Lecture 3: Strongly-coupled Higgs bosons

- Strongly interacting EWSB models
- Longitudinal WW scattering
- A partially strong WW scattering example

Challenge of Strongly-interacting EWSB

- Strong EWSB scenario is not favored by precision data.
- SUSY models always predict a light Higgs boson $\leq$ 150 GeV. Constraints of SUSY will enforce the EWSB sector to be weakly interacting. In contrast, a general 2HDM with a light Higgs can be partially strong.
- Unlike SUSY there may not be any new particles accessible at the LHC for a strong EWSB. It is a big challenge for the LHC.
- Experimental conditions for detecting strong WW scattering is not easy.

Why we are still interested in strong EWSB

- When no light Higgs boson is found, the longitudinal WW scattering will essentially grows with energy until violating unitarity at TeV scale. So the SM without a Higgs boson cannot be a fundamental theory.
- We expect some new dynamics should show up at TeV scale, otherwise the WW scattering will violate unitarity.
- A heavy Higgs boson of order TeV is already a strong interacting model.

$$m_H^2 \sim 2\lambda v^2$$

  where λ is the self-coupling of the ϕ^4 term.

- Other strong EWSB models should be possible, as long as it breaks $SU(2)_L \times U(1)_Y$ the same way as the scalar Higgs boson. Classic examples include technicolor models.

- One powerful approach is to use the effective Lagrangian to parameterize the strong theory. Since all new degrees of freedom are heavy, we can integrate out their effects and parameterize their effects through some higher dimension operators that involve the light fields.

Probes of EWSB

Equivalence Theorem: At high energy, an amplitude involving external longitudinal polarized gauge bosons is equivalent to the amplitude with the external gauge bosons replaced by the corresponding Goldstone bosons, up to corrections of order m_V/E. At high energy, the longitudinal guage boson recalls its identity as the Goldstone bosons.

$$\mathcal{A}(W_L W_L \to W_L W_L) \simeq \mathcal{A}(ww \to ww) + O(m_W/E)$$

Scattering of longitudinal W boson at high energy can reveal the structure of the EWSB sector.

Pion-like dynamics

If we imagine the $W_L W_L$ scatter strongly like the pions in QCD, we can express the $W_L W_L$ scattering amplitudes in terms of isospin amplitudes, exactly as pions in QCD. If we assign isospin indices

$$W_L^a W_L^b \to W_L^c W_L^d$$

where $a, b, c = 1, 2, 3$. And

$$W_L^{\pm} = \frac{1}{\sqrt{2}} \left(W_L^1 \mp i W_L^2 \right), \qquad Z_L = W_L^3$$

then the scattering amplitudes is given by

$$\mathcal{M}(W_L^a W_L^b \to W_L^c W_L^d) = A(s,t,u)\delta^{ab}\delta^{cd} + A(t,s,u)\delta^{ac}\delta^{bd} + A(u,t,s)\delta^{ad}\delta^{bc}$$

The physical amplitudes can be expressed in the function $A(s,t,u)$:

$$\begin{aligned}
\mathcal{M}(W_L^+ W_L^- \to Z_L Z_L) &= \mathcal{M}(Z_L Z_L \to W_L^+ W_L^-) = A(s,t,u) \\
\mathcal{M}(W_L^+ W_L^- \to W_L^+ W_L^-) &= A(s,t,u) + A(t,s,u) \\
\mathcal{M}(Z_L Z_L \to Z_L Z_L) &= A(s,t,u) + A(t,s,u) + A(u,t,s) \\
\mathcal{M}(W_L^{\pm} Z_L \to W_L^{\pm} Z_L) &= A(t,s,u) \\
\mathcal{M}(W_L^{\pm} W_L^{\pm} \to W_L^{\pm} W_L^{\pm}) &= A(t,s,u) + A(u,t,s)
\end{aligned}$$

Strongly Interacting Higgs Sector Models

- SM with a heavy Higgs boson –

$$\mathcal{L} = |D_\mu \Phi|^2 - \mu^2 |\Phi|^2 - \lambda |\Phi|^4$$

After the Φ takes on the VEV, $\Phi = (0, (v+H)/\sqrt{2})$, the Higgs self-interacting parts become

$$\mathcal{L} \in \frac{1}{2}(\partial H)^2 - \frac{1}{2}(2\lambda v^2) H^2 + \frac{\lambda v^4}{4}\left(1 - 4\frac{H^3}{v^3} - \frac{H^4}{v^4}\right)$$

The 4-point vertex is $\sim -\lambda$ and the 3-point vertex is $\sim -\lambda v$. When the m_H becomes large while keeping v fixed, the λ is getting larger.

Strongly Interacting Higgs Sector Models ...

- Chirally-coupled scalar model – consistent with the chiral symmetry $SU(2)_L \times SU(2)_R$, spontaneously broken to diagonal $SU(2)$. The Goldstone fields are described by

$$\Sigma = \exp\left(\frac{2iw^a \tau^a}{v}\right)$$

and a scalar S. Under the transformation

$$\begin{aligned} \Sigma &\longrightarrow L\,\Sigma\,R^\dagger \\ S &\longrightarrow S \end{aligned}$$

so the Lagrangian for the fields is

$$\mathcal{L} = \frac{v^2}{4}\,\mathrm{Tr}\;\partial^\mu \Sigma^\dagger\;\partial_\mu \Sigma + \frac{1}{2}\;\partial^\mu S\;\partial_\mu S - \frac{1}{2} M_S^2 S^2 + \frac{1}{2} g v\;S\;\mathrm{Tr}\;\partial^\mu \Sigma^\dagger\;\partial_\mu \Sigma + \ldots$$

where g and M_S are free parameters. The g can be traded by the width of $S \to ww$

$$\Gamma_S = \frac{3 g^2 M_S^3}{32 \pi v^2}$$

If $g = 1$ S reduces to the SM Higgs.

One can work out the scattering amplitude $A(s,t,u)$:

$$A(s,t,u) = \frac{s}{v^2} - \frac{g^2 s^2}{v^2} \frac{1}{s - M_S^2 + i M_S \Gamma_S \theta(s)}$$

- Chirally-coupled vector field – consistent with chiral symmetry. Parameterize the Goldstone fields as

$$\xi = \exp\left(\frac{iw^a\tau^a}{v}\right)$$

where under the $SU(2)_L \times SU(2)_R$ transformations ξ transforms

$$\xi \longrightarrow \xi' \equiv L\xi U^\dagger = U\xi R^\dagger$$

where L, R, U are $SU(2)$ group elements and U is a function of L, R, w^a. We can construct following currents

$$\begin{aligned} J_{\mu\, L} &= \xi^\dagger\, \partial_\mu\, \xi \longrightarrow U\, J_{\mu\, L}\, U^\dagger + U\, \partial_\mu\, U^\dagger \\ J_{\mu\, R} &= \xi\, \partial_\mu\, \xi^\dagger \longrightarrow U\, J_{\mu\, R}\, U^\dagger + U\, \partial_\mu\, U^\dagger \\ \mathcal{A}_\mu &= J_{\mu\, L} - J_{\mu, R} \\ \mathcal{V}_\mu &= J_{\mu\, L} + J_{\mu, R} + 2igV_\mu \end{aligned}$$

where $V_\mu \longrightarrow U\, V_\mu\, U^\dagger + ig^{-1}\, U\, \partial_\mu\, U^\dagger$ is the vector field of interest. Then one can show that

$$\begin{aligned} \mathcal{A}_\mu &\longrightarrow U\, \mathcal{A}_\mu\, U^\dagger \\ \mathcal{V}_\mu &\longrightarrow U\, \mathcal{V}_\mu\, U^\dagger \end{aligned}$$

The Lagrangian is

$$\mathcal{L} = -\frac{1}{4} V^a_{\mu\nu} \, V^{a\mu\nu} - \frac{1}{4} v^2 \, \mathrm{Tr} \, \mathcal{A}_\mu \, \mathcal{A}^\mu - \frac{1}{4} a v^2 \, \mathrm{Tr} \, \mathcal{V}_\mu \, \mathcal{V}^\mu$$

The V_μ represents a technicolor type vector mesons, e.g., techni-rho.
The mass and the width of the ρ_{TC} fixed the model

$$M_V^2 = a g^2 v^2 , \qquad \Gamma_V = \frac{a M_V^3}{192 \pi v^2}$$

The scattering amplitude can be obtained to be

$$\begin{aligned} A(s,t,u) &= \frac{s}{4v^2}(4-3a) \\ &+ \frac{a M_V^2}{4v^2} \left[\frac{u-s}{t - M_V^2 + i M_V \Gamma_V \theta(t)} + \frac{t-s}{u - M_V^2 + i M_V \Gamma_V \theta(u)} \right] \end{aligned}$$

