UV Completion of Axion

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Axion

SM Lagrangian contains theta angle for QCD.

$$\frac{\theta}{32\pi^2}\epsilon_{\mu\nu\rho\sigma}\mathrm{tr}\,F_{\mu\nu}F_{\rho\sigma} \quad \equiv \theta F^2$$

- P and CP violating.
- Observable $\bar{\theta} \equiv \theta$ + (CP phase of quarks).

Unobserved neutron electric dipole moment implies

$$\bar{\theta} < 10^{-9}$$ So small or zero?! Why?

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Axion [Peccei, Quinn] [Weinberg] [Wilczek] [KSVZ] [DFSZ]

- $\bar{\theta}$ is a dynamical field, $\mathcal{L}_{\rm eff} = \frac{a}{f_a}F^2 + \frac{1}{2}(\partial_\mu a)^2$
- No CP violation except the above:
  Minimum of the potential is CP conserving $\bar{\theta} = 0$.
- Mass $\sim 13/f_a$ MeV, decay constant

$$10^{10}\ \text{GeV} < f_a < 10^{12}\ \text{GeV}.$$

Axion as a Goldstone boson

Consider a global Abelian chiral symmetry

$$\psi_i \longrightarrow e^{i\gamma_5 q^i \alpha}\psi_i, \quad \phi \longrightarrow e^{iq\alpha}\phi,$$

being anomalous,

$$\delta\mathcal{L}_{\rm eff} = 2{\rm tr}\,(qt^a t^a)F^2\alpha.$$

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broken by instanton. ['t Hooft]
If broken also by VEV of a charged scalar field $\langle\phi\rangle = v$, its phase field a, a pseudoscalar Goldstone, has anomalous coupling

$$\mathcal{L}_{\text{eff}} = a\frac{2\text{tr}\,(qt^a t^a)}{qv}F^2 + \frac{1}{2}(\partial_\mu a)^2 + \partial_\mu a\sum q^i\overline{\psi}_i\gamma^\mu\gamma^5\psi_i$$

$$\delta a = qv\alpha$$

Axion is a pseudoscalar Goldstone $f_a = qv$

Many VEVs

Suppose we have identified two Abelian symms. $U(1)_P \times U(1)_Q$ from the potential.

1. Partly broken by $\langle \phi_1 \rangle = v_1$.
   - Unbroken $U(1)'$ direction: under which ϕ_1 is neutral,

     $$q_n^i \equiv q_Q^1 q_P^i - q_P^1 q_Q^i.$$

   - Anomalous coupling of the axion comes from 'broken $U(1)$'?
     cf. any linear combination of global $U(1)$s can be another good $U(1)$.

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     cf. any linear combination of global $U(1)$s can be another good $U(1)$.
2. Completely broken by $\langle \phi_1 \rangle = v_1, \langle \phi_2 \rangle = v_2$.
   - If $\mathrm{tr}\, q_P l \neq 0, \mathrm{tr}\, q_Q l \neq 0$, are there two anomalous $U(1)$s?
     cf. an anomaly free linear combination exists
   - Are anomaly free $U(1)$s irrelevant to axions
   - What is the dominant component of the axion?

These will guide us for embedding axions to UV physics.

- GUT
- string theory

  $\vdots$

One $U(1)$, many VEVs

Many fields developing VEVs $\delta a_1/v_1 = q^1\alpha, \delta a_2/v_2 = q^2\alpha$. It becomes the axion by the linear combination

$$a = \frac{1}{f_a}(a_1 q^1 v_1 + a_2 q^2 v_2),$$

$$\mathcal{L}_{\text{eff}} = a\frac{\operatorname{tr} q^i l^i}{f_a}F^2, \quad \delta a = f_a \alpha$$

Decay constant

$$f_a \equiv \sqrt{(q^1 v_1)^2 + (q^2 v_2)^2}$$

One U(1) broken by many fields, larger VEV dominates.

Invariant direction:

$$\frac{1}{f_a}(a_1 q^2 v_2 - a_2 q^1 v_1)$$

What if we introduce more $U(1)$?

