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# 3-loop corrections to the lightest Higgs boson mass in SUSY-QCD

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Universität Karlsruhe

in collaboration with

R. Harlander, P. Kant, M. Steinhauser

# Motivation

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Experiment    LHC:  $\delta m_h^{\text{exp}} = 100 - 200 \text{ MeV}$     and    ILC:  $\delta m_h^{\text{exp}} = 50 \text{ MeV}$

Theory: Higgs sector of the MSSM perturbatively calculable ( $m_h \leq 135 \text{ GeV}$ )

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- exact 1-loop [Chankowski, Pokorski and Rosiek '92], [Brignole '92], [Dabelstein '94]
- 2-loop  $\mathcal{O}(\alpha_t \alpha_s, \alpha_t^2, \alpha_b \alpha_s, \alpha_b \alpha_t)$  in effective potential approximation ( $p^2 = 0$ )  
[Haber, Hempfling, Hoang '96], [Heinemeyer, Hollik and Weiglein '98], [Degrassi, Slavich, Zwirner '01], [Espinosa and Zang '00], [Brignole, Degrassi, Slavich, Zwirner '02], [Carena et al '00], [Heinemeyer et al '05], [S. Martin '03]
- Momentum-dependent corrections ( $p^2 = m_h^2$ ): 2-loop SUSY-QCD [S.Martin '05]
- 3-loop LL and NLL  $\mathcal{O}(\alpha_t \alpha_s^2, \alpha_t^2 \alpha_s, \alpha_t^3)$  [S. Martin '07]

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- 3-loop LL and NLL  $\mathcal{O}(\alpha_t \alpha_s^2, \alpha_t^2 \alpha_s, \alpha_t^3)$  [S. Martin '07]
- Missing contributions:  $\delta m_h^{\text{th}} \simeq 3 - 5 \text{ GeV}$  [G. Degrassi et al '02], [Allanach et al '04]
  - full 2-loop corrections
  - full 3-loop SUSY-QCD corrections

# Framework

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MSSM:

$$V_0 = m_1^2 H_1 \bar{H}_1 + m_2^2 H_2 \bar{H}_2 - m_{12}^2 (\epsilon_{ab} H_1^a H_2^b + \text{h.c.}) \\ + \frac{g'^2 + g^2}{8} (H_1 \bar{H}_1 - H_2 \bar{H}_2)^2 + \frac{g^2}{2} |H_1 \bar{H}_2|^2$$

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$m_1, m_2, M_{12}$ : soft SUSY breaking terms

$g, g'$ :  $SU(2)$  and  $U(1)$  gauge couplings

$$\epsilon_{12} = -1$$

$$H_1 = \begin{pmatrix} H_1^1 \\ H_1^2 \end{pmatrix} = \begin{pmatrix} v_1 + (\phi_1^0 + i\chi_1^0)/\sqrt{2} \\ \phi_1^- \end{pmatrix}$$
$$H_2 = \begin{pmatrix} H_2^1 \\ H_2^2 \end{pmatrix} = \begin{pmatrix} \phi_2^+ \\ v_2 + (\phi_2^0 + i\chi_2^0)/\sqrt{2} \end{pmatrix}$$

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Additional parameters:  $\tan \beta = v_2/v_1$ ,  $M_A^2 = -m_{12}^2(\tan \beta + \cot \beta)$

$$\mathcal{M}_{H,\text{tree}}^2 = \frac{\sin 2\beta}{2} \times \begin{pmatrix} M_Z^2 \cot \beta + M_A^2 \tan \beta & -M_Z^2 - M_A^2 \\ -M_Z^2 - M_A^2 & M_Z^2 \tan \beta + M_A^2 \cot \beta \end{pmatrix}$$

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Higher order corrections

$$\mathcal{M}_H^2 = \mathcal{M}_{H,\text{tree}}^2 - \begin{pmatrix} \hat{\Sigma}_{\phi_1} & \hat{\Sigma}_{\phi_1\phi_2} \\ \hat{\Sigma}_{\phi_1\phi_2} & \hat{\Sigma}_{\phi_2} \end{pmatrix}$$

$\hat{\Sigma}_{\phi_i}$  = renormalized self-energies



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$V_{\text{eff}}$ -approximation:  $p^2 = 0 \Rightarrow \hat{\Sigma}_i(0) = \Sigma_i(0) - \delta V_i$

$\Sigma_i(0)$  = bare self-energies

$\delta V_i$  = Higgs potential counterterms

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Approximations:

