Non-perturbative moduli superpotential with positive exponents

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based on
Abe, Higaki, T.K., hep-th/0511160
Abe, Higaki, T.K., Seto, 0804.3229[hep-th]
1 Introduction

Superstring theory is a promising candidate for unified theory including gravity. Superstring theory has several moduli fields, i.e. dilaton $S$, Kahler moduli $T$ and complex structure moduli $U$. Moduli correspond to the size and shape of compact space.

$\langle \rangle$ of moduli fields

$\rightarrow$ couplings in low-energy effective theory,

e.g. gauge and Yukawa couplings

Hence, it is important to stabilize moduli $\langle \rangle$ at realistic values.
Moduli-stabilizing potential

(llocal) minimum of potential
  $\Rightarrow$ supersymmetry breaking
  (a specific pattern of SUSY breaking terms)
Moduli potential is also important
  from the cosmological viewpoint,
    e.g. inflation.

Thus, it is important to study moduli
  stabilization from the viewpoint of
  particle physics as well as cosmology
Actually, lots of works have been done so far.
Non-perturbative effect

We usually consider non-perturbative terms like

\[ W_{np} = Ae^{-aT}, \quad A = O(M_P^3), \quad a > 0 \]

Scalar potential \( V \)

- low bump \( \ll M_P^4 \)
- runaway
- overshooting problem
- realization of inflation
- destabilization by finite temperature effects
Non-perturbative effect

Non-perturbative terms like

\[ W_{np} = A e^{aT}, \quad a > 0 \]

can be realized.

We study implications of such non-perturbative terms with positive exponents.
2. Non-perturbative moduli superpotential with positive exponents

Our scenario is based on supergravity, which could be derived from type IIB string. (Our supergravity model might be derived from other strings.)

Flux compactification, Giddings, Kachru, Polchinski, ‘01

The dilaton $S$ and complex structure moduli $U$ are assumed to be stabilized by the flux-induced superpotential

$$W_{\text{flux}} (S, U)$$

That implies that $S$ and $U$ have heavy masses of $O(M_p)$. The Kaher moduli $T$ remain not stabilized.
Non-perturbative moduli superpotential

SU(Na) super Yang-Mills theory
gauge kinetic function $f_a$

For example, when $f_a = T$
like the gauge sector on D7-brane,
the gaugino condensation induces

$$W_{np} = Ae^{-2\pi f_a / Na}$$

That is the well-known form of non-perturbative terms.
Moduli mixing in gauge coupling

In several string models, gauge kinetic function $f$ is given by a linear combination of two or more fields.

Weakly coupled hetero. /heterotic M

$$f = S \pm \beta T$$

Similarly,

IIA intersecting D-branes/IIB magnetized D-branes

$$f = \pm mS \pm wT$$

Gaugino condensation $\Rightarrow \exp[-a f]$  
Moduli mixing superpotential

Lust, et. al. ’04
Positive exponent

Note that S is already stabilized by a heavy mass.

So, we replace S by its VEV.

\[ f_a = m_a S + w_a T \]

\[ W_{np} = A \exp[-2\pi(m_a S_0 + w_a T)/N_a] = A' e^{-aT} \]

\[ f_a = m_a S - w_a T \]

\[ W_{np} = A \exp[-2\pi(m_a S_0 - w_a T)/N_a] = A' e^{aT} \]

non-perturbative superpotential with positive exponent
Positive exponent

Scalar potential $V$

$f_a = m_a S - w_a T$

strong coupling

$f_a < O(1)$

$f_a = 0$

reliable height of potential barrier

$f_a \approx 1$

$W_{np} \approx A \exp[-2\pi / N_a] = O(M_P^3)$

$V \approx |A|^2 e^{-4\pi / N_a} = O(M_P^4)$
3. Potential forms and implications

Let us study the following superpotential

\[ W_{total} = W_0 + \sum A_a e^{-2\pi(m_a S_0 + w_a T)/N_a} + \sum A_b e^{-2\pi(m_b S_0 - w_b T)/N_b} \]

F-term scalar potential

\[ V_F = e^K \left[ D_T W (\overline{D_T W}) K^{T\bar{T}} - 3 |W|^2 \right] \]

\[ D_T W = K_T W + W_T \]

\[ K = -3 \ln(T + \bar{T}) \]

Total scalar potential

\[ V = V_F + \frac{E}{(T + \bar{T})^n} \]

We tune E such that \( V = 0 \) at one of minima.
3-1. W with a single term

\[ W_{\text{total}} = Ae^{-2\pi (mS - wT) / N} \]

This is one of the simplest models to stabilize moduli.