- Nonresonant channels – with chiral symmetry the Lagrangian is

$$\mathcal{L} = \frac{v^2}{4}\,\mathrm{Tr}\,\partial_\mu \Sigma^\dagger \partial^\mu \Sigma + L_1 \left(\frac{v}{\Lambda}\right)^2 \mathrm{Tr}\,(\partial_\mu \Sigma^\dagger \partial^\mu \Sigma)\ \mathrm{Tr}\,(\partial_\nu \Sigma^\dagger \partial^\nu \Sigma)$$
$$+L_2 \left(\frac{v}{\Lambda}\right)^2 \mathrm{Tr}\,(\partial_\mu \Sigma^\dagger \partial_\nu \Sigma)\ \mathrm{Tr}\,(\partial^\mu \Sigma^\dagger \partial^\nu \Sigma)$$

where $\Lambda \lesssim 4\pi v$ denotes the scale of new physics. The scattering amplitude is, to the order p^4,

$$A(s,t,u) = \frac{s}{v^2} + \frac{1}{4\pi^2 v^4}\left(2L_1(\mu)s^2 + L_2(\mu)(t^2+u^2)\right) + \frac{1}{16\pi^2 v^4}\Bigg[$$
$$- \frac{t}{6}(s+2t)\log\left(-\frac{t}{\mu^2}\right) - \frac{u}{6}(s+2u)\log\left(-\frac{u}{\mu^2}\right) - \frac{s^2}{2}\log\left(-\frac{s}{\mu^2}\right)\Bigg]$$

Comments – If ignoring the dynamics like the techni-scalar, techni-rho, or the $L_{1,2}$ terms, the leading contribution is the term

$$A(s,t,u) = \frac{s}{v^2}$$

This is the low energy theorem of pion dynamics. It depends only on v.

Longitudinal weak gauge boson scattering

First, we write the longitudinal polarization 4-vector of a W boson as

$$\epsilon_L^\mu(p) = \frac{p^\mu}{m_W} + v^\mu(p)$$

where

$$v^\mu(p) \simeq -\frac{m_W}{2p^{0\,2}}(p^0, -\vec{p}) \sim O(m_W/E_W)$$

In the CM system of $W_L^+(p_1)W_L^-(p_2) \to W_L^+(k_1)W_L^-(k_2)$, the polarization of the $W_L^+(p_1)$ is

$$\epsilon_L^\mu(p_1) = \left(\frac{|\vec{p}_1|}{m_W}, \frac{p_1^0}{m_W|\vec{p}_1|}\vec{p}_1\right)$$

Then we found that

$$\epsilon_L^\mu(p_1) - \frac{p_1^\mu}{m_W} \simeq -\frac{1}{2}\frac{m_W}{p_1^{0\,2}}(p_1^0, -\vec{p}_1) = -\frac{2m_W}{s}\,p_2^\mu$$

Therefore

$$\epsilon_L^\mu(p_1) = \frac{p_1^\mu}{m_W} - \frac{2m_W}{s}\,p_2^\mu$$

Similarly,

$$\epsilon_L^\mu(p_2) = \frac{p_2^\mu}{m_W} - \frac{2m_W}{s}\ p_1^\mu$$

and for $\epsilon(k_1)$ and $\epsilon(k_2)$.

Amplitudes for $W_L^- W_L^+ \to W_L^- W_L^+$

The gauge part – There are t-channel γ, Z diagrams, s-channel γ, Z diagrams, 4-point vertex, and also s- and t-channel Higgs diagrams.

$$\begin{aligned}
i\mathcal{M}_t^{\gamma+Z} &= -ig^2\left(\frac{s_w^2}{t}+\frac{c_w^2}{t-m_Z^2}\right)\left[(p_1+k_1)^\mu \epsilon(p_1)\cdot\epsilon(k_1) - 2k_1\cdot\epsilon(p_1)\,\epsilon^\mu(k_1) - 2p_1\cdot\epsilon(k_1)\,\epsilon^\mu(p_1)\right] \\
&\quad \times \left[(p_2+k_2)_\mu \epsilon(p_2)\cdot\epsilon(k_2) - 2k_2\cdot\epsilon(p_2)\,\epsilon_\mu(k_2) - 2p_2\cdot\epsilon(k_2)\,\epsilon_\mu(p_2)\right] \\
i\mathcal{M}_s^{\gamma+Z} &= -ig^2\left(\frac{s_w^2}{s}+\frac{c_w^2}{s-m_Z^2}\right)\left[(p_1-p_2)^\mu \epsilon(p_1)\cdot\epsilon(p_2) + 2p_2\cdot\epsilon(p_1)\,\epsilon^\mu(p_2) - 2p_1\cdot\epsilon(p_2)\,\epsilon^\mu(p_1)\right] \\
&\quad \times \left[(k_2-k_1)_\mu \epsilon(k_1)\cdot\epsilon(k_2) - 2k_2\cdot\epsilon(k_1)\,\epsilon_\mu(k_2) + 2k_1\cdot\epsilon(k_2)\,\epsilon_\mu(k_1)\right] \\
i\mathcal{M}_4 &= ig^2\left[2\epsilon(p_2)\cdot\epsilon(k_1)\,\epsilon(p_1)\cdot\epsilon(k_2) - \epsilon(p_2)\cdot\epsilon(p_1)\,\epsilon(k_1)\cdot\epsilon(k_2) - \epsilon(p_2)\cdot\epsilon(k_2)\,\epsilon(p_1)\cdot\epsilon(k_1)\right]
\end{aligned}$$

Then we substitute the longitudinal polarizations:

$$\begin{aligned}
i\mathcal{M}_4 &= i\frac{g^2}{4m_W^4}\left[s^2+4st+t^2-4m_w^2(s+t)-\frac{8m_W^2}{s}ut\right] \\
i\mathcal{M}_t^{\gamma+Z} &= -i\frac{g^2}{4m_W^4}\left[(s-u)t-3m_W^2(s-u)+\frac{8m_W^2}{s}u^2\right] \\
i\mathcal{M}_s^{\gamma+Z} &= -i\frac{g^2}{4m_W^4}\left[s(t-u)-3m_W^2(t-u)\right]
\end{aligned}$$

Note that there are terms proportional to E^4/m_W^4, naively expected from the 4-point vertex. But the gauge structure will make sure cancellation of the E^4/m_W^4 terms. Afterall, we are left with the E^2/m_W^2 terms. At leading order

$$i\mathcal{M}^{\text{gauge}} = -i\frac{g^2}{4m_W^2}u + \mathcal{O}((E/m_W)^0)$$

If there were no Higgs boson, the gauge part will grow indefinitely.

The Higgs part – the sum of the two Higgs diagrams is

$$\begin{aligned} i\mathcal{M}^{\text{Higgs}} &= -i\frac{g^2}{4m_W^2}\left[\frac{(s-2m_W^2)^2}{s-m_h^2} + \frac{(t-2m_W^2)^2}{t-m_h^2}\right] \\ &\simeq i\frac{g^2}{4m_W^2}u + \mathcal{O}((E/m_W)^0) \end{aligned}$$

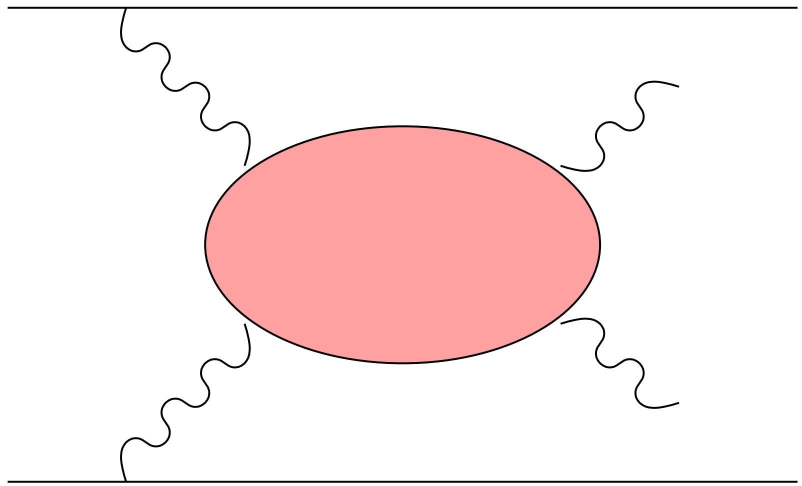
in the limit of $s \gg m_h^2, m_W^2$.

Comments –

- The bad energy growing behavior is tamed by the appearance of the Higgs boson.
- Cancellation of bad high energy behavior occurs when the energy

scale is well beyond the Higgs boson mass.

- If m_h is light ~ 100 GeV, then $W_L W_L$ would be weakly scattered.
- If the Higgs mass is very heavy ~ 1 TeV, the $W_L W_L$ enjoys a period of interaction which grows with energy up to the Higgs boson mass. That is the name of strongly interacting W system.
- The cancellation of the leading energy growing terms is very delicate. if the cancellation from the Higgs diagram is not complete, due to, e.g., the g_{hww} coupling is smaller than the SM value. The $W_L W_L$ scattering amplitude will grow with s – Partially strong $W_L W_L$ scattering.