- $\mathcal{O}(\alpha_t \alpha_s^2)$  corrections  $\rightsquigarrow \mathcal{O}(M_{\text{top}}^4)$
- no-mixing in the top-sector  $\rightsquigarrow M_{\tilde{t}_1} = M_{\tilde{t}_2}$
- $A_t$ -contributions neglected  $\rightsquigarrow A_t = 0$

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Computation of  $\hat{\Sigma}_{\phi_2}(0)$  at 3-loops:

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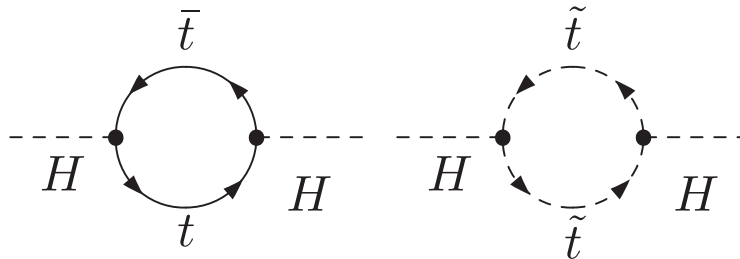
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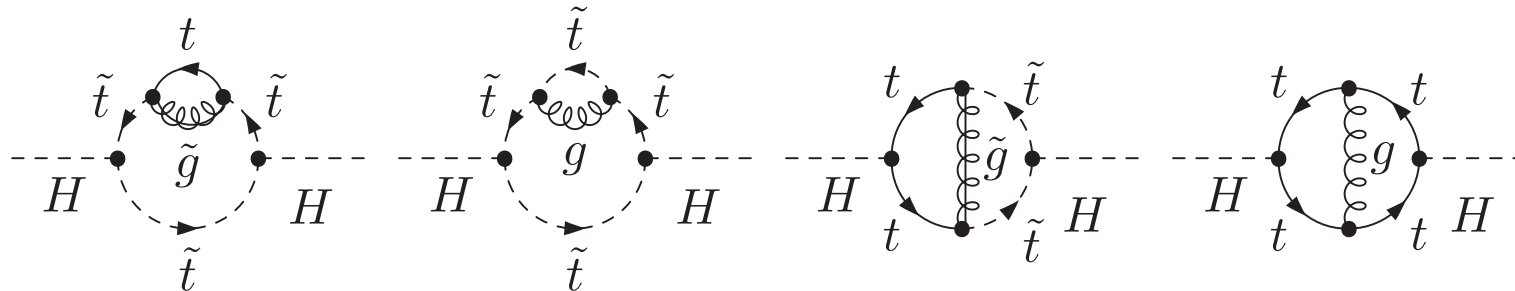
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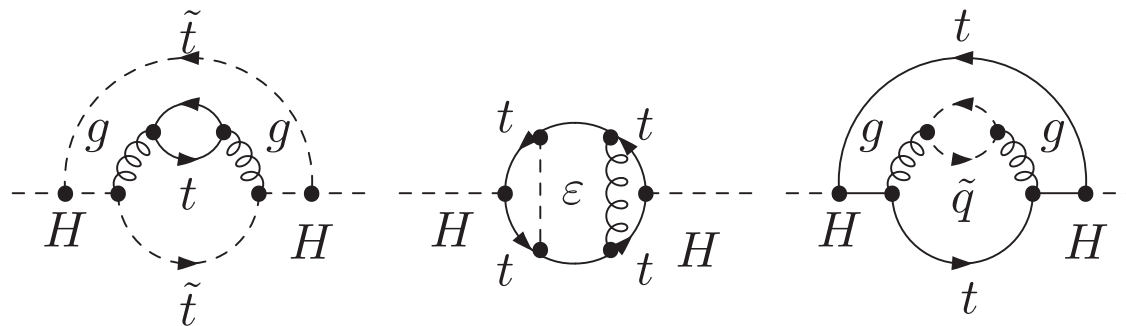
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- $\simeq$  **16.000** diagrams
- Computer programs: QGRAF, PERL, FORM, MINCER, MATAD, EXP, ...  
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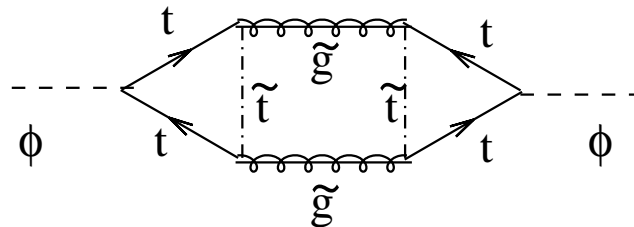
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- Asymptotic expansion  $\rightsquigarrow$  3-loop **tadpole** integrals  $\rightsquigarrow$  MATAD

