

# Constraints of the $B_\mu/\mu$ Solution due to the Hidden Sector Renormalization

Hae Young Cho

Department of physics and astronomy ,Seoul National University

SUSY08 June 17th 2008

- 1 Introduction
  - MSSM
- 2  $B_\mu/\mu$  problem in GMSB
  - generating  $\mu$  in GMSB
  - $\mu$  problem in GMSB
- 3 The Hidden Sector RG Effect as a Solution to  $\mu$  problem in GMSB
- 4 Constraints of the  $B_\mu/\mu$  Solution due to the Hidden Sector RG
  - Visiting the Low Energy Spectrum
  - Consistency Test
- 5 Conclusion

# Supersymmetry

## Why We Need SUSY?

- Gauge hierarchy problem
- Natural candidate for the dark matter
- Gauge coupling unification

To apply to the low energy phenomenology, SUSY should be broken, and it should be transmitted by some messenger interactions. There are many mechanisms to explain this process. e.g. mSUGRA, GMSB, AMSB, Mirage mediation, etc..

# GMSB

## The attractiveness of GMSB

- FCNC problem: The gauge interaction is flavor blind.
- The predictivity: We can get almost SUSY breaking soft terms with few parameters.
- The possibility: In the concept of metastable vacua, the model building is made easier.

# $\mu$ problem in MSSM

## The problem

- It is not natural that  $\mu$  should be electroweak scale.
- $\mu$  term is not allowed if we consider  $U(1)_{PQ}$  at tree level.

## A possible solution

- Giudice - Masiero mechanism  
In the mSUGRA setup, via non-trivial *kähler* potential  $\mu$  as well as  $B_\mu$  can be obtained to be of a proper scale.

## $\mu$ generation in GMSB

- The basic philosophy of GMSB is the soft terms can be obtained by gauge interaction.
- $U(1)_{PQ}$  can not be broken by gauge interaction.
- To generate  $\mu$ , we introduce the superpotential as follows.

$$W_{H_u H_d} = \xi_u H_u \psi_1 \bar{\psi}_2 + \xi_d H_d \bar{\psi}_1 \psi_2.$$

- As a result, we can obtain

$$\mu = \frac{\xi_u \xi_d}{16\pi^2} \Lambda f(\lambda_1/\lambda_2) \left[ 1 + \mathcal{O}\left(\frac{F^2}{M_{mess}^4}\right) \right]$$

$$B_\mu = \frac{\xi_u \xi_d}{16\pi^2} \Lambda^2 f(\lambda_1/\lambda_2) \left[ 1 + \mathcal{O}\left(\frac{F^2}{M_{mess}^4}\right) \right] = \Lambda \mu.$$

## Phenomenological Requirement

At the electroweak scale, the higgs sectors have several constraints.

- From the requirement of  $\det M_{higgs}^2 < 0$ ,

$$B_\mu^2 > (|\mu_{eff}|^2 + M_{H_u}^2)(|\mu_{eff}|^2 + M_{H_d}^2).$$

- For the stability of the Higgs potential,

$$2B_\mu < 2|\mu|^2 + M_{H_u}^2 + M_{H_d}^2.$$

- These lead us to a statement,  $B_\mu \sim \mu^2$ .

To solve  $B_\mu/\mu$  problem in GMSB, one needs a natural way to explain the phenomenologically unacceptable hierarchy.

## The Basic Idea of the HSRG Solution

- To introduce the hidden sector effect, it is required to assume that the hidden sector is strongly coupled.
- The operators proportional to  $SS^\dagger$  suffer suppression via the hidden sector RG effects.
- There can be mixing between operators, and this affects on the boundary condition.
- No need to introduce additional degree of freedom like NMSSM.



The soft terms of GMSB can be represented as follows.

$$O_\phi : \int d^4\theta c_\phi^q \frac{q^\dagger q}{M^2} \phi^\dagger \phi, \quad \int d^4\theta c_\phi^s \frac{S^\dagger S}{M^2} \phi^\dagger \phi,$$

$$O_{B_\mu} : \int d^4\theta c_{B_\mu}^q \frac{q^\dagger q}{M^2} H_u H_d + h.c., \quad \int d^4\theta c_{B_\mu}^s \frac{S^\dagger S}{M^2} H_u H_d + h.c.,$$

$$O_\lambda : \int d^4\theta c_\lambda^s \frac{S}{M} W^{a\alpha} W_\alpha^a + h.c.,$$

$$O_A : \int d^4\theta c_A^s \frac{S}{M} \phi^\dagger \phi + h.c.,$$

$$O_\mu : \int d^4\theta c_\mu^s \frac{S^\dagger}{M} H_u H_d + h.c.,$$

If we consider the hidden sector RG effects, then for the operators which are proportional to  $S^\dagger S$  or  $q^\dagger q$ , can be represented as follows.

