Using Dimensional Reduction for Hadronic Collisions

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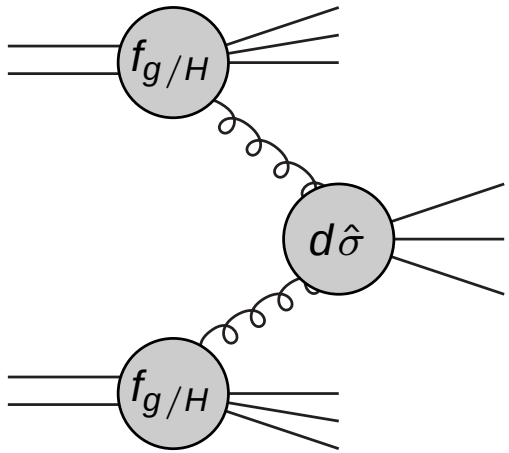

SUSY 2008, Seoul

Outline

1 Introduction

2 Definition of DREG and DRED

3 Main Results — singularity structure and RS dependence

4 Conclusions

Motivation

- Dimensional Reduction: usually best regularization for SUSY — however not without problems
- Understanding of DRED has recently improved:
  1. Definition without mathematical inconsistency, proof of SUSY in important cases [DS '05, Hollik, DS '05]
  2. Application of DRED to SUSY and non-SUSY processes at $\geq$ 3-loops [Harlander,Jones,Kant,Mihaila,Steinhauser '06-'08]
  3. Resolution of "factorization problem" of van Neerven, Smith; Beenakker, Hopker, Zerwas [Signer, DS '05]
- Here: application of DRED to real + virtual NLO corrections of arbitrary hadronic processes

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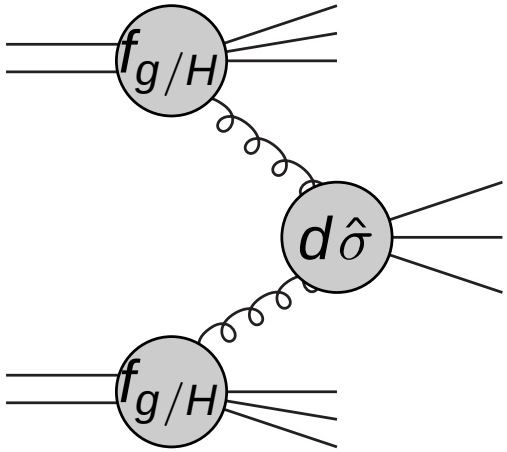
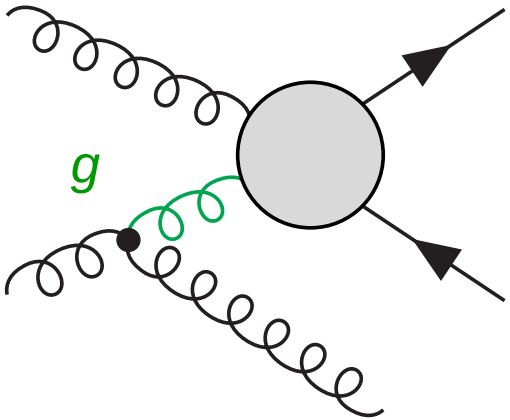
Hadronic Processes



$$d\hat{\sigma} = d\sigma_{\mathrm{RS}}^{\mathrm{Born}} + d\sigma_{\mathrm{RS}}^{\mathrm{real}} + d\sigma_{\mathrm{RS}}^{\mathrm{virt}} + d\sigma_{\mathrm{RS}}^{\mathrm{coll}}$$

- IR (+ UV) singularities $\Rightarrow$ RS dependence, must cancel
- IR sing. must factorize

Factorization — old results

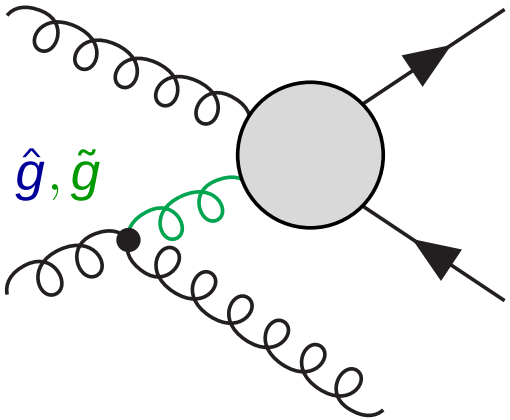


- apparent factorization problem

[Beenakker, Kuijf, van Neerven, Smith '88] [van Neerven, Smith '04] [Beenakker, Höpker, Spira, Zerwas '96]

$$\sigma^{\mathrm{DRED}}(gg \to t\bar{t}g) \to P_{g\to gg}\ \sigma_{gg} + \text{non-factorizing term!}$$

