \frac{1}{2}\left(\frac{V'(\phi)}{V}\right)^2 \approx \frac{-\frac{dH}{dt}}{H^2} \ll 1 \qquad \eta \equiv \frac{V''(\phi)}{V} \ll 1 \qquad N(\phi(t_i)) \equiv \left| \int_{\phi(t_i)}^{\phi(t_f)} \frac{d\phi}{\sqrt{2\epsilon}} \right| > 60$$

- End of inflation: $\epsilon(\phi(t_f)) \approx 1$ with $V(\phi_{min}) \approx 0$ at the minimum of the potential

- Density perturbation amplitude: $\sqrt{P_{\mathcal{R}}(k)} \approx \sqrt{\frac{V(\phi)}{24\pi^2\epsilon(\phi)}} \sim 10^{-5}$

never Planckian

For adiabatic, $P_\Phi(k) = \frac{4}{9} P_{\mathcal{R}}(k)$

$$\mathcal{R} \equiv \Phi - \frac{1}{3}\frac{\delta\rho}{\rho + P} \qquad \langle \mathcal{R}(\vec{x})\mathcal{R}(\vec{y})\rangle = \int \frac{d^3k}{(2\pi)^3} P_{\mathcal{R}}(k) e^{i\kappa\cdot(x-y)}$$

scale invariance nearly automatic! $\Delta^2_{\mathcal{R}}(k) \equiv \frac{k^3}{2\pi^2} P_{\mathcal{R}}(k) \propto k^{n_s - 1}$

$$n_s - 1 = 2\eta(\phi) - 6\epsilon(\phi)$$

$$n_s - 1 = 0 \longrightarrow \text{"scale invariance"}$$

! indicates source of fine tuning

Shape and Magnitude

- Can map $n_s(k) - 1$ to the shape of the potential

Field to k map

$$n_s(k) - 1 = 2M_p^2 \frac{V''(\phi)}{V(\phi)} - 3M_p^2 \left(\frac{V'(\phi)}{V(\phi)}\right)^2 \ll 1$$

$$\frac{d \ln[k/k_i(T_{RH}, g_*(T_{RH}), \phi_\epsilon)]}{d(\phi/M_p)} = -\left(\frac{1}{M_p}\frac{V(\phi)}{V'(\phi)} - \frac{M_p}{2}\frac{V'(\phi)}{V(\phi)}\right)$$

- Solution to the diff eqs depend on initial conds which in turn depend on reheating.

- Gravity waves are generated during inflation because $\sqrt{g}R$ is not classically conformally invariant. On the other hand, $\sqrt{g}F_{\mu\nu}F^{\mu\nu}$ is classically conformally invariant.

- Can map the amplitude of the gravity waves to the height of the potential

$$\frac{\Delta_E^2}{\Delta_\mathcal{R}^2} = 16\epsilon$$

$$\Delta_E^2 = \frac{k^3}{\pi^2} P_E = \frac{2V(\phi)}{3\pi^2 M_p^4}$$

Sources the B-mode polarization discussed in the last lecture.

$$\Delta_E^2 \propto k^{n_T}$$

$$n_T = \frac{-\Delta_E^2}{8\Delta_\mathcal{R}^2} \qquad n_T \geq \frac{-\Delta_E^2}{8\Delta_\mathcal{R}^2}$$

“Lyth Bound”

Measurable tensor perturbations typically require large field variations (near Planckian or larger)

$$\frac{\Delta_h^2}{\Delta_S^2} = 16\epsilon$$

To be measurable, can’t be too small

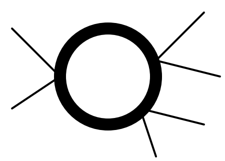
$$\epsilon \gtrsim 10^{-4}$$

$$\Delta N = \int_{\phi_i}^{\phi_f} \frac{d\phi}{M_p} \frac{1}{\sqrt{2\epsilon}} \sim 60$$

Hence, $\Delta\phi \gtrsim M_p$

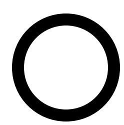
Does that Mean Tensor Perturbations Should not be Measurable?

- People often quibble over whether large field variations are natural, but that is not really the issue. It is simply
  - the dynamics may not be reliably captured by a simple potential



Can no longer integrate out.