Signals for a strongly interacting Higgs sector

One can look for excess of the $W_L W_L$ pairs that come off from the strong EWSB sector. Possible channels are

$$Z_L Z_L, \qquad W_L^+ W_L^-, \qquad W_L^\pm W_L^\pm, \qquad W_L^\pm Z_L$$

We look for the leptonic decay modes of these channels.

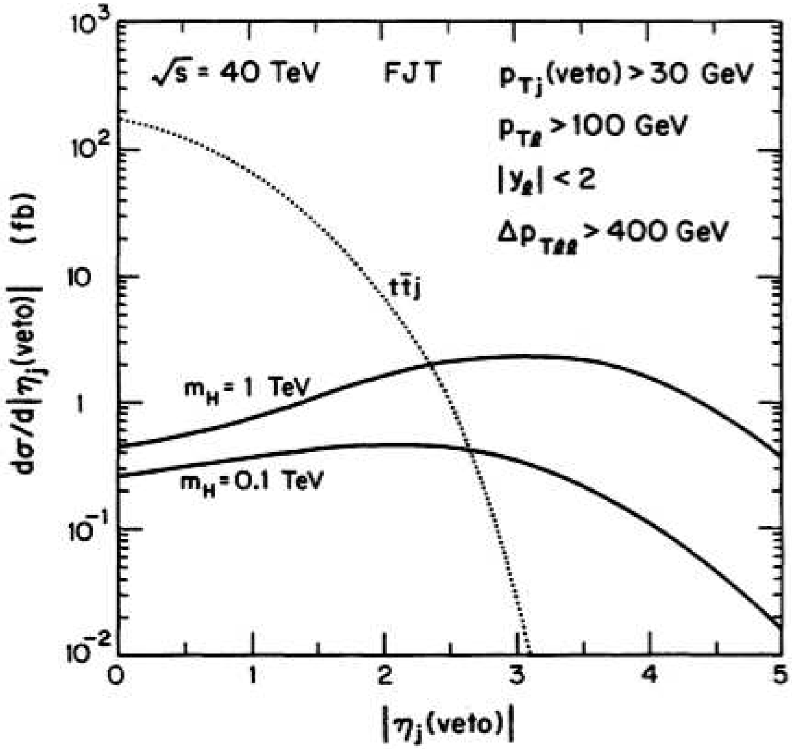
Features

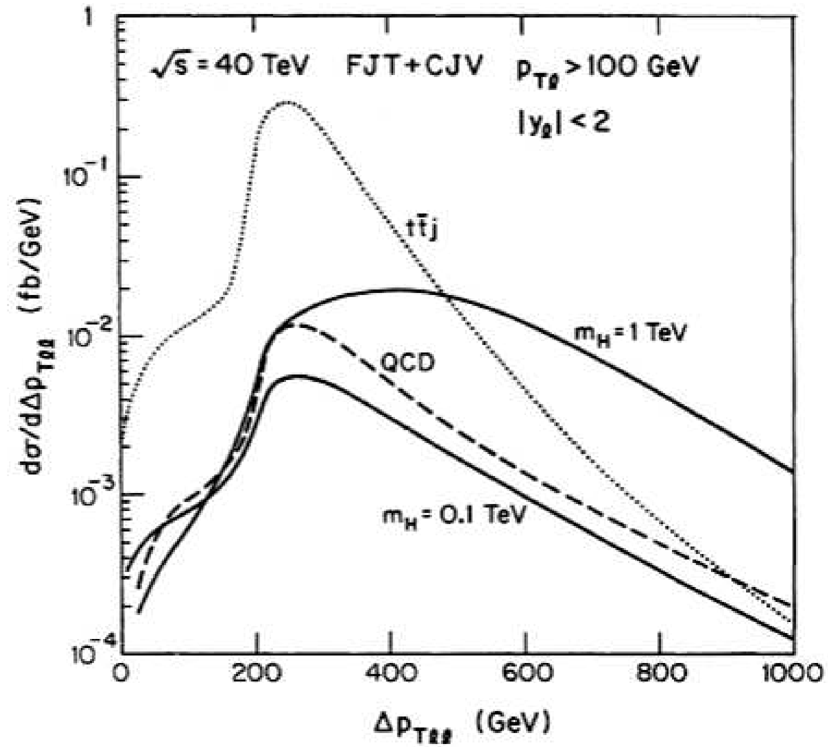
- Enhancement in large invariant mass region because of the strong scattering at TeV scale.
- Hadronic activities only come from the "spectator" jets when we demand pure leptonic decays of the W_L's.
- The kinematics of these jets is independent of the EWSB interactions.
- Since these jets radiate a W_L, based on the *effective W approximation*,

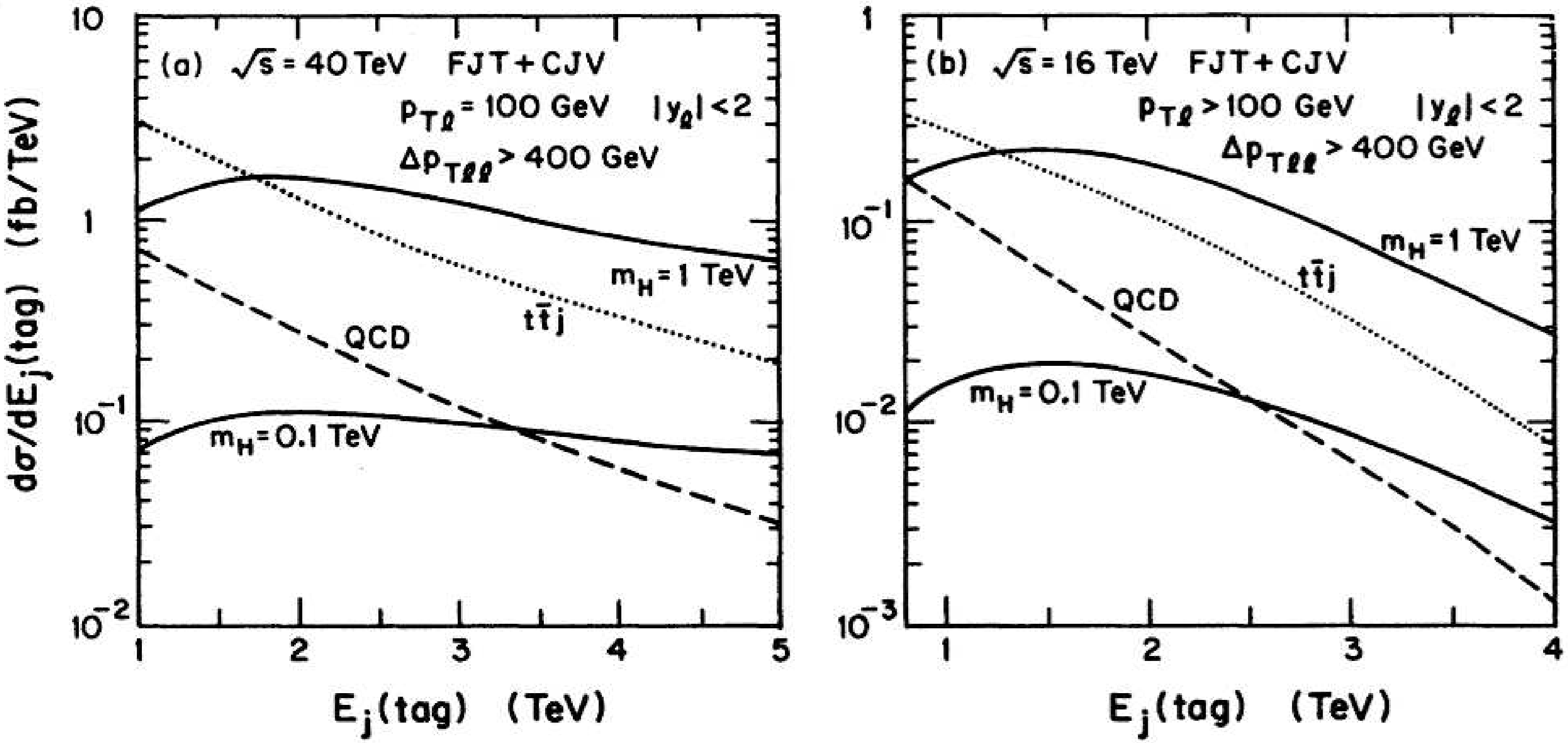
$$f_{q \to W_L}(x) \sim \frac{1-x}{x}$$

  the majority of the jets are very forward and carry most of the initial energy – energetic forward jet.
- Large rapidity gap – central jet veto.