$$\begin{aligned}
 O_\phi : & \int d^4\theta \left( \frac{\Lambda_{CFT}}{M_{mess}} \right)^{\alpha_q} Z_q^{-1}(\mu_R) c_\phi^q \frac{q^\dagger q}{M^2} \phi^\dagger \phi, \\
 & \int d^4\theta \left( \frac{\Lambda_{CFT}}{M_{mess}} \right)^{\alpha_s} Z_s^{-1}(\mu_R) c_\phi^s \frac{S^\dagger S}{M^2} \phi^\dagger \phi, \\
 O_{B_\mu} : & \int d^4\theta \left( \frac{\Lambda_{CFT}}{M_{mess}} \right)^{\alpha_q} Z_q^{-1}(\mu_R) c_{B_\mu}^q \frac{q^\dagger q}{M^2} H_u H_d + h.c., \\
 & \int d^4\theta \left( \frac{\Lambda_{CFT}}{M_{mess}} \right)^{\alpha_s} Z_s^{-1}(\mu_R) c_{B_\mu}^s \frac{S^\dagger S}{M^2} H_u H_d + h.c.,
 \end{aligned}$$

And those proportional to either  $S$  or  $S^\dagger$  are

$$O_\lambda : \int d^4\theta Z_s^{-1/2}(\mu_R) c_\lambda^s \frac{S}{M} W^{a\alpha} W_\alpha^a + h.c.,$$

$$O_A : \int d^4\theta Z_s^{-1/2}(\mu_R) c_A^s \frac{S}{M} \phi^\dagger \phi + h.c.,$$

$$O_\mu : \int d^4\theta Z_s^{-1/2}(\mu_R) c_\mu^s \frac{S^\dagger}{M} H_u H_d + h.c.,$$

## Mixing between the operators

As mentioned in the previous slide, the mixing between operators is important in this case.

- Between the quadratic: This appears to be an anomalous dimension of the goldstino multiplet, and give a suppression effect to the quadratic operators in the boundary conditions.
- Between the linear operator and the quadratic: This give corrections to the boundary condition such as non-zero trilinear coupling and the soft scalar masses.

As a result, we can get

$$\delta m_{H_{u,d}}^2 = -\mu^2$$

$$\delta A = -\mu$$

$$(\text{suppression factor}) = \left( \frac{\Lambda_{CFT}}{M_{mess}} \right)^\alpha$$

## Basic things

- Accept the basic idea of the solution.
- The RG effects of hidden sectors appear as the boundary condition.
- Under the 'effective' messenger scale, the RG evaluation is that of MSSM.
- Make a use of `softsusy` to get MSSM RG running.

# Our strategy 1

- Set the scale where the hidden sectors are integrated out as  $\Lambda_{CFT}$ , i.e. 'effective' messenger scale as  $10^8\text{GeV}$ .
- Set the messenger scale as  $10^{14}\text{GeV}$ .
- At  $\Lambda_{CFT}$  the scalar masses are suppressed as  $16\pi^2$ .
- The sign of  $\mu$  is positive.
- Set the universal trilinear coupling to satisfy  $A = -\mu$ .
- Set the correction to higgs mass as  $\delta m_{H_{u,d}} = -\mu^2$ .
- Set the gravity contribution as zero.
- Set  $m_t = 170.9\text{GeV}$ .
- Set free parameters as  $\tan\beta$  and  $\frac{F}{M_{mess}}$ .
- Scan  $\tan\beta$  from 4 to 50 and  $\frac{F}{M_{mess}}$  from  $5.0 \times 10^4\text{GeV}$  to  $2.0 \times 10^5\text{GeV}$  for case of 1 messenger.

# The low energy spectra

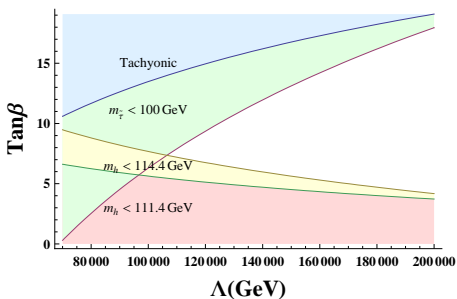


Figure: Plot of forbidden region in  $16\pi^2$  suppression case, with  $A = -\mu$ ,  $\Lambda_{CFT} = 10^8 \text{ GeV}$  and the number of messengers = 1.

## Our strategy 2

- We choose the parameter section which survives the low energy constraints.
- Once we get the values of  $\mu$  and  $B_\mu$  which make the proper EWSB at the electroweak scale.
- We trace them back to the 'effective' messenger scale which is denoted as  $\Lambda_{CFT}$ .
- We compare the suppression factor obtained by this process with that of the boundary condition.
- We define  $\rho$  as follows.

$$\rho = \frac{B'_\mu \times (\text{suppression factor})}{\mu \times \Lambda}$$



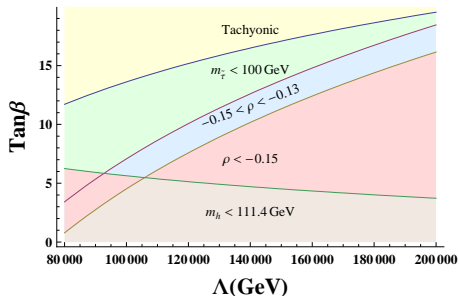


Figure: The set up is the same as Fig. 1 except that the yellow part is excluded and we provide the ratio  $\rho$  between the theoretical prediction and the result of the trace-back RG.