  - we have no confidence about physics above the Planck scale

What is  ?

- Calculability should not be taken as a fundamental requirement of physics

Why 60 efolds?

- Largest scale that we see homogeneous and isotropic:

$$L = a_0 \int dx = a_0 \int_{a_{dec}}^{a_0} \frac{da}{H a^2} \approx a_0 \int_{a_{dec}}^{a_0} \frac{da}{H_0 (a_0/a)^{3/2} a^2} \approx \frac{2}{H_0} \equiv a_0 X$$

- inflation can take place only if homogeneous (small patch of comoving coordinate size > X became the observable universe):

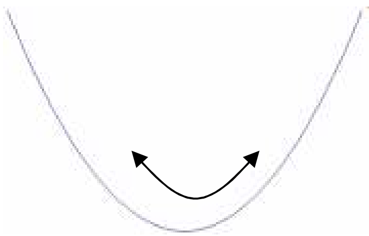
(also for curvature) ► $\frac{1}{H_I} > X a_I \quad \Rightarrow \quad \frac{1}{H_I} > \overbrace{a_0 X}^{L} \overbrace{\left(\frac{a_I}{a_0}\right)}^{\text{sufficiently small}}$ ← need enough efolds

$$\frac{a_I}{a_0} = \frac{a_I}{a_e}\frac{a_e}{a_0} \equiv e^{-N}\frac{a_e}{a_0} \qquad max\left(\frac{a_e}{a_0}\right) \approx \frac{a_{RH}}{a_0} = \left(\frac{g_{*S}(t_{RH})}{g_{*S}(t_0)}\right)^{1/3} \frac{T_0}{T_{RH}} \approx \frac{T_0}{T_{RH}}$$

$$H_I \approx \frac{T_{RH}^2}{\sqrt{3}} \qquad \ln\left(\frac{T_{RH}}{T_0}\right) + \frac{1}{2}\ln(\Omega_{R0}) \approx 60 + \ln\left(\frac{T_{RH}}{10^{15}\,GeV}\right) < N$$

Standard Reheating

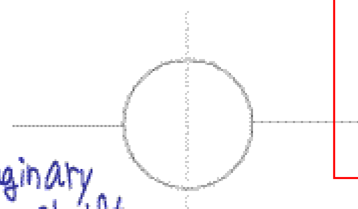
- Inflaton field decays: e.g.



$$L=\frac{1}{2}[(\partial\phi)^2+m^2\phi^2+L_i]$$

T_{RH} defined to be when radiation domination is achieved.

$$L_i=-\lambda\phi\bar{\Psi}\psi$$



$\Gamma \sim \frac{m\lambda^2}{4\pi}$

imaginary shift $e^{i\omega t}$

$$\Gamma_{tot}=\frac{\Im(\Sigma(m^2))}{m}$$

$$\partial_t^2\phi+(3H+\Gamma_{tot})\partial_t\phi+(m^2+\frac{\Gamma_{tot}^2}{4})\phi\approx 0$$

use following approx: $\rho_\phi\approx\frac{1}{2}((\partial_t\phi)^2+m^2\phi^2)$ $\quad \frac{(\partial_t\phi)^2}{2}\approx\frac{m^2\phi^2}{2}$ $\quad \Gamma^2/4\ll m^2$

$\dot{\phi}\ddot{\phi}\sim-\Gamma\dot{\phi}^2$

$$\partial_t\rho_R+4H\rho_R=\Gamma_{tot}\rho_\phi$$

estimate:

$\Rightarrow$ reheating temperature as a function of time $\quad \rho_R\equiv\frac{\pi^2}{30}g_*(T)T^4 \quad T_{RH}\approx 0.2\left(\frac{200}{g_*}\right)^{1/4}\sqrt{\Gamma_{tot}M_{pl}}$

Entropy becomes large during this time.

Preheating

- Depending on the initial conditions and coupings, other paths to reheating exists
- For example,

$$\mathcal{L}_I = \frac{1}{2} g^2 \chi^2 \phi^2$$

oscillates leading to parametric resonance

$$\frac{1}{\sqrt{g}} \partial_\mu (\sqrt{g} \partial^\mu \chi) + (m^2 + g^2 \langle \phi \rangle^2) \chi = 0$$

- In some cases, this and other non-linear dynamics can lead to non-perturbative breakup of homogeneous inflaton condensates on a time scale much faster than the perturbative decay time scale.