Factorization — old results

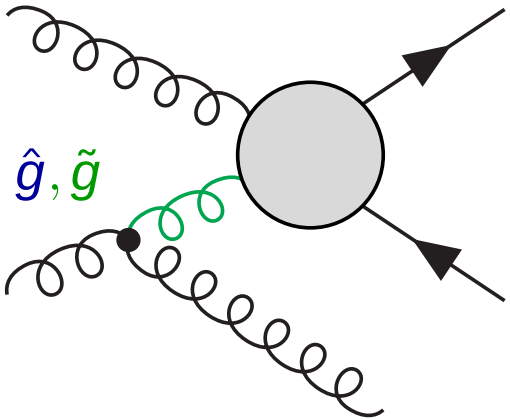


- resolution: split $g = \hat{g} + \tilde{g}$ (gauge field + ϵ-scalar)

[Signer, DS '05]

$$\sigma^{\text{DRED}}(gg \to t\bar{t}g) \to P_{g\to\hat{g}g}\ \sigma_{g\hat{g}} + P_{g\to\tilde{g}g}\ \sigma_{g\tilde{g}}$$

Factorization — old results



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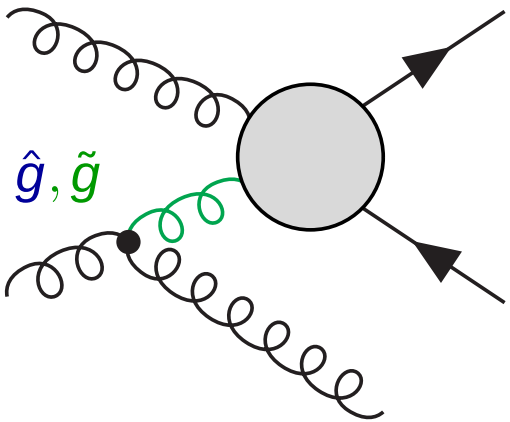
- However, no factorization problem found in [Kunszt, Signer, Trocsanyi '94] [Catani, Seymour, Trocsanyi '97]

Factorization — old results



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[Signer, DS '05]

$$\sigma^{\text{DRED}}(gg \to t\bar{t}g) \to P_{g\to\hat{g}g}\ \sigma_{g\hat{g}} + P_{g\to\tilde{g}g}\ \sigma_{g\tilde{g}}$$

- However, no factorization problem found in [Kunszt, Signer, Trocsanyi '94] [Catani, Seymour, Trocsanyi '97]

Aims: clarify! Precise definitions of DRED (different versions!)
General sing. structure in DRED

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Definition of DRED

dimensional schemes: momenta D-, gluons either D- ($\hat{g}$) or 4-dimensional ($g = \hat{g} + \tilde{g}$)

	CDR	DRED
"internal" gluon	$\hat{g}^{\mu\nu}$	$g^{\mu\nu}$
"external" gluon	$\hat{g}^{\mu\nu}$	$g^{\mu\nu}$

Definition of DRED

dimensional schemes: momenta D-, gluons either D- ($\hat{g}$) or 4-dimensional ($g = \hat{g} + \tilde{g}$)

3 spaces: $\underset{\bar{g}^{\mu\nu}}{\text{4S}} \subset \underset{\hat{g}^{\mu\nu}}{\text{Q}D\text{S}} \subset \underset{g^{\mu\nu} = \hat{g}^{\mu\nu} + \tilde{g}^{\mu\nu}}{\text{Q4S}}$