Partly broken

$U(1)_P \times U(1)_Q$ broken by $\langle \phi_1 \rangle = v$,

- Unbroken $U(1)'$ direction: under which ϕ_1 is neutral,

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Axion term? 'Broken direction'??

$$\frac{\mathrm{tr}\,(q_P l)}{q_P^1} \neq \frac{\mathrm{tr}\,(q_Q l)}{q_Q^1}\ ?$$

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- Be gauged away from the residual $U(1)'$, $e^{\mathrm{tr}\,(q_n l)\alpha i} = e^{-\mathrm{tr}\,(q_P l) i / q_P^1}$.
- No axion term generated.

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If $U(1)'$ is not anomalous,

- $\frac{\mathrm{tr}\,(q_P l)}{q_P^1} = \frac{\mathrm{tr}\,(q_Q l)}{q_Q^1} - \frac{\mathrm{tr}\,(q_n l)}{q_P^1 q_Q^1}$ is the unique coefficient of a generated axion term.

Axion term generation depends on the charge q_1,
vacuum configuration

Completely broken

$U(1)_P \times U(1)_Q$ completely broken by $\langle\phi_1\rangle = v_1, \langle\phi_2\rangle = v_2$.
The variation $\delta a_1/v_1 = q_P^1\alpha + q_Q^1\beta, \delta a_2/v_2 = q_P^2\alpha + q_Q^2\beta$ must reproduce the anomaly $2(\mathrm{tr}\, q_P l\alpha + \mathrm{tr}\, q_Q l\beta)F^2$.

$$\mathcal{L}_{\rm eff} = \left(a_1\frac{c_1}{v_1} + a_2\frac{c_2}{v_2}\right)F^2,$$

$$c_1 = \mathrm{tr}\,[(q_P^i q_Q^2 - q_Q^i q_P^2)l^i]/(q_P^1 q_Q^2 - q_Q^1 q_P^2),$$
$$c_2 = \mathrm{tr}\,[(q_P^1 q_Q^i - q_Q^1 q_P^i)l^i]/(q_P^1 q_Q^2 - q_Q^1 q_P^2).$$

Axion term depends on $\{q_P^i, q_Q^i, v_i\}$: vacuum configuration

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$$\mathcal{L}_{\text{eff}} = \left(-a_1 \frac{q_f^2}{v_1} + a_2 \frac{q_f^1}{v_2} \right) \frac{1}{q_P^1 q_Q^2 - q_Q^1 q_P^2} F^2.$$

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Axion term depends on q_f^1, q_f^2, v_1, v_2: vacuum configuration

Interpretation: we always have anomaly free direction

$$q_f^i = q_P^i \mathrm{tr}\,(q_Q l) - q_Q^i \mathrm{tr}\,(q_P l).$$

The axion is the unique, orthonormal combination to Goldstone boson $\propto q_f^1 v_1 a_1 + q_f^2 v_2 a_2$.
The decay constant is

$$M = \left((c_1/v_1)^2 + (c_2/v_2)^2 \right)^{-1/2} < \min(v_1/c_1, v_2/c_2),$$

More U(1) broken, smaller VEV dominates.

Dominant component

r U(1)s broken by n scalar field VEVs:

- $r-1$ anomaly free symmetries, and as many Goldstone bosons
- $n-r$ invariant combinations
- 1 axion combination:

the unique orthonormal combination to the others.

We can show

Decay constant is dominated by $(n-r+1)$th largest VEV.

Hirarchy is possible.

- Higher energy scale physics, low scale axion decay constant.

$$10^9 \text{ GeV} < M < 10^{12} \text{ GeV},$$

- The original PQ: Two Higgs model $v_1^2 + v_2^2 = 246$ GeV.

Light pseudoscalars

Light pseudoscalars

- axion $\propto -a_1 \frac{q_f^2}{v_1} + a_2 \frac{q_f^1}{v_2}$ couples to F^2,
- Goldstone $\propto q_f^1 v_1 a_1 + q_f^2 v_2 a_2$,

are orthonormal combinations.
No light pseudoscalar has been observed. The Goldstone boson must be either

1. eaten by gauge boson
   of anomaly free symmetry, e.g. from GUT,
2. couple to another hidden sector gauge field,
3. 'universality.'