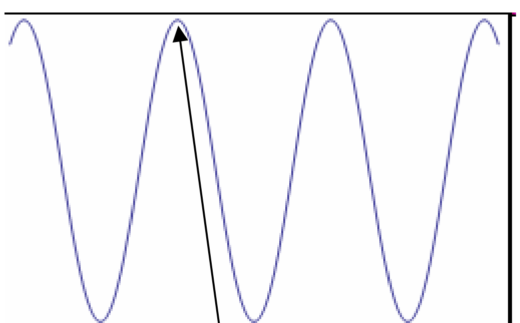
Some "Well Motivated" Models Exist

Almost nobody doubts something like slow-roll inflation could have happened:

- Global U(1) neglecting natural Planck-scale violations

$$V(\phi) = V_0(1 - \cos(\frac{\phi}{f})) + c\mu^4 \exp(-S_I)\cos(\frac{\phi}{f} + \Psi)$$

Typically Planckian



If Planckian instanton violates the global symmetry, can destabilize slow-roll

Bad: need to start close to the top unless f is large

SUSY η problem

SUGRA potential

$$U = \frac{1}{2}\Re[f_{ab}^{-1}]D_a D_b + e^{K/M_{\rm Pl}^2} g^{ij*}(D_i W)(D_j W)^* - \frac{3}{M_{\rm Pl}^2} e^{K/M_{\rm Pl}^2} W^* W$$

$K(\phi, \bar{\phi}) = |\phi|^2 + \ldots$ generates minimal kinetic term.

$$V \sim \frac{3M_S^6}{M_p^2} \exp\left[\frac{|\phi|^2}{M_p^2}\right] \qquad \eta \equiv \frac{M_p^2 V''(\phi)}{V(\phi)} \sim \mathcal{O}(1)$$

- D-term inflation can evade the η problem at the expense of a trans-planckian mass scale

$$V_D = \frac{1}{2}\left(\frac{g}{2}\sum_n q_n |\phi_n|^2 + \xi\right)^2$$

- This problem can be tuned away

$$W = S(A_1 A_2 - m^2)$$

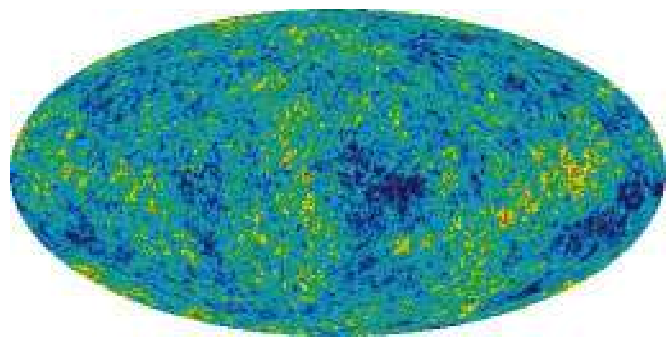
$$K = S^\dagger S + A_1^\dagger A_1 + A_2^\dagger A_2$$

$$\eta \sim \frac{1}{2}\frac{|S|^2}{M_p^2} \ll 1$$

Beyond 2-point Function

Single field slow roll predicts approximately Gaussian statistics.

$$\ln P \propto \left[\frac{\Delta T}{T} - 3 f_{NL}\left(\left(\frac{\Delta T}{T}\right)^2 - \langle\left(\frac{\Delta T}{T}\right)^2\rangle\right)\right]^2 \qquad f_{NL} \approx 0$$



More generally, a Gaussian theory is a non-interacting field theory (i.e. N-point function expressible as products of 2-point function)

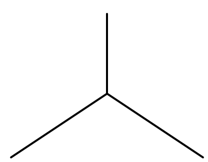
In particular, $\langle\zeta\zeta\zeta\rangle = 0$ $\qquad \left[\zeta_k \approx \mathcal{R}_k \quad \text{for } \frac{k}{aH} \ll 1\right]$