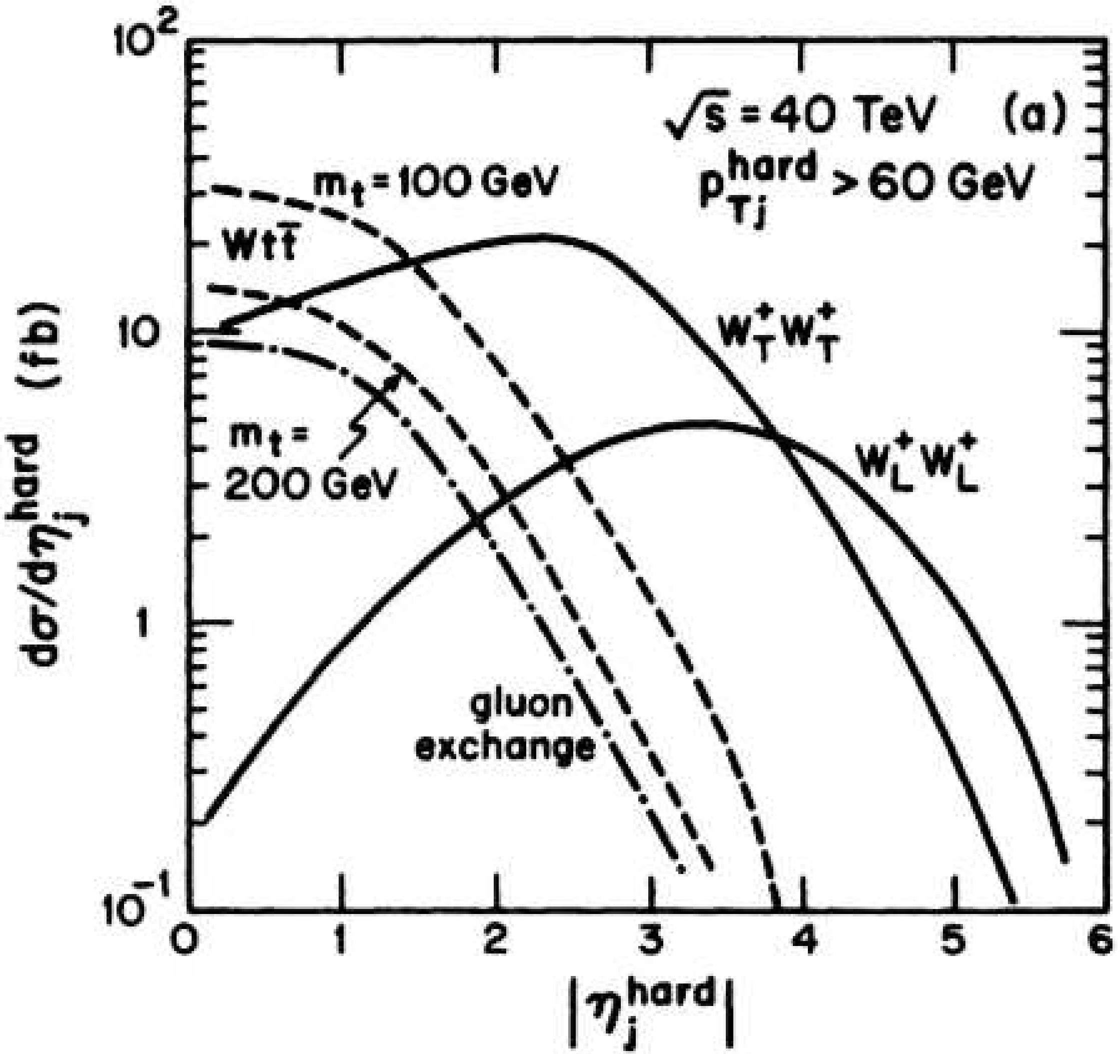
Heavy Higgs resonance $W_L^+ W_L^- \to W_L^+ W_L^-$

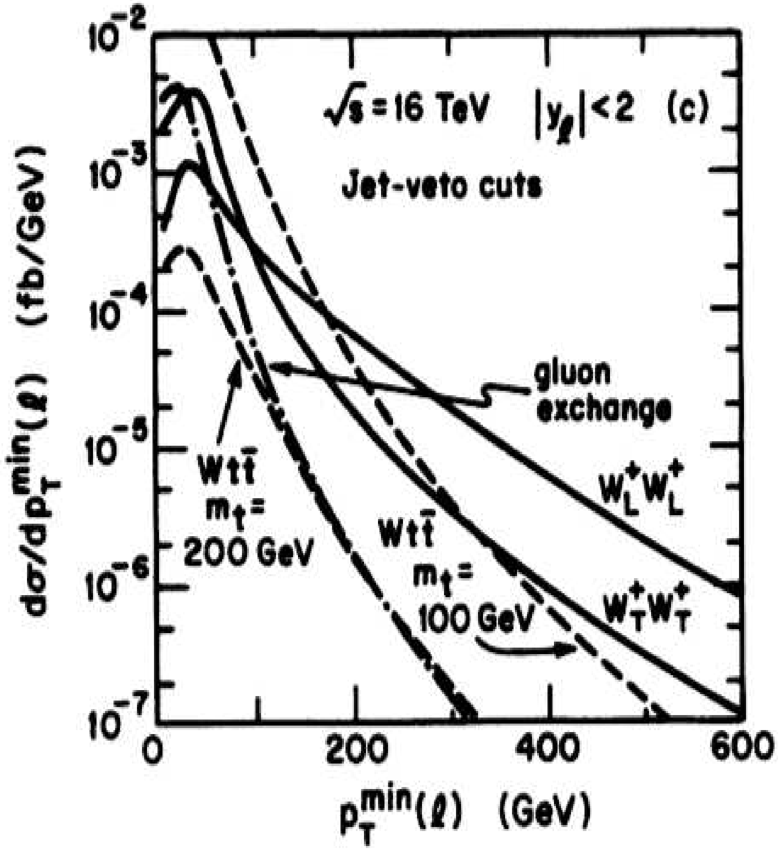




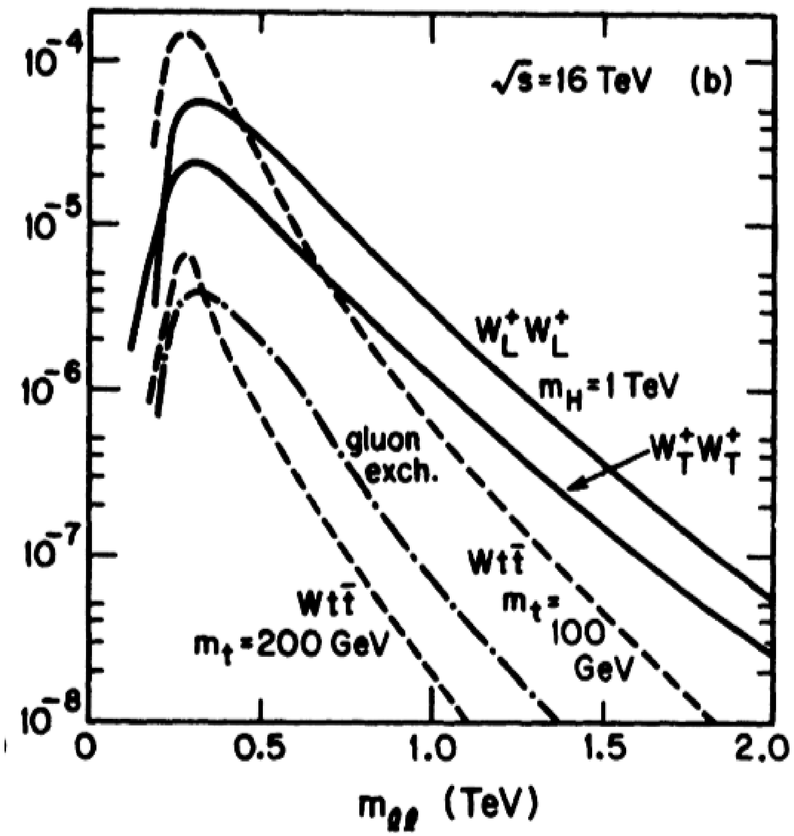

(a) √s = 40 TeV FJT + CJV
pTℓ = 100 GeV |yℓ| < 2
ΔpTℓℓ > 400 GeV
mH = 1 TeV
tt̄j
QCD
mH = 0.1 TeV
dσ/dEj(tag) (fb/TeV)
Ej(tag) (TeV)
(b) √s = 16 TeV FJT + CJV
pTℓ > 100 GeV |yℓ| < 2
ΔpTℓℓ > 400 GeV
mH = 1 TeV
tt̄j
QCD
mH = 0.1 TeV
Ej(tag) (TeV)

Nonresonant $W_L^+W_L^+ \to W_L^+W_L^+$





√s = 16 TeV |y_ℓ|<2 (c)
Jet-veto cuts
gluon exchange
$W_L^+W_L^+$
$W_T^+W_T^+$
$Wt\bar{t}$ m_t = 200 GeV
$Wt\bar{t}$ m_t = 100 GeV
$d\sigma/dp_T^{min}(\ell)$ (fb/GeV)
$p_T^{min}(\ell)$ (GeV)



√s = 16 TeV (b)
$W_L^+W_L^+$
m_H = 1 TeV
$W_T^+W_T^+$
gluon exch.
$Wt\bar{t}$ m_t = 100 GeV
$Wt\bar{t}$ m_t = 200 GeV
$m_{\ell\ell}$ (TeV)

Event rates per LHC year (100 fb^{-1}) of strong WW scattering

From Bagger et al. 1993

channel	Bkgd	1 TeV Higgs	Scalar	Vec 1TeV	Vec 2.5TeV	LET
$ZZ(4\ell)$	0.7	9	4.6	1.4	1.3	1.5
$ZZ(2\ell 2\nu)$	1.8	29	17	4.7	4.4	5.0
W^+W^-	12	27	18	6.2	5.5	5.8
$W^\pm Z$	4.9	1.2	1.5	4.5	3.3	3.5
$W^\pm Z(*)$	0.82	-	-	2.3	-	-
$W^\pm W^\pm$	3.7	5.6	7.0	12	11	13

(*) $0.8 < (M_T(WZ) < 1.1$ TeV.