# Regularization and Renormalization

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- Regularization: Dimensional Reduction  $\rightsquigarrow$   $\epsilon$ -scalars
  - anti-commuting  $\gamma_5$
- Renormalization
  - $\alpha_s$  in  $\overline{\text{DR}}$  - scheme to 1-loop
  - $M_t, M_{\tilde{t}_1}, M_{\tilde{t}_2}$  in OS - scheme to 2-loops  
[Bednyakov, Onishchenko, Velizhanin and Veretin '02], [S. Martin '03,'05]  
re-computed for specific mass hierarchies
  - $M_{\tilde{g}}$  in OS - scheme to 1-loop [Pierce, Bagger, Matchev and Zahng '96]
  - $M_\epsilon$  in OS - scheme to 1-loop [Bednyakov, Onishchenko, Velizhanin and Veretin '02]  
 $M_\epsilon = 0 \Leftrightarrow \overline{\text{DR}'}$
  - $\theta_{\tilde{t}}$  in OS - scheme to 1-loop [Pierce, Bagger, Matchev and Zahng '96]
  - $A_t \rightsquigarrow 2M_t A_t = (M_{\tilde{t}_1}^2 - M_{\tilde{t}_2}^2) \sin 2\theta_t + 2M_t \mu_{\text{SUSY}} \cot \beta$

# Results

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- Cross Checks

- exact 2-loop results: agreement with [\[FeynHiggs\]](#), [\[Degrassi, Slavich, Zwirner '01\]](#)

- 2- and 3-loop results: gauge independent

- SUSY-limit (  $M_t = M_{\tilde{t}}$ ,  $M_{\tilde{g}} = M_q = M_{\tilde{q}} = 0$  ) :  $\delta M_h^{(3)} = 0$

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$$\begin{aligned} \hat{\Sigma}_{\phi_2} = & \frac{3G_F M_t^4}{\sqrt{2}\pi^2 \sin^2 \beta} \left\{ L_{tS} + \frac{\alpha_s}{\pi} [-4L_{tS} + 2L_{tS}^2] + \left(\frac{\alpha_s}{\pi}\right)^2 \left[ \frac{671}{324} + \frac{1}{27}\pi^2 + \frac{1}{9}\zeta_3 \right. \right. \\ & + \left( -\frac{1591}{108} - 3L_{\mu t} + \frac{1}{3}\pi^2 - \frac{4}{9}\pi^2 \ln 2 + \frac{55}{18}L_{t\tilde{q}} + \frac{5}{6}L_{t\tilde{q}}^2 \right) L_{tS} \\ & + \left( \frac{13}{18} + \frac{3}{2}L_{\mu t} - \frac{5}{3}L_{t\tilde{q}} \right) L_{tS}^2 + \frac{53}{18}L_{tS}^3 \\ & \left. \left. + \left( \frac{475}{108} - \frac{5}{9}\pi^2 \right) L_{t\tilde{q}} - \frac{25}{36}L_{t\tilde{q}}^2 - \frac{5}{18}L_{t\tilde{q}}^3 + \mathcal{O}\left(\frac{M_{\text{SUSY}}^2}{M_{\tilde{q}}^2}\right) \right] \right\} \end{aligned}$$

$$L_{\mu t} = \ln(\mu^2/M_t^2), \quad L_{tS} = \ln(M_t^2/M_{\text{SUSY}}^2), \quad L_{t\tilde{q}} = \ln(M_t^2/M_{\tilde{q}}^2)$$