3-point function from self-interaction is negligible

- Slow roll parameters → flat potential → weak interaction → almost quadratic theory for $\delta\phi$ → energy density is linear in $\delta\phi$

$$\delta\rho \sim V'(\phi_0)\delta\phi$$

- → almost Gaussian field $\zeta \sim \dfrac{\delta\rho}{\rho + P} \propto \delta\phi$
- Non-gravitational short distance contribution:



$$\mathcal{L}_{int} = \frac{1}{6}V'''(\phi_0)\delta\phi^3$$

$$\langle\zeta\zeta\zeta\rangle \propto V'''(\phi_0)\langle\delta\phi\delta\phi\rangle^3(\frac{H}{\dot{\phi}})^3\frac{1}{H} \propto (\frac{d\eta}{d\ln k} + ...)P_\zeta^2$$

- This would contribute $\mathcal{O}(\epsilon^2)$ to f_{NL}.
- Gravitational contribution dominates, however, as we now review.

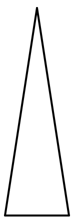
General Slick Argument

Maldacena's slick argument for gravitationally induced NG
In the squeezed limit.

Key: Suppose a field theory is a decoupling theory.

If $\phi(k_3) \approx \phi_0$ then

$$\langle \phi(\vec{k}_1)\phi(\vec{k}_2)\phi(\vec{k}_3)\rangle = \langle\ \langle \phi(\vec{k}_1)\phi(\vec{k}_2)\rangle_{\text{short distance in the presence of } \phi_0}\ \phi_0\ \rangle_{\text{long distance}}$$



$$\langle \zeta(\vec{k}_1)\zeta(\vec{k}_2)\zeta(\vec{k}_3)\rangle$$

$\vec{k}_3 = -\vec{k}_1 - \vec{k}_2 \to 0$ $\qquad \exp(2\zeta_G(0))|d\vec{x}|^2$

$$\langle \zeta(\vec{k}_1)\zeta(\vec{k}_2)\zeta(-\vec{k}_1 - \vec{k}_2)\rangle \sim \langle \zeta_G\zeta_G(e^{-\zeta_G(0)}|\vec{k}_1 - \vec{k}_2|)\zeta_G(0)\rangle$$

$$\langle \zeta_G\zeta_G(e^{-\zeta_G(0)}k)\zeta_G(0)\rangle \sim \langle \zeta_G\zeta_G(k - \zeta_G(0)k)\zeta_G(0)\rangle$$

$$-\partial_{\ln k}\langle \zeta_G\zeta_G\rangle(k)\langle \zeta_G(0)\zeta_G(0)\rangle \propto (n_s - 1)[P_\zeta(k)]^2$$

$f_{NL}^{squeeze} \sim (n_s - 1)$ Much smaller than order unity.

Hence, if $|f_{NL}^{\text{squeeze}}| > 1$, we must violate decoupling and/or perturbative expansion and/or standard metric/gravity ansatz or have curvature perturbations evolve in a particular way.

Non-Gaussianities = “large” interactions

“large” interactions in the potential typically oppose slow roll

- Large derivative interactions can exist without ruining negative pressure

$$\mathcal{L}_{\rm intuition} = f((\partial\phi)^2, \phi)(\partial\phi)^2 - m^2\phi^2$$

$$\mathcal{L}_{\rm intuition} \sim (\partial\tilde{\phi})^2 - \frac{m^2}{Z}\tilde{\phi}^2$$

- To have a calculable EFT description, for point-like fields, one must have

$$Z \sim \mathcal{O}(1)$$

- However, DBI can in principle get around this because of the extended object nature e.g. membrane-like object coupled to gauge fields

How Well Motivated are Non-minimal Kinetic terms?

- Minimal kinetic term is a symptom of a linear wave description of a particle:

$$(\partial\phi)^2 - m^2\phi^2 \to e^{-ip\cdot x} \to p^2 = m^2$$

- Integrating out momentum shells to derive Wilsonian EFT generate higher powers

$$(\partial\phi)^2 \frac{(\partial\phi)^{2n}}{\Lambda^{4n}}$$

- "Non-particle" description for dynamics corresponds to non-minimal kinetic terms

e.g. membrane-like object coupled to gauge fields

$$S = \mu \int d^{p+1}\sigma \sqrt{|\det(G_{\alpha\beta} + k\mathcal{F}_{\alpha\beta})|}$$