The event rates are not great.

Partially-Strong $W_L W_L$ Scattering

Motivations

Suppose a light Higgs boson is discovered at the CERN LHC. Conventional wisdom tells us that the scattering of longitudinal weak gauge bosons would not grow strong at high energies.

We can show that the presence of a light Higgs boson does not guarantee the complete unitarization of the WW scattering.

We analyze how the LHC experiments can reveal this interesting possibility of partially strong WW scattering.

Partially-Strong $W_L W_L$ Scattering models

- Two-Higgs-doublet model – in a general setting there are parameters in the model such that the mass of the Higgs bosons, $\tan\beta$, and α can be chosen freely. If the light Higgs boson couples to gauge boson with a strength

$$g_{hww} = \sin(\beta - \alpha)\, g_{hww}^{\rm sm}$$

  and the mass of the heavy CP-even Higgs boson very heavy $\sim$ TeV. Note that $g_{Hww} = \cos(\beta - \alpha) g_{hww}^{\rm sm}$. One would find that when $s_{ww} > m_H^2$ the energy-growing behavior of the $W_L W_L$ amplitudes is tamed. But still when the energy in between m_h and m_H is large, growing behavior is expected.

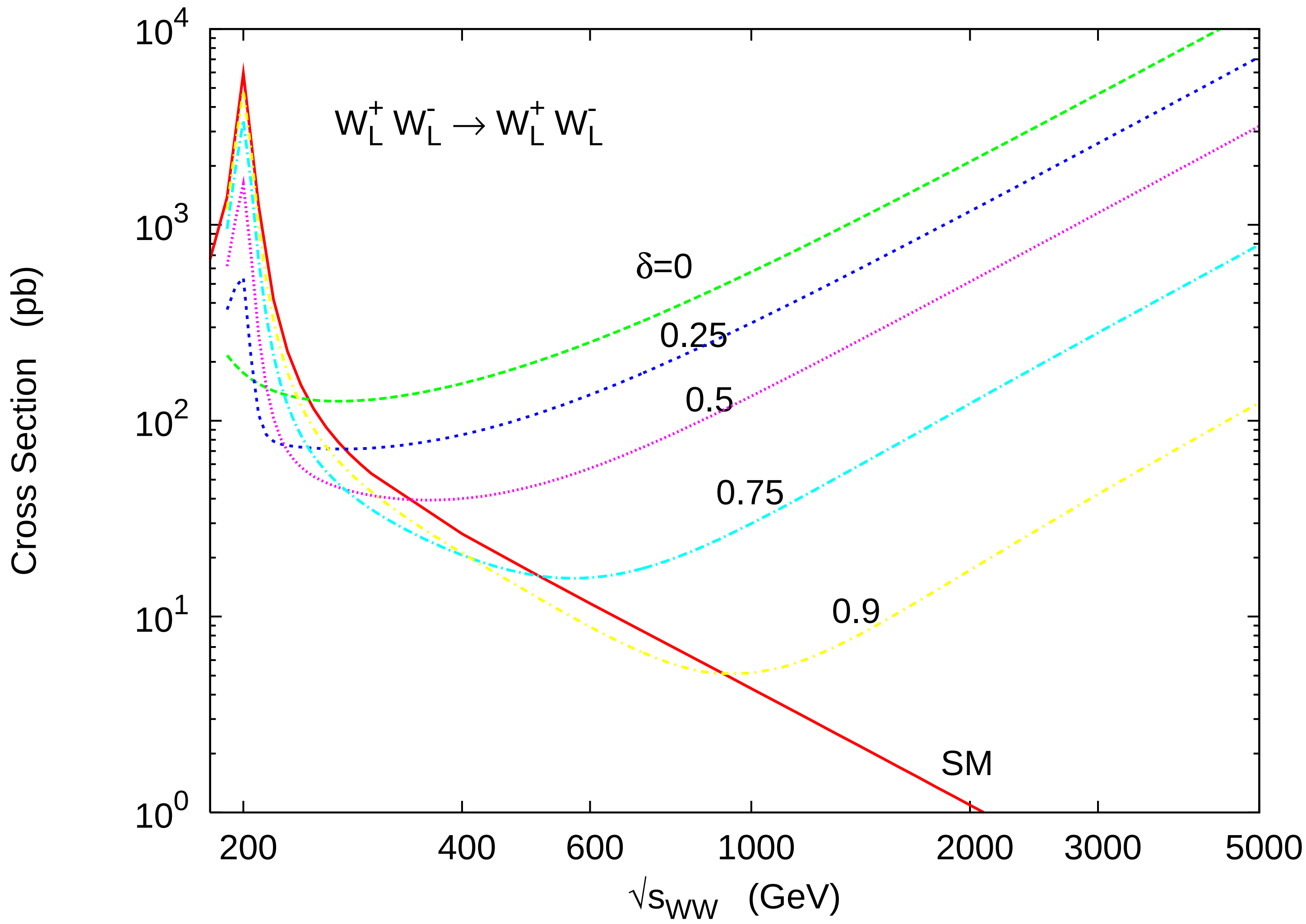
- Strongly-interacting light Higgs model – Giudice et al. a composite-like model for the light Higgs boson is assumed with the size of the ratio $g_{hWW}/g_{hWW}^{\rm SM}$ smaller than 1. All other heavier degrees of freedom are integrated out and the effects are parameterized as an effective Lagrangian with an explicit UV cutoff.

Partially Strong $W_L W_L$ Scattering

If the cancellation from the Higgs diagrams is not complete, due to, e.g., the g_{hww} coupling is smaller than the SM value. The $W_L W_L$ scattering amplitude will grow with s.

Suppose the Higgs-W-W coupling is $\sqrt{\delta}$ of the SM value, then amplitudes become

$$
\begin{aligned}
i\mathcal{M}^{\rm gauge} &= -i\frac{g^2}{4m_W^2}\,u + \mathcal{O}((E/m_W)^0) \\
i\mathcal{M}^{\rm higgs} &= i\frac{g^2}{4m_W^2}\,u\,\delta + \mathcal{O}((E/m_W)^0) \\
i\mathcal{M}^{\rm all} &= -i\frac{g^2}{4m_W^2}u(1-\delta) + \mathcal{O}((E/m_W)^0)
\end{aligned}
$$