For example, \( n=2 \)

\[ 4\pi w \text{Re}(T) = 5N \]

Similar results for \( n=3 \)

\[ W \text{ is R-symmetric.} \]

- global SUSY: Nelson, Seiberg, ‘94
- supergravity: Abe, T.K., Omura

\[ \frac{F^T}{T + \overline{T}} = \frac{2}{3} m^{3/2} \]

\( \text{Im}(T) \) is still flat.
3.2 W with two terms

**KKLT type**

\[ W_{total} = W_0 + Ae^{-(mS+nT)}, \]

\[ W_0 = W_{flux} \quad \text{or} \quad e^{-kS} \]

**Racetrack type**

\[ W_{total} = A_1 e^{-2 \pi (m_1S + w_1 T) / N_1} + A_2 e^{-2 \pi (m_2S + w_2 T) / N_2} \]

These are well-known.
Cosmology

Height of bump is determined by gravitino mass \[ \approx m_{3/2}^2 M_P^2 \]

Overshooting problem

Inflation?

Destabilization due to finite temperature effects

\[ \Delta V = (\alpha_0 + \alpha_2 g^2)\hat{T}^4 \]

\[ \Delta V = [\alpha_0 + \alpha_2 / (mS + wT)]\hat{T}^4 \]

Brustein, Steinhardt, ’93

Buchmuller, et. al. ‘04
New models

Abe, Higaki, T.K., ‘05

KKLT-like model

\[ W = W_0 + e^{-8\pi^2 (m_b S - w_b T) / N_b} \]

\[ \Delta V = \left[ \alpha_0 + \alpha_2 \left/ (mS - wT) \right. \right] T^4 \]

Racetrack-like model

\[ W = e^{-8\pi^2 (m_a S + w_a T) / N_b} + e^{-8\pi^2 (m_b S - w_b T) / N_b} \]

The above problems may be avoided.
3.3 Application: racetrack inflation

\[ W = W_0 + \sum_{a=1,2} A_a e^{-2\pi (m.a.S_0 + w_a T) / N_a} + A_3 e^{-2\pi (m_3 S_0 - w_3 T) / N_3} \]

When \( A_3 = 0 \), this corresponds to the superpotential of racetrack inflation. Blanco-Pillado, et al, ‘04

\[ \frac{N_1}{w_1} = 100, \quad \frac{N_2}{w_2} = 90, \quad W_0 = -1/25000 \]
\[ A_1 e^{-2\pi m_1 S_0 / N_1} = 1/50, \quad A_2 e^{-2\pi m_2 S_0 / N_2} = -35/1000 \]

\[ \varepsilon = \frac{M_P^2}{2V^2} \left( \frac{dV}{d\phi} \right)^2, \quad \eta = \frac{M_P^2}{V} \frac{d^2V}{d\phi^2} \]

slow-roll inflation
around the saddle point
Racetrack inflation

Slow roll parameters and e-folding

\[ \epsilon = 0, \quad \eta = -0.006097 \]

\[ N = 130 \]
Racetrack inflation

\[ W = W_0 + \sum_{a=1,2} A_a e^{-2\pi (m_a S_0 + w_a T) / N_a} + A_3 e^{-2\pi (m_3 S_0 - w_3 T) / N_3} \]

\[ A_3 = 1, \quad m_3 S_0 = 68.8 \pi, \]
\[ w_3 = 1, \quad N_3 = 20 \]

slow-roll inflation
around the saddle point
Racetrack inflation

Slow roll parameters and e-folding

\[ \varepsilon = 0, \quad \eta = -0.006850 \]

\[ N = 130 \]
Summary

We have studied moduli superpotential with positive exponents.

Scalar potential has a high barrier of the Planck scale.

This has significant implications from the viewpoints of cosmology and particle phenomenology, e.g. realization of inflation models, avoiding the overshooting problem, etc.

Further studies would be important.