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“internal” gluon	$\hat{g}^{\mu\nu}$	$g^{\mu\nu}$
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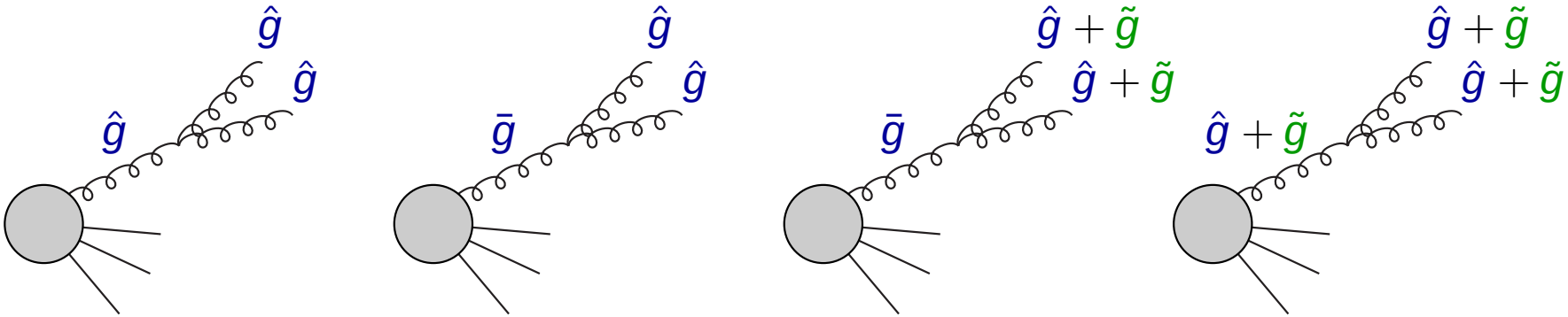
	CDR	HV	FDH	DRED
"internal" gluon	$\hat{g}^{\mu\nu}$	$\hat{g}^{\mu\nu}$	$g^{\mu\nu}$	$g^{\mu\nu}$
"external" gluon	$\hat{g}^{\mu\nu}$	$\bar{g}^{\mu\nu}$	$\bar{g}^{\mu\nu}$	$g^{\mu\nu}$

Definition of DRED

dimensional schemes: momenta D-, gluons either D- ($\hat{g}$) or 4-dimensional ($g = \hat{g} + \tilde{g}$)

3 spaces: $\underset{\bar{g}^{\mu\nu}}{4S} \subset \underset{\hat{g}^{\mu\nu}}{QDS} \subset \underset{g^{\mu\nu} = \hat{g}^{\mu\nu} + \tilde{g}^{\mu\nu}}{Q4S}$

	CDR	HV	FDH	DRED
"internal" gluon	$\hat{g}^{\mu\nu}$	$\hat{g}^{\mu\nu}$	$g^{\mu\nu}$	$g^{\mu\nu}$
"external" gluon	$\hat{g}^{\mu\nu}$	$\bar{g}^{\mu\nu}$	$\bar{g}^{\mu\nu}$	$g^{\mu\nu}$



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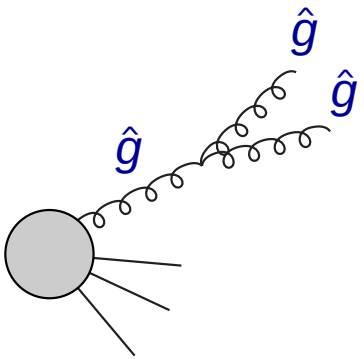
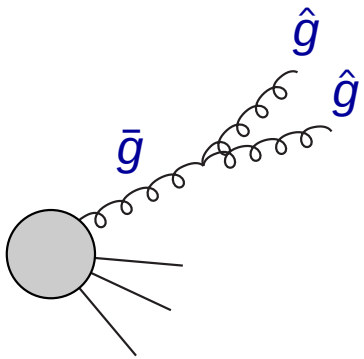
Main Results



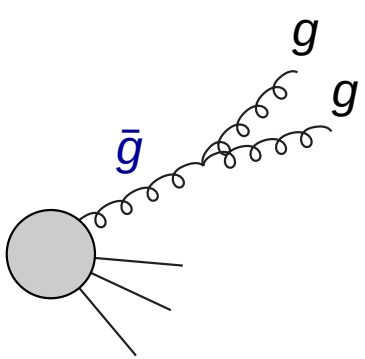
CDR

$P_{\hat{g}\to\hat{g}\hat{g}}$



HV

$\ldots$



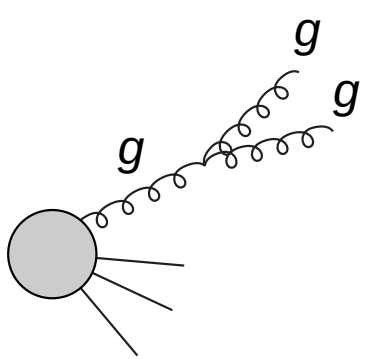
FDH

$\ldots$

$+P_{\hat{g}\to\tilde{g}\tilde{g}}$

[Catani, S., T. '97]