## 'A sign problem?'

Some questions may arise in this situation.

- Is this an illusion at the 'effective messenger scale'?
- Is this originated from the rather small 'effective messenger scale'?
- What will happen if  $B_\mu$  carries different sign to  $\mu$ ?

Is this an illusion at the 'effective messenger scale'?

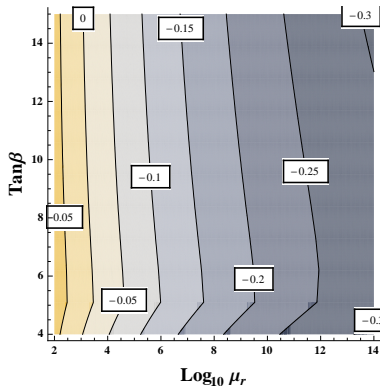


Figure: The RG running of  $B'_\mu$  within the visible sector only. We should note that the value of  $\frac{B'_\mu}{\mu^2}$  over  $\Lambda_{CFT}$  (in this plot  $10^8 GeV$ ).

Is this originated from the rather small 'effective messenger scale'?

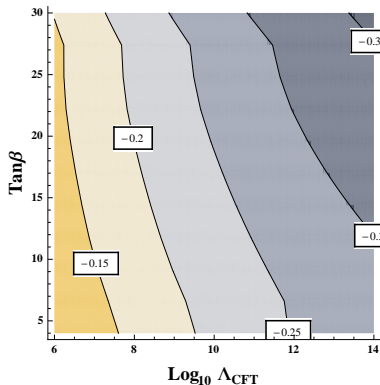


Figure: The effect of varying  $\Lambda_{\text{CFT}}$  scale with  $\Lambda = 1.5 \times 10^5 \text{ GeV}$ . Here we can see that the sign problem is generic in this mechanism. The contours represent the value of  $\frac{B'_\mu}{\mu^2}$ .

What will happen if  $B_\mu$  carries a different sign to  $\mu$ ?

From the symmetric property,  $\mu$  and  $B_\mu$  can have different signs when they are generated, i.e. at the messenger scale. Since the linear operators can be rotated by  $U(1)_R$  symmetry, however,  $A$  and  $M_{\frac{1}{2}}$  can be rotated as well as  $\mu$ . Therefore the boundary conditions change as follows

$$\text{sgn}(\mu) = -1,$$

$$\Lambda = -|\Lambda|.$$

Because  $B_\mu = \Lambda\mu$  should hold,  $B_\mu$  should be positive.

What will happen if  $B_\mu$  carries a different sign to  $\mu$ ?

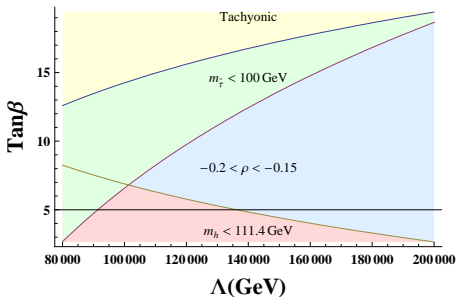
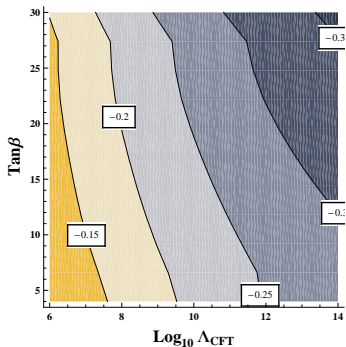
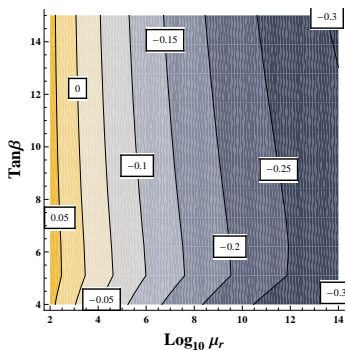


Figure: The case that  $B_\mu$  is generated with a different sign to  $\mu$ .

What will happen if  $B_\mu$  carries a different sign to  $\mu$ ?



# Conclusion

- We studied  $B_\mu/\mu$  solution via hidden sector RG effect by a numerical analysis.
- At the low energy spectrum we get the valid region is small.
- As a consistency test, we used trace back RG evolution.
- As a result we find that there is a sign problem.