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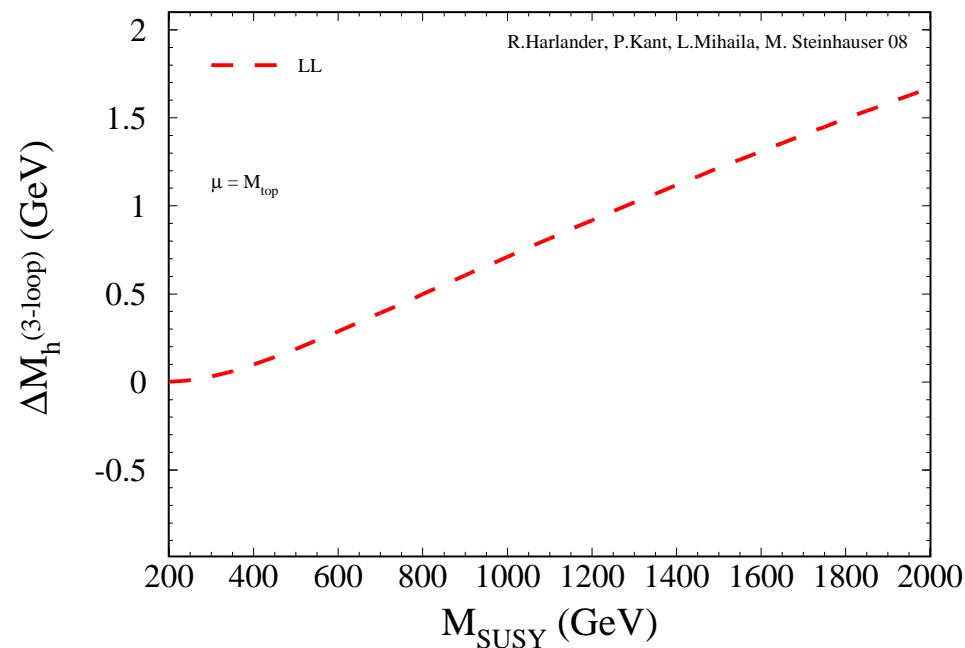
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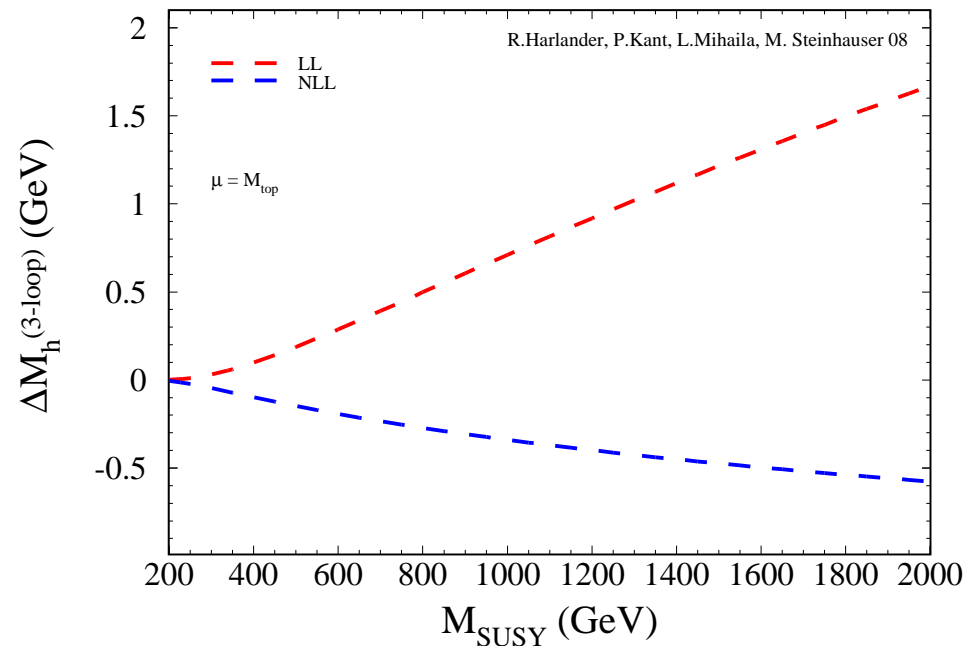
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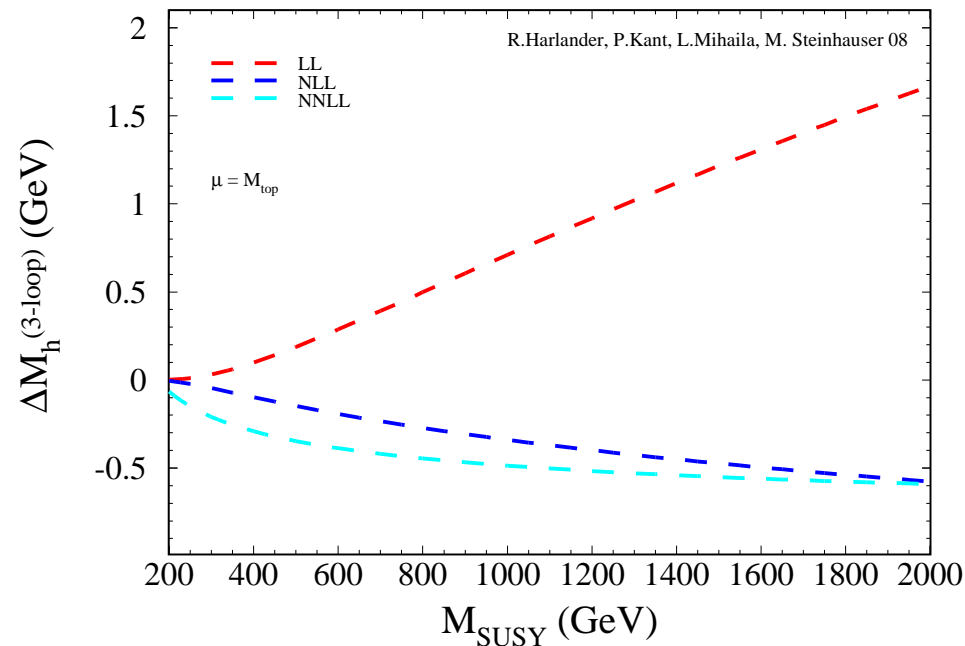