'Anomalous' $U(1)$

Another source of axion 'anomalous' $U(1)_a$ [Atick, Dixon, Sen] [Dine, Seiberg, Witten] [Dine, Ichinose, Seiberg]

- Antisymmetric tensor: dual to axion $\epsilon_{\mu\nu\rho\sigma}\partial_\rho B_{\mu\nu} = \partial_\sigma a_0$.

$$K = -M_{Pl}^2 \ln(S + \bar{S} - \frac{1}{6}\mathrm{tr}\, q_a V), \quad S = 1/g^2 + ia_0/8\pi f_a$$

- $\mathrm{tr}\, q_a \neq 0$, generating nontrivial Fayet–Illiopoulos term.
- Universality $\frac{1}{12}\mathrm{tr}\, q_a = \frac{1}{2}\mathrm{tr}\,_{G_a} q_a l = \frac{1}{3}\mathrm{tr}\, q_a^3 = \ldots$
- Anomaly from gauge $U(1)$ is canceled by transformation

$$S \to S + i\mathrm{tr}\, q_a \theta.$$

- Natural in string theories with Green–Schwarz mechanism.

$$f_a \sim M_{Pl}/100$$

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- Natural in string theories with Green–Schwarz mechanism.

$$f_a \sim M_{Pl}/100$$

If this $U(1)_a$ is again broken by VEV $\langle\phi\rangle = v$, again its phase contributes a_1,

$$\left(\frac{a_0}{f_a} + \frac{a_1}{qv}\right) F^2$$

whose decay constant is

$$M = (f_a^{-2} + (qv)^{-2})^{-1/2} < \min(f_a, qv)$$

May obtain smaller axion coupling?

D-term constraint

Because of SUSY, D-term constraint

$$D_a = q_a^1 |\langle \phi_1 \rangle|^2 + \xi_{\rm FI} = 0, \quad \xi_{\rm FI} = \frac{g M_{Pl}^2}{192\pi^2} {\rm tr}\, q_a$$

The VEV must be of the same order as $\sqrt{\xi_{\rm FI}} \sim M_{Pl}/100$.
Introducing more VEV does not improve it. The largest dominates.

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If we have more global $U(1)$ broken, we can have the axion coupling

$$M = (f_a^{-2} + M_1^{-2} + M_2^{-2})^{-1/2}, \quad M_i \propto q_i v_i$$

and

$$D_a = q_a^1 |\langle \phi_1 \rangle|^2 + q_a^2 |\langle \phi_2 \rangle|^2 + \xi_{\rm FI} = 0.$$

The smallest dominant, we can have much smaller VEV, evading D-term constraint.

Axion from heterotic orbifold

Heterotic string compactification on orbifolds.

- Large symmetries predicted by heterotic string: gauge group, SUSY...
- Break them by associating symmetries of orbifolds.
- Promising route to MSSM: gauge group, chiral fermions... axion.
- Axion from anomalous $U(1)$ natural.

Other symmetries?

- String theory predicts no global symmetry.
- Global symmetry broken by quantum gravity. [Krauss, Wilczek]

Accidental symmetries at a given orders of perturbation. [Lazarides, Panagiotakopoulos, Shafi]

Superpotential at up to order $\sim$ 7-10 we have a number of accidental anomalous $U(1)$s. [KSC, IW Kim, JE Kim], [KSC, Nilles, Ramos-Sanchez, Vaudrevange]

We can have smaller ($\ll M_{Pl}$) axion decay constat.

Conclusions

We have analyzed axions from multiple anomalous global $U(1)$s.

- Axion depends on charges and VEVs: vacuum configuration.
- r $U(1)$s broken by n fields: $(n - r + 1)$th largest VEV dominates.
- Axion is the unique orthonormal component of anomaly free combinations of $U(1)$s.
- Guide the embeddability to GUT or string

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String theory

- contains ‘anomalous’ $U(1)_a$.
- plus accidental $U(1)$s at some given order of superpotential.
- We can obtain lower axion decay constant than the string scale.
- Evade D-term constraint to have lower scale axion.