$$S = \mu \int d^{p+1}\sigma \sqrt{|\det(\eta_{\alpha\beta} + k^2\partial_\alpha\Phi^i\partial_\beta\Phi^i + kF_{\alpha\beta})}$$

$$\mathcal{L} = \frac{-1}{f(\phi)}\sqrt{1-2f(\phi)X} - [V(\phi) - \frac{1}{f(\phi)}] \qquad \frac{1}{a^3}\frac{d}{dt}(a^3\gamma\dot{\phi}) + [\frac{f'(\phi)}{f(\phi)}\frac{X}{\gamma} + V'(\phi) + \frac{f'(\phi)}{f^2}(1+\frac{1}{\gamma})] = 0$$

How Might Non-minimal Kinetic Term Generate Larger Non-Gaussianities (away from squeezed limit)?

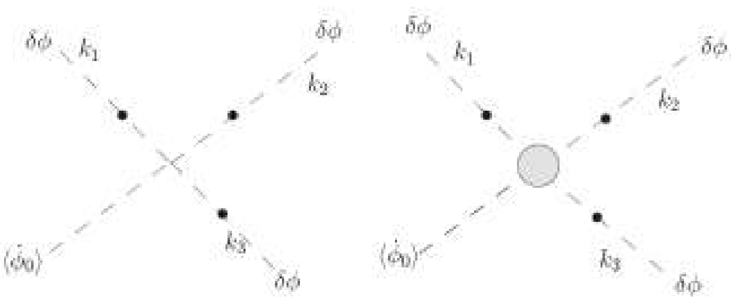
[Chen et al 06, Seerey and Lidsey 05]

$$\langle \zeta(\tau,\vec{k}_1)\zeta(\tau,\vec{k}_2)\zeta(\tau,\vec{k}_3)\rangle = (2\pi)^7\delta^{(3)}(\sum_i \vec{k}_i)[P^\zeta(k_1+k_2+k_3)]^2\frac{\mathcal{A}(\vec{k}_1,\vec{k}_2,\vec{k}_3)}{\prod_i k_i^3}$$

$$\mathcal{A}(\vec{k}_1,\vec{k}_2,\vec{k}_3) = \frac{3}{10} f_{NL}^{\text{equil or local}} \sum_i k_i^3$$

Consider, for example, $S_{int} = \int d^4x\sqrt{g}\frac{c}{\Lambda^4}(\partial_\mu\phi\partial^\mu\phi)^2$

Break Z2



$$S_{int} \ni \int d^4x a^3 \frac{4c}{\Lambda^4}\dot{\phi}_0(t)\delta\dot{\phi}^3(x)$$

$$\langle \zeta(t,\vec{k}_1)\zeta(t,\vec{k}_2)\zeta(t,\vec{k}_3)\rangle \sim (2\pi)^3\delta^{(3)}(\vec{k}_1+\vec{k}_2+\vec{k}_3)\times \frac{1}{H}\times\frac{c\dot{\phi}_0}{\Lambda^4}\times(\frac{H}{\dot{\phi}_0})^3\times(H^2)^3 h(\vec{k}_1,\vec{k}_2,\vec{k}_3)$$

$$= (2\pi)^7\delta^{(3)}(\sum_i\vec{k}_i)\frac{c\dot{\phi}_0^2}{\Lambda^4}\left[\left(\frac{H}{\dot{\phi}_0}\frac{H}{2\pi}\right)^2\right]^2 h(\vec{k}_1,\vec{k}_2,\vec{k}_3)$$

$$f_{NL}^{\text{equil}} \sim c\epsilon_V\frac{H^2}{\Lambda^2}\frac{M_{pl}^2}{\Lambda^2}$$

Easy to Obtain Non-Gaussian Isocurvature

In passing, one should note that Gaussianity is as much of the feature of slow roll as it is the feature of a purely the quadratic theory:

$$\langle \delta\rho_\chi \rangle \sim m^2 \langle : \chi^2 : \rangle \neq 0$$

$$\langle \delta\rho_\chi \delta\rho_\chi \delta\rho_\chi \rangle \sim m^6 \langle : \chi^2 :: \chi^2 : \rangle \langle : \chi^2 : \rangle \sim m^6 \langle \chi\chi \rangle^2 \langle : \chi^2 : \rangle \neq 0$$

even for a pure quadratic theory without gravitational constraint equation induced couplings. However, note that interaction coupling still played an important role to produce the isocurvature fluctuations.