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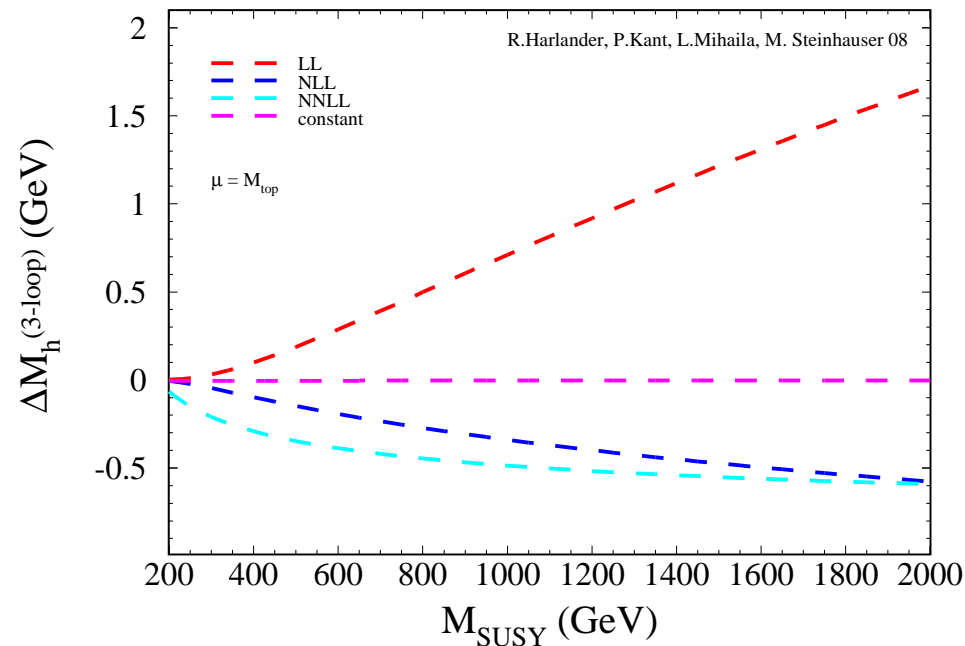
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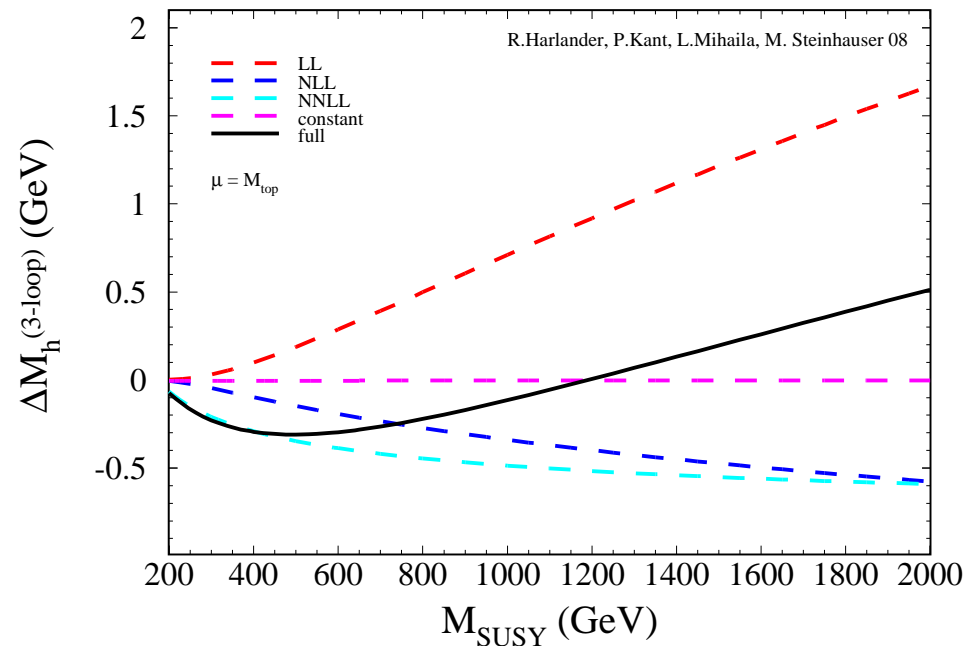
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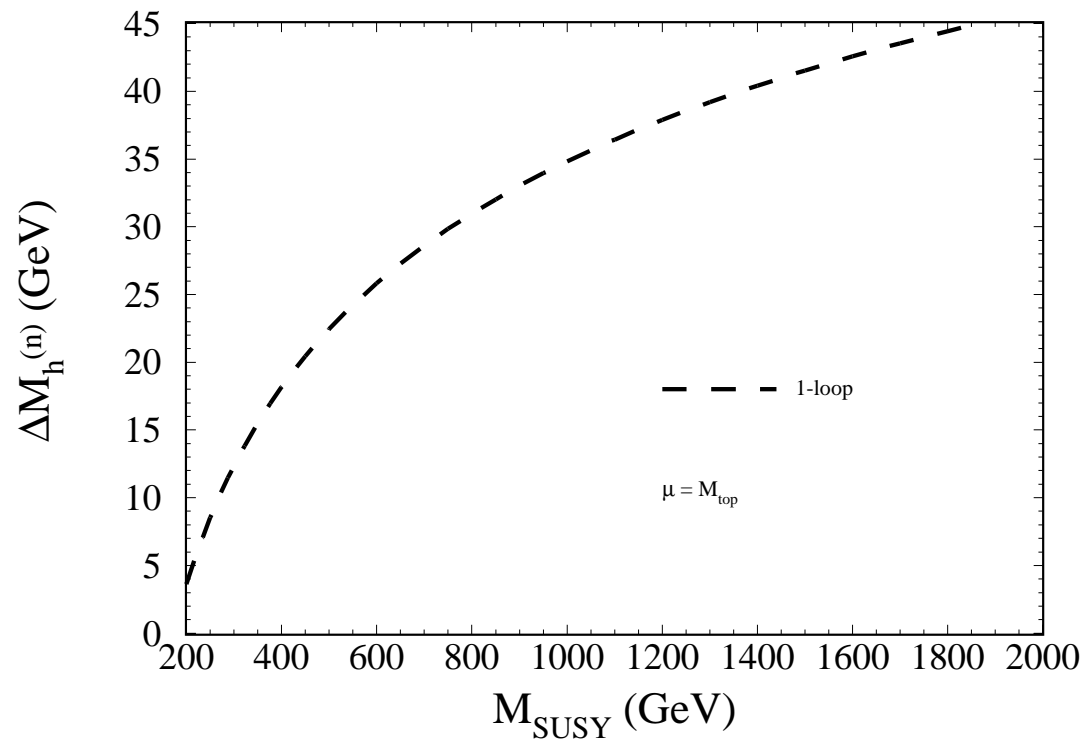
$$\begin{array}{llll} \mu = M_t = 170.9 \text{ GeV} & G_F & = & 1.16637 \times 10^{-5} \text{ GeV}^{-2} \\ M_Z = 91.1876 \text{ GeV} & \alpha_s^5(M_Z) = 0.1189 & \Rightarrow & \alpha_s(M_t) = 0.0926 \\ M_A = 1 \text{ TeV} & \tan \beta = 40 & & M_{\tilde{q}} = 2 \text{ TeV} \end{array}$$