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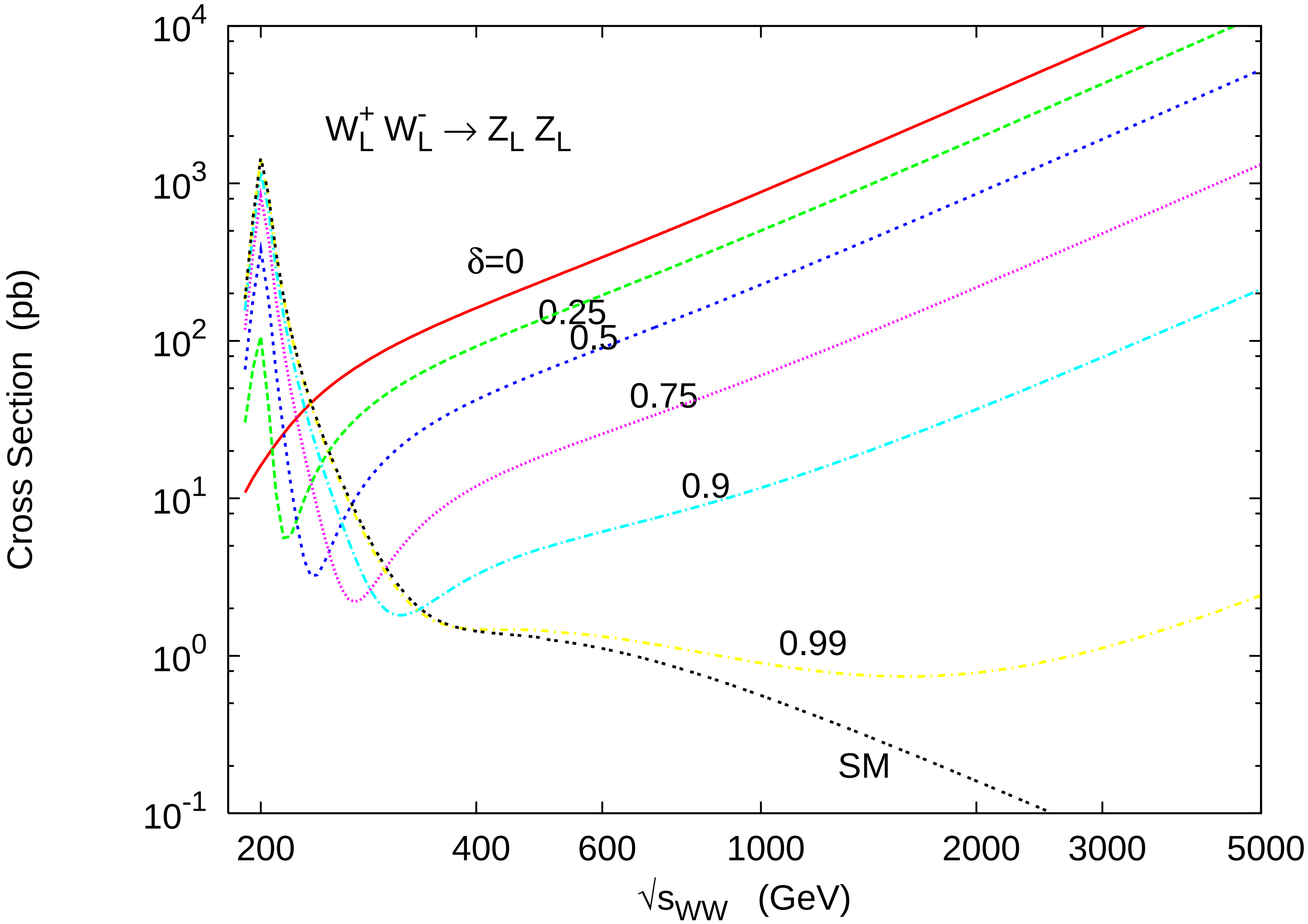

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A nonresonant channel $W_L^+ W_L^+ \to W_L^+ W_L^+$

There are t- and u-channel γ, Z diagrams, 4-point vertex, as well as the t and u channel Higgs diagrams. The sum of gauge parts is

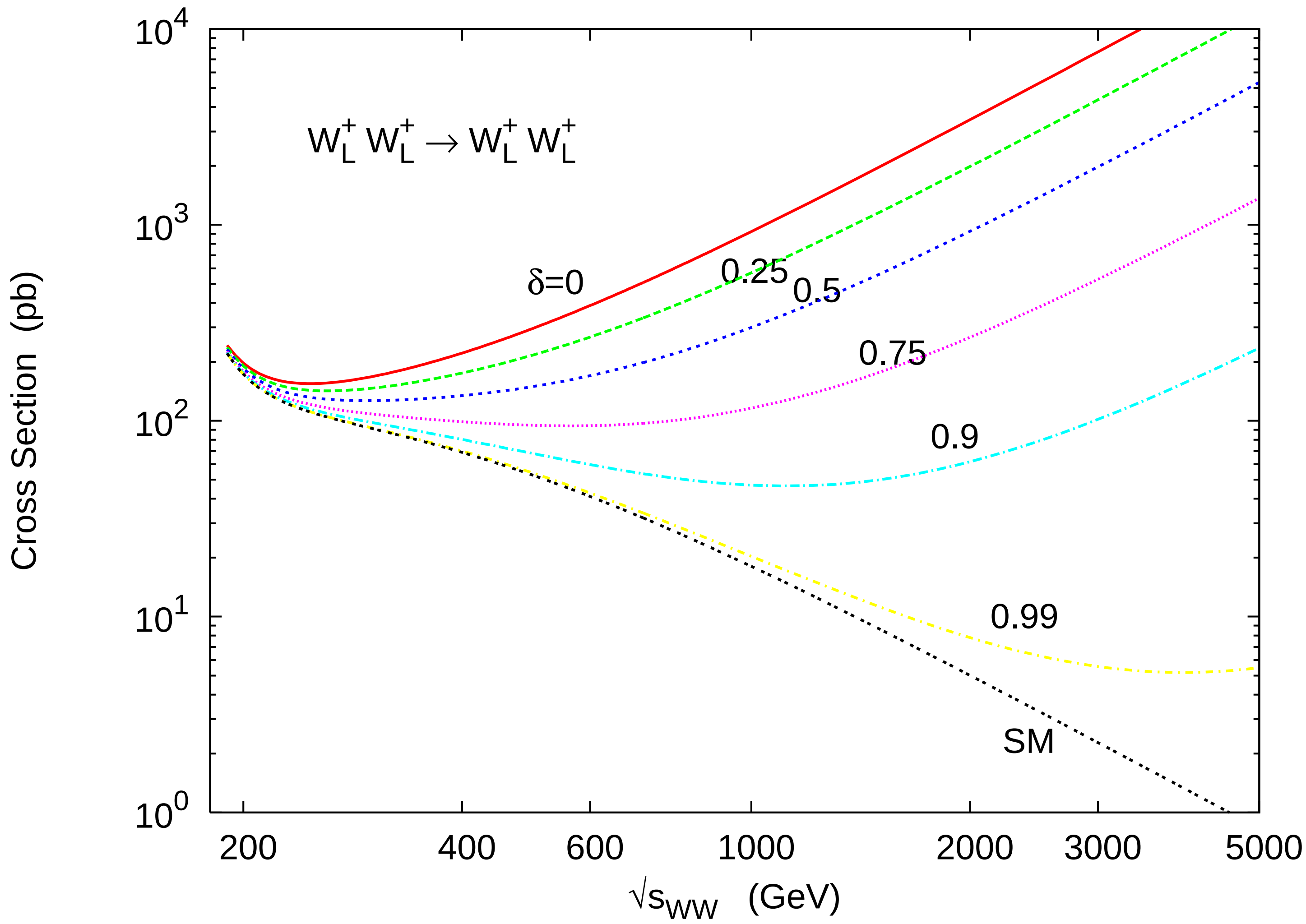
$$i\mathcal{M}^{\rm gauge} = i\frac{g^2}{4}\left[\frac{u+t}{m_W^2} + O((E/m_W)^0)\right]$$

While the Higgs amplitude is given by, in the limit $|t|, |u| \gg m_W^2, m_h^2$,

$$i\mathcal{M}^{\rm higgs} = -i\frac{g^2}{4}\left[\frac{u+t}{m_W^2} + O((E/m_W)^0)\right]$$

Suppose the Higgs amplitude is δ times the SM value, the grand sum will be

$$i\mathcal{M}^{\rm gauge} + i\mathcal{M}^{\rm higgs} = i\frac{g^2}{4}\frac{u+t}{m_W^2}(1-\delta)$$



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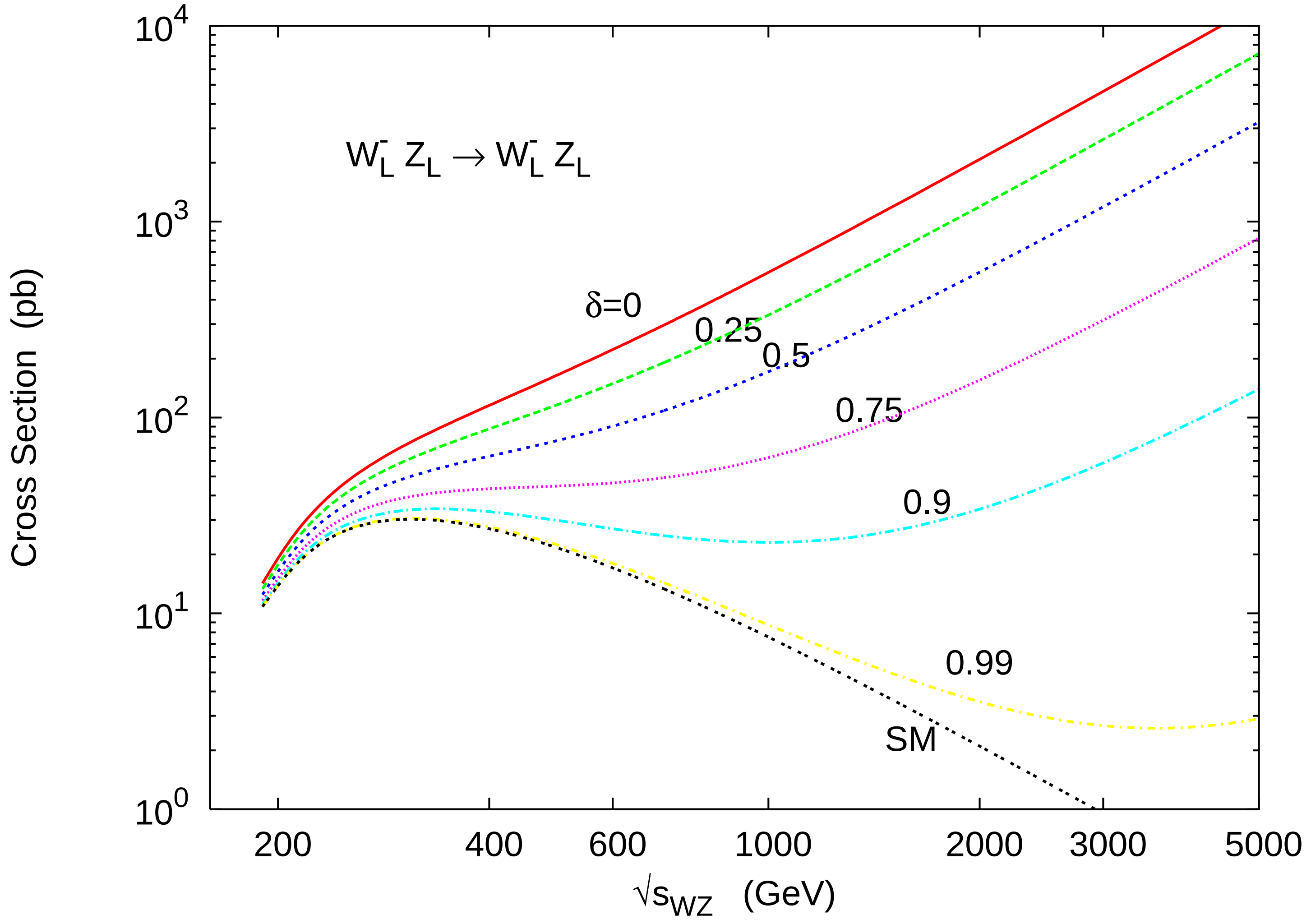