Hence, such isocurvatures can also contaminate curvature perturbations:

Curvature can mix with isocurvature through non-adiabatic Pressure:

$$\dot{\mathcal{R}} = \frac{1}{3(\rho + P)^2} \underbrace{\{-\delta\rho \frac{dP}{dt} + \delta P \frac{d\rho}{dt}\}}$$

This non-adiabatic pressure vanishes for single field.

Definition of Isocurvature Perturbations

There are several subtly different definitions used in the literature, but the main important points are:

1) If there are N distinct light field degrees of freedom in the effective field theory, all will fluctuate during inflation. Hence, you need N perturbation variables.
2) Keep track of which degrees of freedom eventually makeup radiation and dark matter at matter radiation equality.

“Curvaton” Mechanism for Generating Large Non-Gaussianity in the squeezed limit.

Curvaton scenarios convert isocurvature perturbations into curvature perturbations through

$$\dot{\mathcal{R}} = \frac{1}{3(\rho+P)^2}\{-\delta\rho\frac{dP}{dt} + \delta P\frac{d\rho}{dt}\}$$

This generates large non-Gaussianities since

$$\rho_\sigma \sim m_\sigma^2\sigma^2$$

$$\sigma = \sigma_0 + \delta\sigma$$

$$\frac{\delta\rho_\sigma}{\rho_\sigma} \sim \frac{2\delta\sigma}{\sigma_0} + \frac{\delta\sigma^2}{\sigma_0^2}$$

$$\zeta \sim \left[\frac{\rho_\sigma}{\rho_R}|_{\mathrm{decay}}\right]\frac{\delta\rho_\sigma}{\rho_\sigma} = \left[\frac{\rho_\sigma}{\rho_R}|_{\mathrm{decay}}\right]\left(\frac{2\delta\sigma}{\sigma_0} + \frac{\delta\sigma^2}{\sigma_0^2}\right)$$

Measurability

Why can't we probe usual inflationary scenarios at near future colliders?

Colliders probe vacuum today. Since current energy reach is near a TeV, only TeV scale changes in the scalar vevs can be probed.

Slow roll inflationary scenarios typically require a large field variation (much larger than TeV) either during or at the end of inflation because of a combination of e-fold and density perturbation constraints.

Since parameters to maintain slow roll are also typically fine tuned, people resort to constructions of models without explicit connection to SM.

Hopeful but Limited Potential Information Even for Single Field With Canonical Kinetic Term

- Bottom line: Ideal measurements may yield $V(\phi(k))$ over a range of ϕ if we assume single field minimal kinetic term models 
- Since slow-roll, tiny field range maps to largely varying Fourier scale range

$$\frac{d\ln[k/k_i(T_{RH}, g_*(T_{RH}), \phi_e)]}{d(\phi/M_p)} = -\left(\frac{1}{M_p}\frac{V(\phi)}{V'(\phi)} - \frac{M_p}{2}\frac{V'(\phi)}{V(\phi)}\right)$$

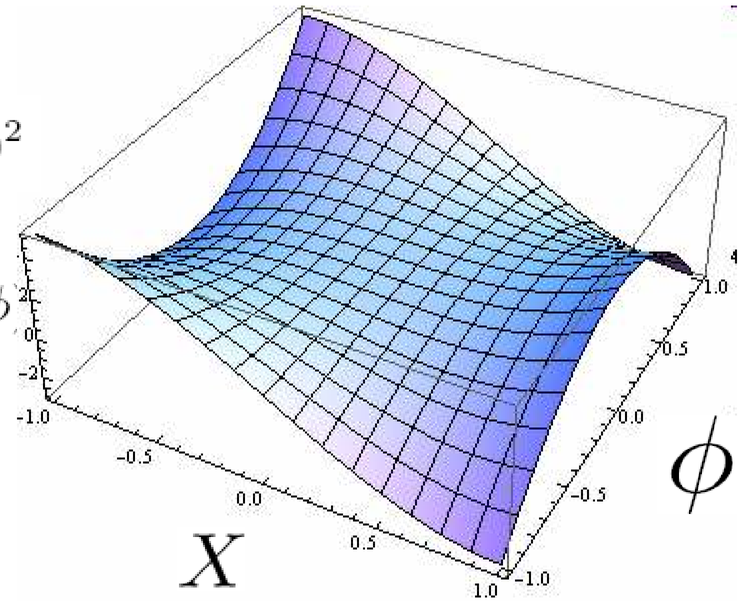
- When not strongly running, the potential well captured by a Taylor expansion to quartic order. (Analytic form.)
- 1 D manifold constrained by 1D manifold of data (ideal).
- Well known consistency relationship can still rule out slow roll even with limited knowledge. 