$$\Delta M_h^{(n)} \equiv M_h^{(n\text{-loop})} - M_h^{\text{tree}}$$

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Input parameters:  $\mu = M_t = 170.9 \text{ GeV}$   $G_F = 1.16637 \times 10^{-5} \text{ GeV}^{-2}$   
 $M_Z = 91.1876 \text{ GeV}$   $\alpha_s^5(M_Z) = 0.1189 \Rightarrow \alpha_s(M_t) = 0.0926$   
 $M_A = 1 \text{ TeV}$   $\tan \beta = 40$   $M_{\tilde{q}} = 2 \text{ TeV}$

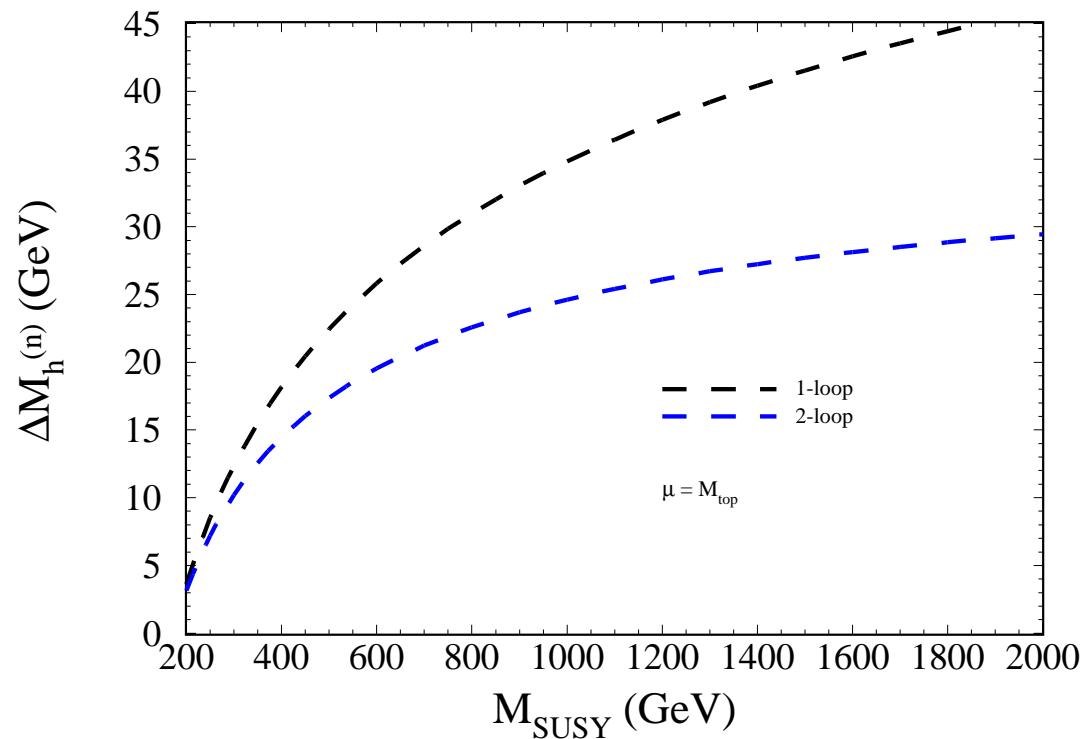
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Input parameters:  $\mu = M_t = 170.9 \text{ GeV}$   $G_F = 1.16637 \times 10^{-5} \text{ GeV}^{-2}$   
 $M_Z = 91.1876 \text{ GeV}$   $\alpha_s^5(M_Z) = 0.1189 \Rightarrow \alpha_s(M_t) = 0.0926$   
 $M_A = 1 \text{ TeV}$   $\tan \beta = 40$   $M_{\tilde{q}} = 2 \text{ TeV}$

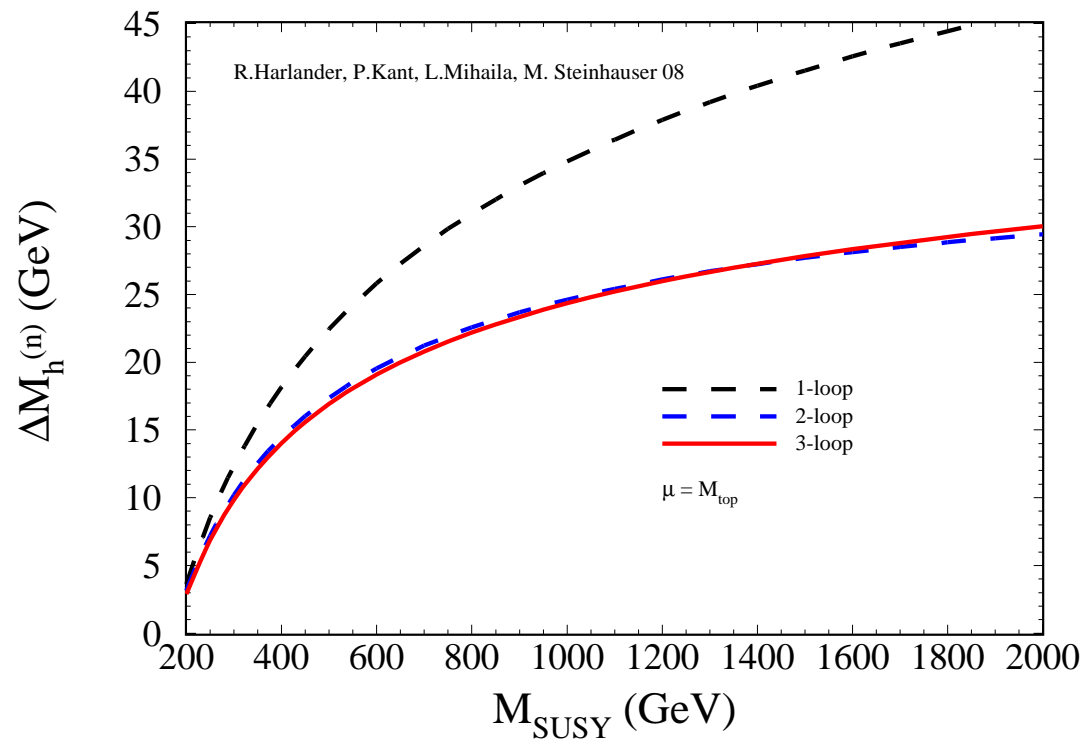
$$\Delta M_h^{(n)} \equiv M_h^{(n\text{-loop})} - M_h^{\text{tree}}$$