[Kunszt, S., T. '94]



DRED

$\ldots$

$\ldots$

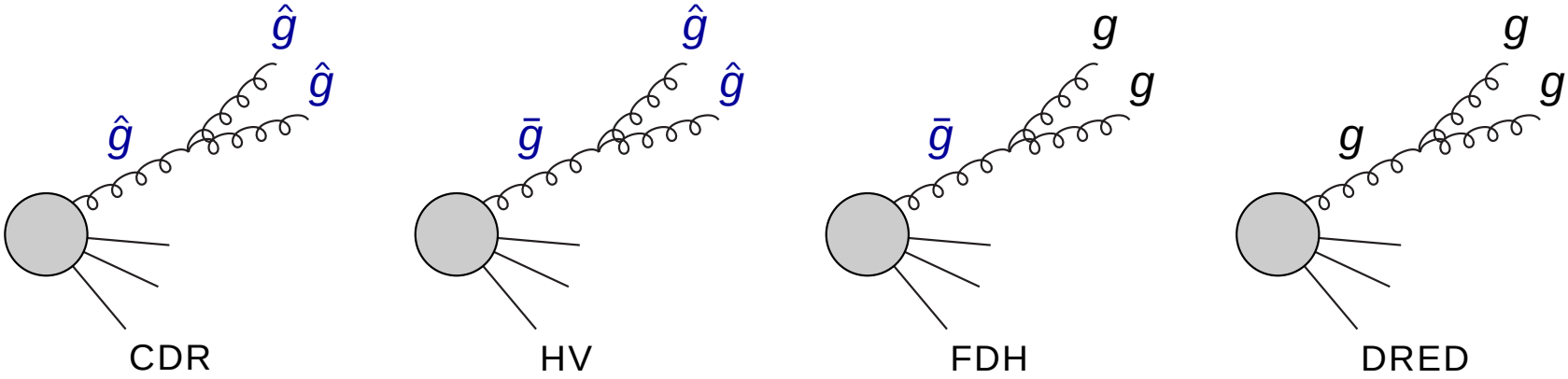
$P_{\tilde{g}\to\hat{g}\tilde{g}}$

$+P_{\tilde{g}\to\tilde{g}\hat{g}}$

[Signer, DS '08]

understand real coll. sing.:

Main Results



$P_{\hat{g}\to\hat{g}\hat{g}}$ ... ...

$+P_{\hat{g}\to\tilde{g}\tilde{g}}$ ...

[Catani, S., T. '97]

[Kunszt, S., T. '94]

