Curvature perturbation spectrum from false vacuum inflation

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Based on

- JG and M. Sasaki, arXiv:0804.4488[astro-ph]

Outline

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2. Two-point Correlation functions
   - Inflaton field 2-point correlation function
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Predictions of slow-roll inflation

Scale invariant spectrum $\mathcal{P}_{\mathcal{R}}$

1. One of the greatest triumphs of inflation
2. Naturally generated
3. Confirmed by recent observations e.g. WMAP5: $n_{\mathcal{R}} \approx 0.96$

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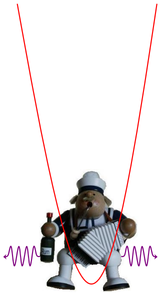
Inflation = nearly scale invariant $\mathscr{P}_{\mathscr{R}}$: **NOT necessarily true**

e.g. false vacuum inflation

$\mathcal{R}_c$ during false vacuum inflation



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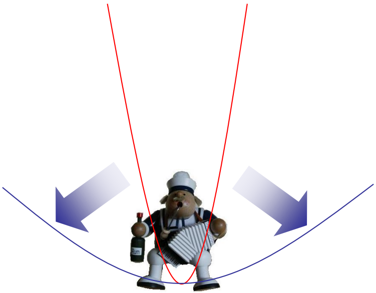

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$$m_{\rm eff}^2 = m_\phi^2 + \frac{\mu^2}{a^2} = m_{\rm eff}(t)$$

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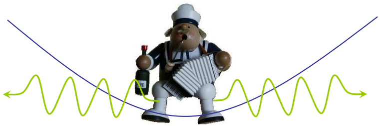

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Breaking the perfect dS phase

- He can go home for more rum

How can we proceed?

Situation is OK, but the method is inadequate

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Full quantum computation goes as:

1. Calculate the inflaton 2-point correlation function $G(x,x') = \langle\phi(x)\phi(x')\rangle$
2. Calculate the energy density 2-point correlation function $D(x,x') \sim \langle\delta\rho(x)\delta\rho(x')\rangle/\rho^2 \sim D[G(x,x')]$
3. Calculate $\mathscr{P}_\Phi$ using $\nabla^2\Phi \sim \delta\rho/\rho$
4. Calculate $\mathscr{P}_\mathscr{R}$ using $\Phi \sim \int \mathscr{R}_c d\eta$

Inflaton field 2-point correlation function

Given

$$V = V_0 + \frac{1}{2}\left(m_\phi^2 + \frac{\mu^2}{a^2}\right)\phi^2$$

$$\begin{aligned} G(x,x') &= \langle \phi(x)\phi(x') \rangle \\ &= \left(\frac{H}{2\pi}\right)^2 \int_0^\infty ds \cosh(\nu s) \frac{1 + p\sqrt{2\cosh s - 2(1-u)}}{[2\cosh s - 2(1-u)]^{3/2}} e^{-p\sqrt{2\cosh s - 2(1-u)}} \end{aligned}$$

$$p = \sqrt{\mu^2 \eta\eta'} \quad , u = \frac{r^2 - (\eta - \eta')^2}{2\eta\eta'} \quad , r^2 = |\mathbf{x} - \mathbf{x}'|^2 , \nu^2 = \frac{9}{4} - \frac{m_\phi^2}{H^2}$$

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$$p = \sqrt{\mu^2\eta\eta'} \gg 1\,,\; u = \frac{r^2 - (\eta-\eta')^2}{2\eta\eta'} \gg 1\,,\; r^2 = |\mathbf{x}-\mathbf{x}'|^2\,,\; \nu^2 = \frac{9}{4} - \frac{m_\phi^2}{H^2}$$

Early times

Super-horizon separation

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Early times | Super-horizon separation

$$G(x,x') \approx \left(\frac{H}{2\pi}\right)^2 \sqrt{\frac{\pi}{2}} \frac{\sqrt{\mu}\eta\eta'}{\left[r^2-(\eta-\eta')^2\right]^{3/4}} e^{-\mu\left[r^2-(\eta-\eta')^2\right]^{1/2}}$$

Energy density 2-point correlation function (1/2)

Density perturbation on the comoving hypersurface

$$\nabla^2(\rho\Delta) = \nabla^2\left(-T^0{}_0\right) + 3H\partial^i\left(-T^0{}_i\right)$$

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2-point correlation function

$$\begin{aligned} D(x,x') =& \left\langle \nabla_x^2\left[\rho\Delta(x)\right]\nabla_{x'}^2\left[\rho\Delta(x')\right]\right\rangle \\ =& f_i^{\rho\mu\nu}(t) f_{j'}^{\sigma'\alpha'\beta'}(t)\partial^i\partial^{j'}\left\{\left[\partial_\rho\partial_{\alpha'}\partial_{\beta'}G(x,x')\right]\left[\partial_{\sigma'}\partial_\mu\partial_\nu G(x,x')\right]\right. \\ & \left. + \left[\partial_\rho\partial_{\sigma'}G(x,x')\right]\left[\partial_\mu\partial_\nu\partial_{\alpha'}\partial_{\beta'}G(x,x')\right]\right\} \end{aligned}$$

with the time dependent coefficients

$$\begin{aligned} f_i^{00j} = f_i^{0j0} =& \frac{1}{2}\delta_i{}^j \\ f_i^{j00} =& -\delta_i{}^j \\ f_i^{jkl} =& a^{-2}\left[\delta_i{}^j\delta^{kl} + \frac{1}{2}\left(\delta_i{}^k\delta^{jl} + \delta_i{}^l\delta^{jk}\right)\right] \end{aligned}$$

Energy density 2-point correlation function (2/2)

Useful properties of $G(x,x')$:

1. Function of $r = |\mathbf{x} - \mathbf{x}'|$: $G = G(r)$
2. Anti-symmetric w.r.t. spatial derivative: $\partial_{x'} = -\partial_x$
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$\cdots$ After some calculations, we find that to leading order

$$D(r,\eta) \approx \left(\frac{H}{2\pi}\right)^4 16\pi(H\eta)^4 \frac{(\mu\eta)^4}{(\mu r)^3}\mu^8 e^{-2\mu r}$$

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Φ: gauge invariant curvature perturbation in the Newtonian gauge

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$$\begin{aligned} D(x,x') =& 4m_{\rm Pl}^4(H\eta)^4\left\langle\nabla_x^2\left[\nabla_x^2\Phi(x)\right]\nabla_{x'}^2\left[\nabla_{x'}^2\Phi(x')\right]\right\rangle \\ \approx& 4m_{\rm Pl}^4(H\eta)^4(2\mu)^8\left\langle\Phi(x)\Phi(x')\right\rangle \end{aligned}$$

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Thus the 2-point correlation function of Φ in *configuration space*

$$\begin{aligned} \xi_\Phi(x) \equiv& \langle\Phi(\mathbf{x})\Phi(\mathbf{x}+\mathbf{r})\rangle \\ =& \frac{\pi}{64}\left(\frac{H}{2\pi m_{\rm Pl}}\right)^4\frac{(\mu\eta)^4}{(\mu r)^3}e^{-2\mu r} \end{aligned}$$

Power spectrum of Φ

By inverse Fourier transform

$$\mathscr{P}_\Phi = \frac{k^3}{2\pi^2}\int d^3 r \xi_\Phi(r) e^{-i\mathbf{k}\cdot\mathbf{r}}$$

We have to integrate

$$\int_0^\infty dr \frac{e^{-2\mu r}}{r} j_0(kr)\text{: blows up to infinity at } r = 0$$

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$$\int_0^\infty dr \frac{e^{-2\mu r}}{r} j_0(kr) \to \int_{1/\mu}^\infty dr \frac{e^{-2\mu r}}{r} j_0(kr) \approx -\mathrm{Ei}(-2)$$

$$\mathcal{P}_\Phi(k;\eta) \approx \frac{-\mathrm{Ei}(-2)}{32}\left(\frac{H}{2\pi m_{\mathrm{Pl}}}\right)^4 (\mu\eta)^4 \left(\frac{k}{\mu}\right)^3$$

$\mathscr{R}_c$ and Φ

On super-horizon scales the general solution for Φ

$$\Phi = \frac{3}{2} C_1 \frac{\mathscr{H}}{a^2} \int_{\eta_i}^{\eta} (1+w) a^2(\eta') d\eta'$$

Given Φ, $\mathscr{R}_c$ is expressed as

$$\mathscr{R}_c = \frac{2\Phi' + (5+3w)\mathscr{H}\Phi}{3(1+w)\mathscr{H}}$$

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1. a: perfect dS expansion $a = -1/(H\eta)$
2. $1+w$: we need to evaluate $\langle \rho + p \rangle =$???

$\langle \rho + p \rangle$ during dS stage

$$\langle \rho + p \rangle = \left\langle \dot{\phi}^2 + \frac{(\nabla\phi)^2}{3a^2} \right\rangle$$

We take into account···

- Fourier mode expansion

$$\phi(x) = \int \frac{d^3k}{(2\pi)^{3/2}} \left[a_{\mathbf{k}} \phi_k(\eta) e^{i\mathbf{k}\cdot\mathbf{x}} + a^\dagger_{\mathbf{k}} \phi^*_k(\eta) e^{-i\mathbf{k}\cdot\mathbf{x}} \right]$$

- Cutoff at a large physical momentum, $k/a \leq (k/a)_c = H\Lambda$
- Evaluate in the limit $\eta \to -\infty$

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We subtract the dS invariant contribution: $\mu = 0$

$$\langle \rho + p \rangle_{\text{ren}} = \lim_{\Lambda \gg 1} \left[\langle \rho + p \rangle(\Lambda, \mu) - \langle \rho + p \rangle(\Lambda, 0) \right]$$

$$= A \frac{H^4}{16\pi^2} \frac{\left| m^2_\phi \right|}{H^2} (\mu\eta)^2 \ ; \qquad A = \mathcal{O}(1) > 0$$

Power spectrum of $\mathscr{R}_c$

$$1 + w = \frac{A}{48\pi^2} \frac{\left|m_\phi^2\right|}{m_{\rm Pl}^2} (\mu\eta)^2$$

We can explicitly evaluate the integral

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On large scales,

$$\begin{aligned} \mathscr{P}_{\mathscr{R}} &= \frac{\mathscr{P}_\Phi(k;\eta)}{\kappa^2(\mu\eta)^4} \\ &\approx \frac{-2{\rm Ei}(-2)}{A^2} \left(\frac{H^2}{m_\phi^2}\right)^2 \left(\frac{k}{\mu}\right)^3 , \end{aligned}$$

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$$n_{\mathscr{R}} = 4$$

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Conclusions

1. NOT all inflation models incorporate slowly rolling inflaton field
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   - Difficulty: comoving hypersurfaces are not well defined
2. We have calculated the power spectrum $\mathcal{P}_{\mathcal{R}}$ and the spectral index $n_{\mathcal{R}}$
   - Purely quantum field theory approach: regularizing mass μ
   - Highly scale dependent

$$\mathcal{P}_{\mathcal{R}} \sim \mathcal{O}(0.1) \left(\frac{H^2}{m_\phi^2} \right)^2 \left(\frac{k}{\mu} \right)^3$$

$$n_{\mathcal{R}} = 4$$