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Checking of Unitarity: partial-wave coefficients

The $W_L W_L$ scattering amplitudes can be written in terms of isospin amplitudes, exactly as pions in QCD. If we assign isospin indices

$$W_L^a W_L^b \to W_L^c W_L^d$$

where $a, b, c = 1, 2, 3$. And

$$W_L^{\pm} = \frac{1}{\sqrt{2}} \left(W_L^1 \mp i W_L^2 \right), \qquad Z_L = W_L^3$$

then the scattering amplitudes is given by

$$\mathcal{M}(W_L^a W_L^b \to W_L^c W_L^d) = A(s,t,u)\delta^{ab}\delta^{cd} + A(t,s,u)\delta^{ac}\delta^{bd} + A(u,t,s)\delta^{ad}\delta^{bc}$$

The physical amplitudes can be expressed in terms of the function A. The isospin amplitudes $T(I)$ for isospin I are

$$\begin{aligned} T(0) &= 3A(s,t,u) + A(t,s,u) + A(u,t,s) \\ T(1) &= A(t,s,u) - A(u,t,s) \\ T(2) &= A(t,s,u) + A(u,t,s) \end{aligned}$$

In terms of isospin amplitudes, the physical scattering amplitudes can be written as

$$\mathcal{M}(W_L^+ W_L^- \to Z_L Z_L) = \frac{T(0) - T(2)}{3}$$
$$\mathcal{M}(Z_L Z_L \to W_L^+ W_L^-) = \frac{T(0) - T(2)}{3}$$
$$\mathcal{M}(W_L^+ W_L^- \to W_L^+ W_L^-) = \frac{2T(0) + 3T(1) + T(2)}{6}$$
$$\mathcal{M}(Z_L Z_L \to Z_L Z_L) = \frac{T(0) + 2T(2)}{3}$$
$$\mathcal{M}(W_L^\pm Z_L \to W_L^\pm Z_L) = \frac{T(1) + T(2)}{2}$$
$$\mathcal{M}(W_L^\pm W_L^\pm \to W_L^\pm W_L^\pm) = T(2)$$

We can compute the partial-wave according to

$$T(I) = 32\pi \sum_{l=0}^{\infty} (2l+1) P_l(\cos\theta) a_l^I$$

therefore

$$a_l^I = \frac{1}{64\pi} \int_{-1}^{1} d(\cos\theta)\ P_l(\cos\theta)\ T(I)$$

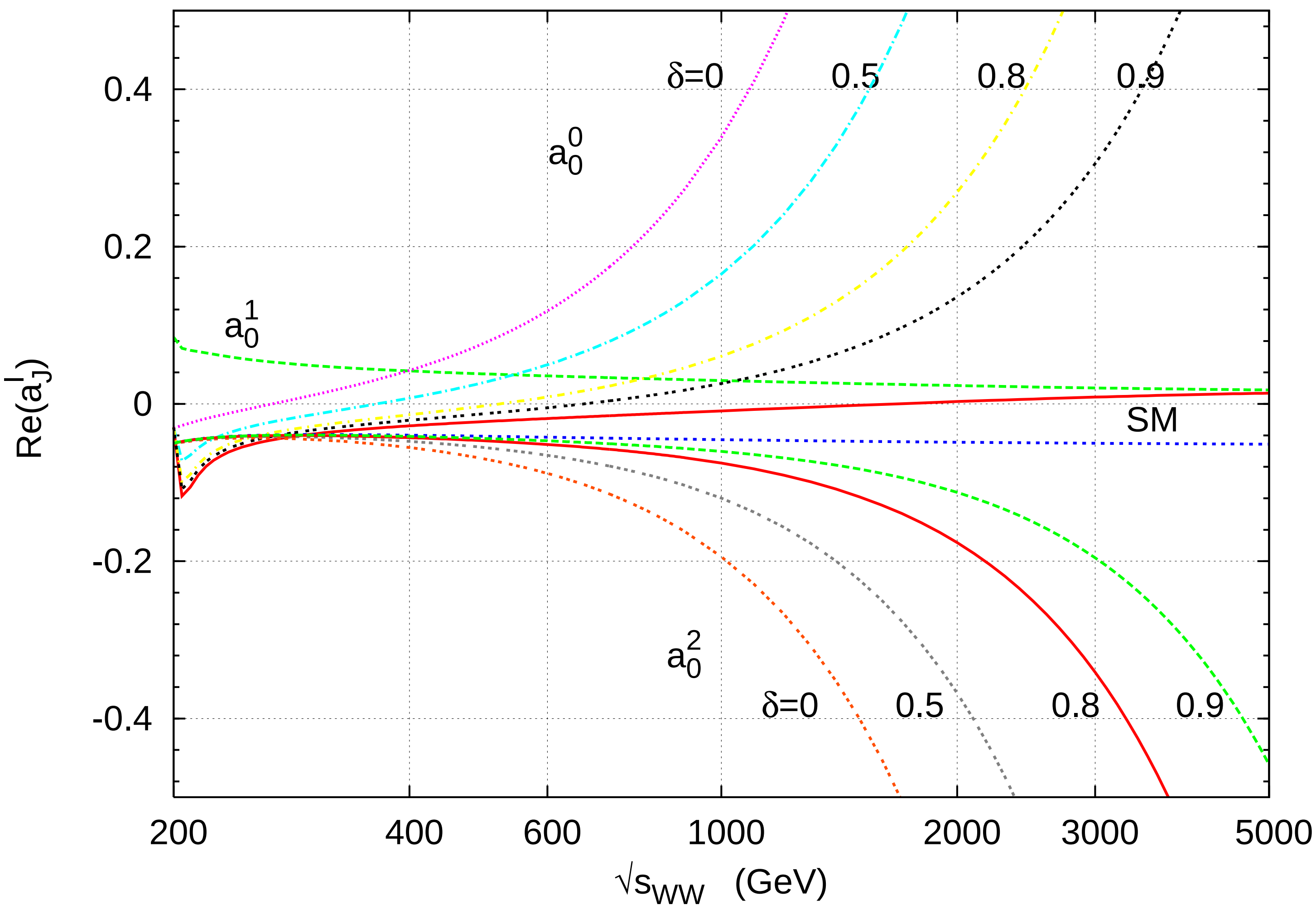
Two body elastic unitarity circle is equivalent to requiring