What If the Canonical Kinetic Term Assumption is Relaxed?

- The general action is simply a 2-D manifold parameterized by (X, ϕ) where

$$X \equiv \frac{1}{2}(\partial\phi)^2$$

$$\mathcal{L}(X, \phi)$$



- The question is to find the general form that satisfies the constraints from the data.
    - Ease of constructing inflationary models numerically for fitting
    - Parameterizing objects that encode data in a transparent way
    - See if there are general theoretical restrictions

Higher Order Correlation Functions

Every N-point function will depend at tree level on a finite set

$$\{\frac{\partial}{\partial\phi^m}\frac{\partial}{\partial X^n}\mathcal{L}(\tilde{\phi},\tilde{X})\}$$

evaluated on the background solution $\{\tilde{\phi}(N_e),\tilde{X}(N_e)\}$

Strategy for action reconstruction:

1) Find these functions of N_e
2) Write down an action consistent with these functions.

Example: Suppose you are given an ideal data set determining $\{\epsilon(N_e),c_s(N_e)\}$

$$P^{\zeta} = \frac{1}{8\pi^2 M_p^2}\frac{H^2}{c_s\epsilon}|_{c_s k=aH} \qquad \left.\frac{d\ln k}{dN_e}\right|_{k=k_s} = -(1-\epsilon-\kappa),$$

$$H(N_e) = H_i \exp\left(\int_{N_i}^{N_e}\epsilon dN\right) \qquad P_h = \frac{2H^2}{\pi^2 M_p^2}|_{k=aH} \qquad \left.\frac{d\ln k}{dN_e}\right|_{k=k_t} = -(1-\epsilon).$$

(optimistically fix a reheating scenario) $\kappa \equiv \frac{\dot{c}_s}{Hc_s}$

Which Models are Consistent with Data?

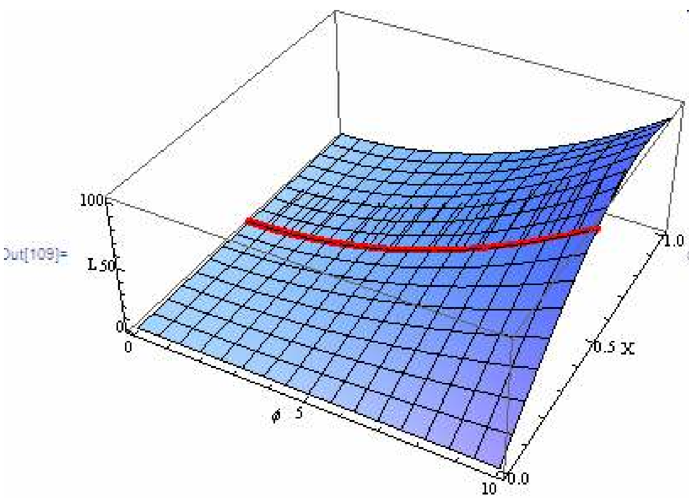
$$\frac{d\tilde{\phi}}{dN_e} = \frac{1}{H_i} \exp\left(- \int_{N_i}^{N_e} \epsilon dN \right) \longrightarrow N_e(\phi)$$

$$H(N_e) = H_i \exp\left(\int_{N_i}^{N_e} \epsilon dN \right)$$

$$\mathcal{L}^{obs}(\tilde{\phi}, \frac{M_p^2}{2}) = 3 M_p^2 H^2 \left(-1 + \frac{2}{3}\epsilon(N_e) \right)$$

$$\mathcal{L}_X^{obs}(\tilde{\phi}, \frac{M_p^2}{2}) = 2\epsilon(N_e) H^2(N_e) M_p^2$$