$P_{\tilde{g}\to\hat{g}\tilde{g}}$

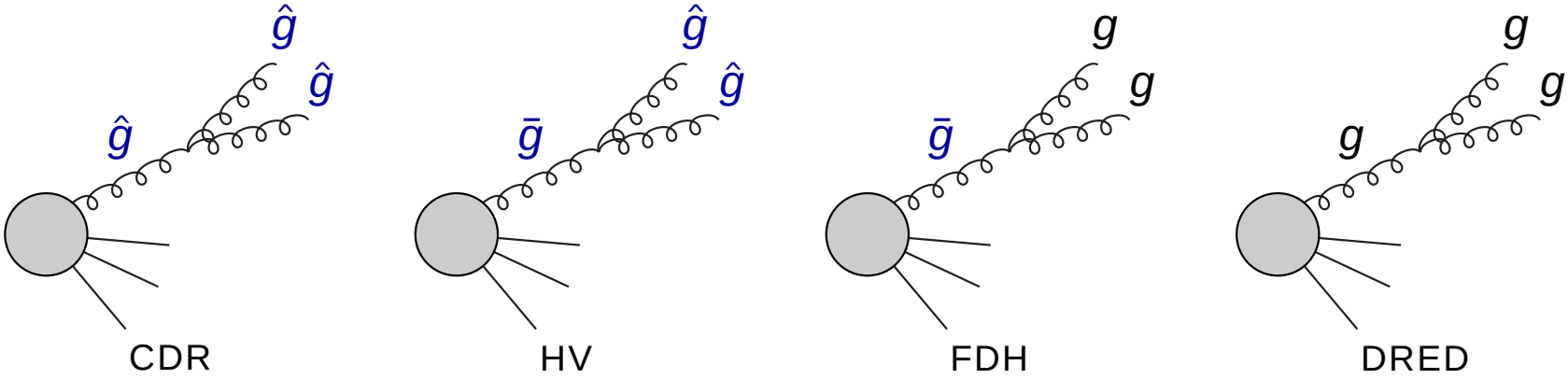
$+P_{\tilde{g}\to\tilde{g}\hat{g}}$

[Signer, DS '08]

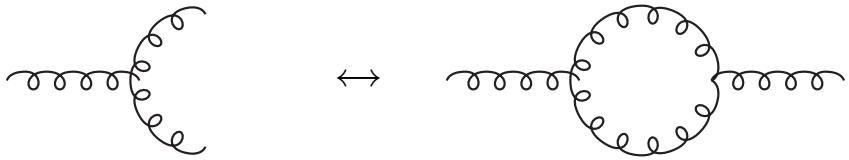
understand real coll. sing.:

$$\mathcal{M}(a_1,a_2;\dots a_l(p_l)\dots a_k(p_k)\dots a_{n+1}) \overset{p_k\|p_l}{=} \frac{2\,g_s^2}{s_{kl}}\sum_{\check{a}_{(kl)}} P_{(kl)^*\to kl}(z)\,\mathcal{M}(a_1,a_2;\dots \check{a}_{(kl)}(p_k+p_l)\dots a_n).$$

Main Results



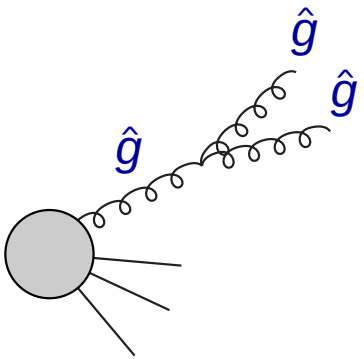
Unitarity:



real $\leftrightarrow$ virtual coll. sing.

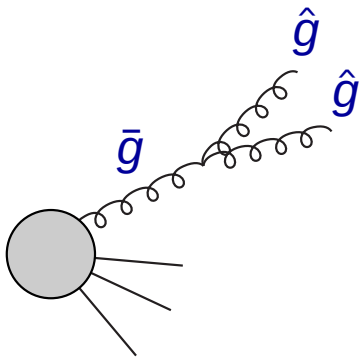
$$\int P_{i\to\text{anything}} = \gamma(i)$$

Main Results



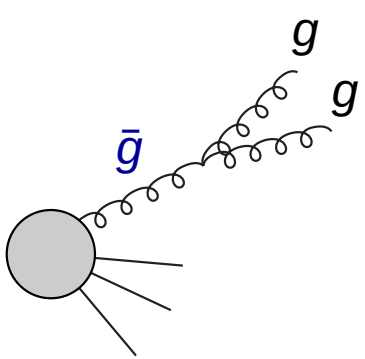
CDR

$$\gamma(\hat{g}) = \int P_{\hat{g}\to\hat{g}\hat{g}}$$



HV

$$\cdots$$



FDH

$$\cdots$$

$$+ \int P_{\hat{g}\to\tilde{g}\tilde{g}}$$

[Catani, S., T. '97]

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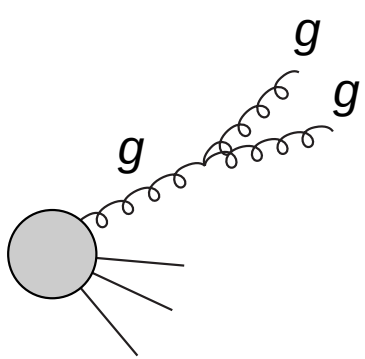