$$\left| a_l^I - \frac{i}{2} \right| = \frac{1}{2}$$

In case of real a_l^I we require

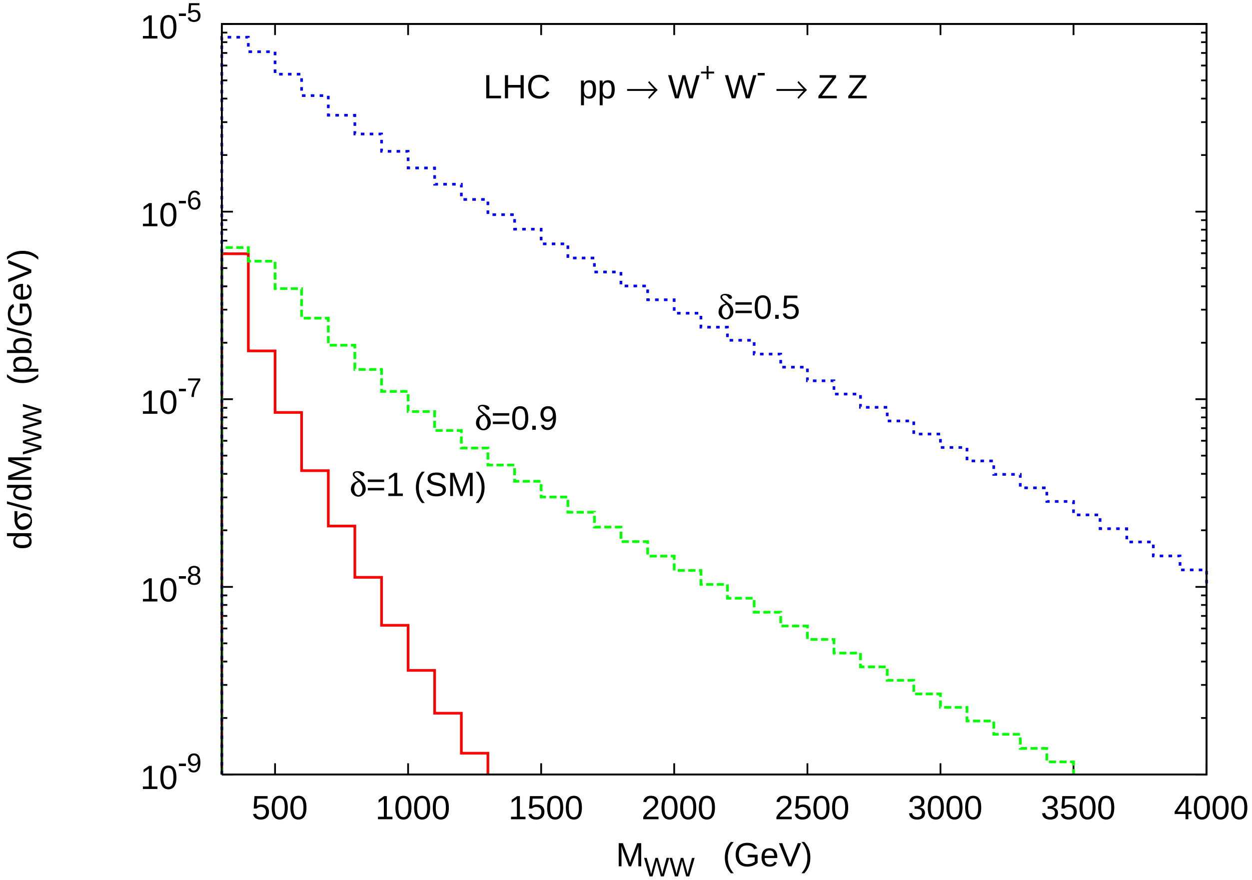
$$\Re e[a_l^I] < \frac{1}{2}$$

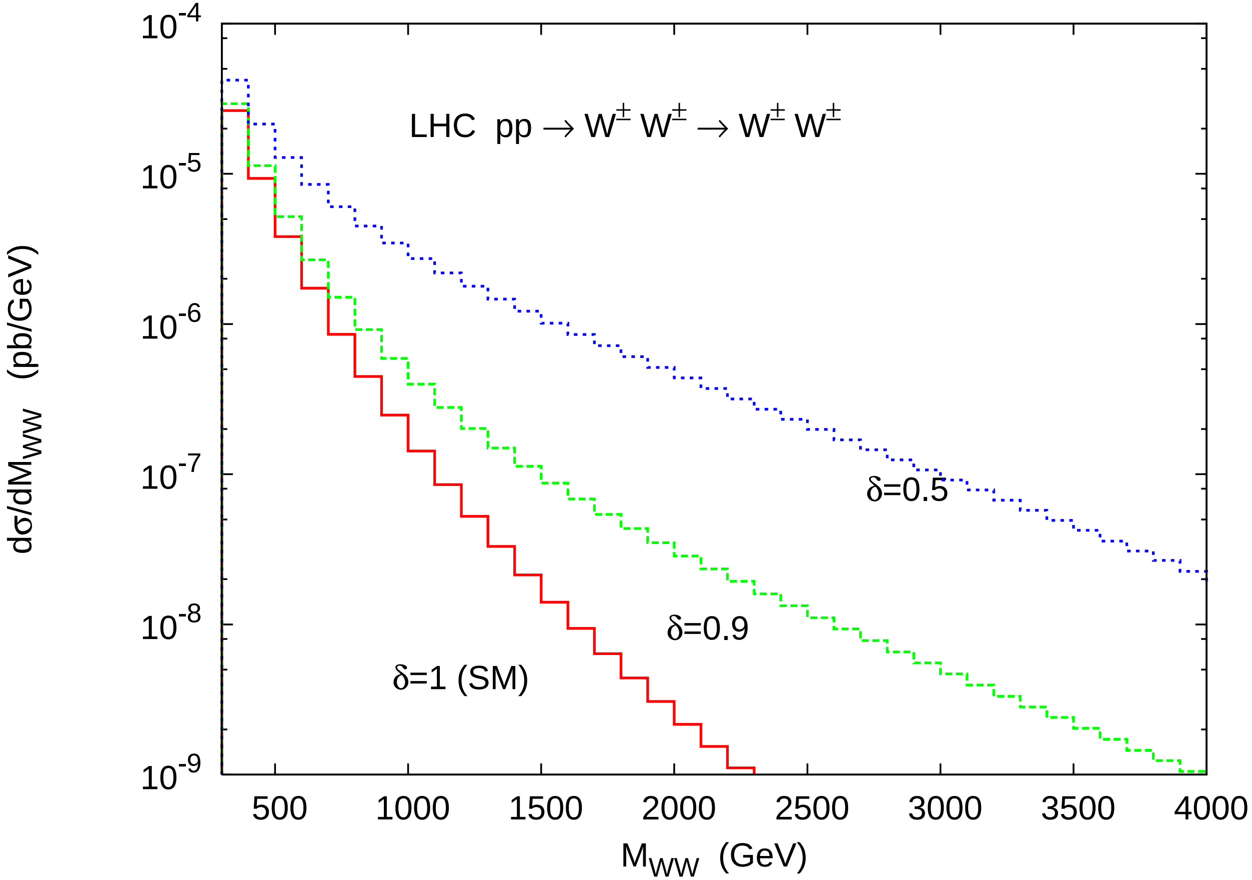
to ensure unitarity.


0.4
0.2
0
-0.2
-0.4
Re(a^I_J)
200
400
600
1000
2000
3000
5000
$\sqrt{s_{WW}}$ (GeV)
δ=0
0.5
0.8
0.9
a^0_0
a^1_0
SM
a^2_0
δ=0
0.5
0.8
0.9

Experimental signals at the LHC

- Enhancement in the large invariant mass region.





LHC pp → W± W± → W± W±
δ=0.5
δ=0.9
δ=1 (SM)
dσ/dM_WW (pb/GeV)
M_WW (GeV)
10^-4
10^-5
10^-6
10^-7
10^-8
10^-9
500
1000
1500
2000
2500
3000
3500
4000

Event rates for a LHC year of 100 fb^{-1}

Subprocess	Number of Events $\delta = 1$ (SM)	0.9	0.5	0 (No Higgs)
$W_L^\pm W_L^\pm \to W_L^\pm W_L^\pm \to \ell^\pm \nu \ell^\pm \nu$	21	26	57	118
$W_L^+ W_L^- \to W_L^+ W_L^- \to \ell^+ \bar{\nu} \ell^- \nu$	8	7	17	67
$W_L^\pm Z_L \to W_L^\pm Z_L \to \ell^\pm \nu \ell^+ \ell^-$	4	5	13	33
$W_L^+ W_L^- \to Z_L Z_L \to \ell^+ \ell^- \ell^+ \ell^-$	0.04	0.12	2	9
$W_L^+ W_L^- \to Z_L Z_L \to \ell^+ \ell^- \nu \bar{\nu}$	0.25	0.74	12	50
$Z_L Z_L \to Z_L Z_L \to \ell^+ \ell^- \ell^+ \ell^-$	0.4	0.32	0.08	0
$Z_L Z_L \to Z_L Z_L \to \ell^+ \ell^- \nu \bar{\nu}$	2.4	2	0.5	0

Summary

- If we do not find a light Higgs boson, we should prepare to look for strong WW scattering. Signatures are excessive production of longitudinal WW pairs in the large invariant mass region.
- Even a light Higgs is found, we should test if this Higgs is completely responsible for EWSB.
- Besides checking its branching ratios, one can use $W_L W_L$ scattering.
- If the scattering turns around and becomes strong, it means the light Higgs boson is not the whole story. The light Higgs is only partially responsible for EWSB.
- The unitarity violation will be delayed relative to the no-Higgs scenario. In the no-Higgs scenario, unitarity violation occurs at 1.2 TeV while with $\delta = 0.5$ it occurs at 1.75 TeV.