$$\mathcal{L}_{XX}^{obs}(\tilde{\phi}, \frac{M_p^2}{2}) = 2 \left(\frac{1}{c_s^2(N_e)} - 1 \right) \epsilon(N_e) H^2 M_p^2$$



$$\begin{aligned} \mathcal{L}(\phi, X) \;=\; & q(\phi, X) + \mathcal{L}^{obs}(\phi, \frac{M_p^4}{2}) - q(\phi, \frac{M_p^4}{2}) \\ & + \left[\partial_X \mathcal{L}^{obs}(\phi, \frac{M_p^4}{2}) - \partial_X q(\phi, \frac{M_p^4}{2}) \right] (X - \frac{M_p^4}{2}) \\ & + \frac{1}{2} \left[\partial_X^2 \mathcal{L}^{obs}(\phi, \frac{M_p^4}{2}) - \partial_X^2 q(\phi, \frac{M_p^4}{2}) \right] (X - \frac{M_p^4}{2})^2 \end{aligned}$$

$q(\phi, X)$ is arbitrary! [0801.0742]

Simple Example

Suppose measurements give

$$\epsilon \sim \frac{1}{2N_e} \ll 1 \qquad c_s = 1 - \delta \qquad \delta \ll 1$$

$$H = H_1 \exp\left(\int_1^{N_e} \frac{dN}{2N}\right) = H_1 N_e^{1/2}$$

$$\mathcal{L}\left(\frac{1}{2}, \phi\right) = H_1^2(1 - H_1^2\phi^2)$$

$$\mathcal{L}_X\left(\frac{1}{2}, \phi\right) = H_1^2$$

$$\mathcal{L}_{XX}\left(\frac{1}{2}, \phi\right) \sim 2H_1^2\delta \qquad q = 0 \longrightarrow \tilde{\mathcal{L}}_1(X, \phi) \sim H_1^2\left[-\frac{3}{4}(H_1\phi)^2 + X + \delta\ X^2\right]$$

$$q = \lambda X^3 \longrightarrow \tilde{\mathcal{L}}_2(X, \phi) = \tilde{\mathcal{L}}_1(X, \phi) + \lambda\left[X^3 - \frac{1}{8} - \frac{3}{4}(X - \frac{1}{2}) - \frac{3}{2}(X - \frac{1}{2})^2\right]$$

Note that these Lagrangians do not give the same c_s unless equation of motion is used to evaluate the background.
At this level (2 X derivatives), the two are **observationally indistinguishable**.

More Explicitly

- Before putting backgd fields "on shell":
  - With $\tilde{\mathcal{L}}_1(X,\phi)$, $$c_s = \left(1 + \frac{4X\delta}{1+2X\delta}\right)^{-1/2}$$

    $\tilde{\mathcal{L}}_2(X,\phi)$ $$c_s = \left(1 + \frac{8X[\lambda(6X-3)+2H_1^2\delta]}{3\lambda(1-2X)^2+(4+8X\delta)H_1^2}\right)^{-1/2}$$

- After putting backgd fields "on shell", both cases give: $$c_s = \left(1 + \frac{2\delta}{1+\delta}\right)^{-1/2}$$

Possibilities to break degeneracy:

1) Other tree-lev terms in 3-point func.
1) Higher order correlation functions.
2) Loop corrections (probably too small)

Near future prospects

- Running of the spectral index will be better known with future experiments such as Planck.
- Polarization:
  - B polarization comes from tensor and lensing (contaminant as far as inflation is concerned).
  - B polarization has no contribution from scalar perturbations.
  - measuring tensor is important for checking consistency condition (to know if it really is inflation!)
  - Unfortunately, typically less than 1% of the scalar spectrum
- Nongaussianities
- Isocurvature
- Theoretical Problems
  - What is the inflaton? Are there truly natural models?
  - Stability of de Sitter space and back reaction.
  - More observables to experimentally ascertain inflation.

Summary of Part 2

- Inflation models are fine tuned because of a combination of density perturbation and efold constraints whose required tuning cannot be easily protected by symmetries
- Consistency conditions + higher N-point functions offer the best tests of inflation
- Reconstruction of potential requires only a 1-D manifold of ideal data, while the reconstruction of kinetic terms is infinitely more difficult.