DRED

$$\cdots$$

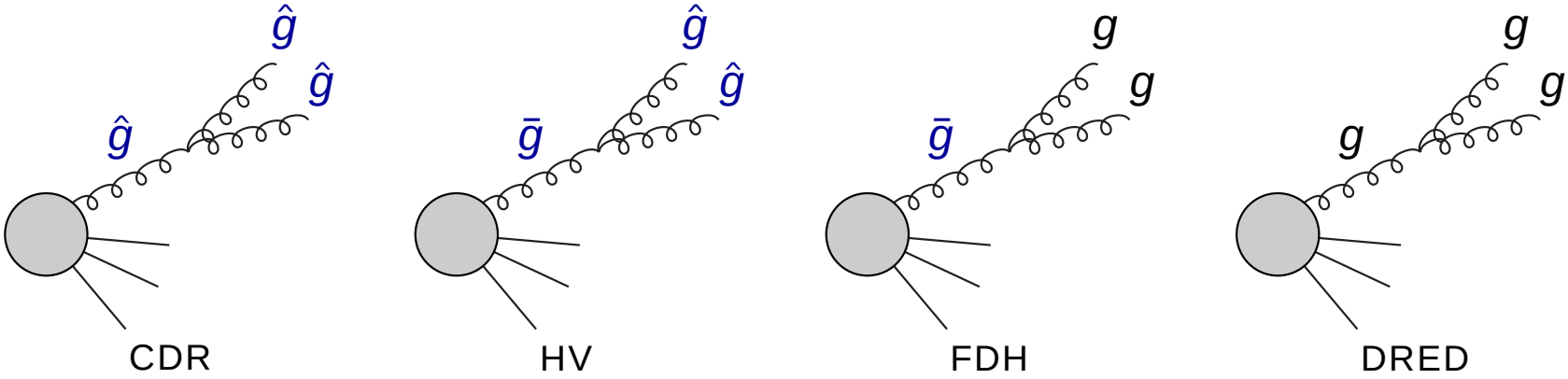
$$\cdots$$

$$\gamma(\tilde{g}) = \int P_{\tilde{g}\to\hat{g}\tilde{g}} + \int P_{\tilde{g}\to\tilde{g}\hat{g}}$$

[Signer, DS '08]

understand virtual coll. sing.:

Main Results



$$\gamma(\hat{g}) = \int P_{\hat{g}\to\hat{g}\hat{g}} \quad \ldots \quad \ldots \quad \ldots$$

$$+ \int P_{\hat{g}\to\tilde{g}\tilde{g}} \quad \ldots$$

[Catani, S., T. '97]

[Kunszt, S., T. '94]

$$\gamma(\tilde{g}) = \int P_{\tilde{g}\to\hat{g}\tilde{g}} + \int P_{\tilde{g}\to\tilde{g}\hat{g}}$$

[Signer, DS '08]

understand virtual coll. sing.:

$$\mathcal{M}^{(1)}(a_1 \ldots a_n) \to \frac{\alpha_s}{2\pi} c_\Gamma \sum_i \sum_{\check{a}_i} \mathcal{M}^{(0)}(a_1 \ldots \check{a}_i \ldots a_n) \left(-\frac{1}{\epsilon} \gamma(\check{a}_i) \right)$$

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Practical Calculations

- singularity structure $\Rightarrow$ RS dependence of real/virtual corrections
- all elements of hadronic processes can be computed in any RS and translated into any other RS with simple formulae
- coll. counterterms $d\sigma^{\text{coll}}$ can be defined in any RS such that e.g. $\overline{\text{MS}}$ PDFs can be used
- in DRED: no PDF for ϵ-scalars $\tilde{g}$ required (of $\mathcal{O}(\epsilon)$ and contributes only of $\mathcal{O}(\epsilon)$)

Summary: Properties of DREG and DRED

	consistent	SUSY-violation	factorization
DREG:	$+$	$-$	$+$
DRED:	$+$	$(+)$	$+$

Summary & Outlook

Comparison of DREG and DRED:

- Factorization: holds in all schemes CDR, HV, FDH, DRED
- RS dependence: distinguish FDH and DRED

  HV$\to$FDH: additional final state $\tilde{g}$: *value* of $\gamma(\hat{g})$ changes
  FDH$\to$DRED: additional splitting $\tilde{g}$: additional $\gamma(\tilde{g})$
  only in DRED: split $g = \hat{g} + \tilde{g}$ required to understand factorization

- Practical calculations: translation rules,
  hadronic processes can be computed in all schemes