# Numerical Results

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# Numerical Results

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Input parameters:

$$\begin{array}{llll} \mu = M_t = 170.9 \text{ GeV} & G_F & = & 1.16637 \times 10^{-5} \text{ GeV}^{-2} \\ M_Z = 91.1876 \text{ GeV} & \alpha_s(M_Z) = 0.1189 & \Rightarrow & \alpha_s(M_t) = 0.0926 \\ M_A = 1 \text{ TeV} & \tan \beta = 40 & & M_{\tilde{q}} = 2 \text{ TeV} \end{array}$$

$$M_{\text{SUSY}} = 0.3 - 1 \text{ TeV} : \quad \Delta M_h^{(3)} \simeq \Delta M_h^{(2)} \pm 500 \text{ MeV}$$

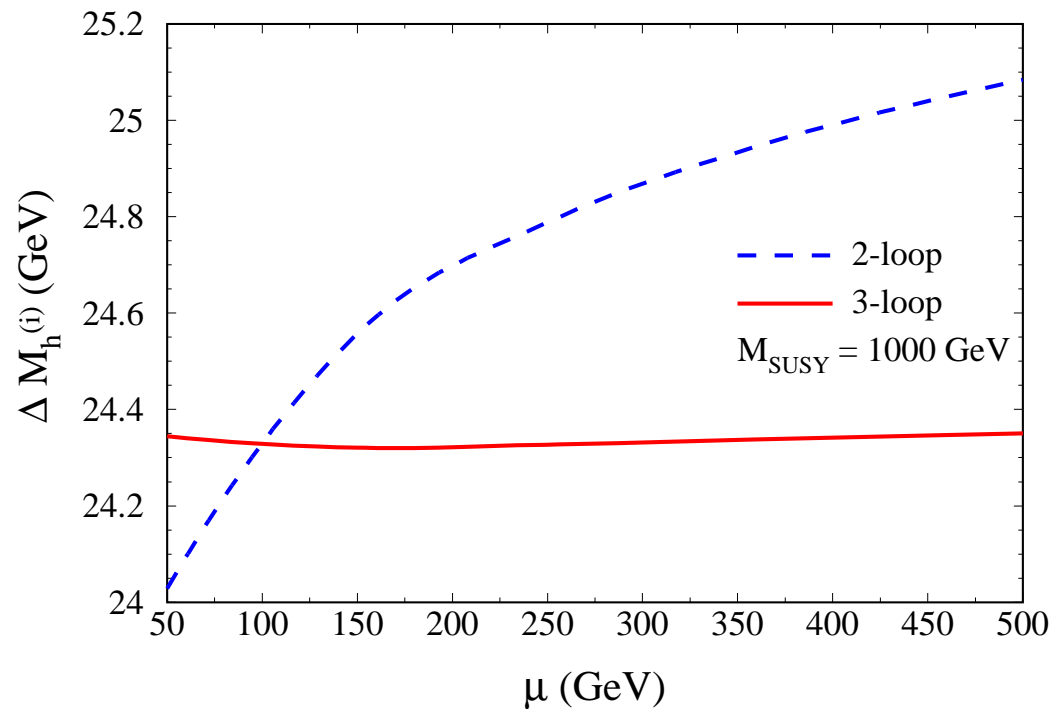
# Numerical Results

Input parameters:

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$$M_{\text{SUSY}} = 0.3 - 1 \text{ TeV} : \quad \Delta M_h^{(3)} \simeq \Delta M_h^{(2)} \pm 500 \text{ MeV}$$

Renormalization scale dependence



# Conclusions

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- $m_h$  to 3-loop accuracy
  - 3-loop effects larger than experimental accuracy expected at LHC & ILC
  - 3-loop corrections stabilize the perturbative series
  
- ToDo:
  - Effects induced by the stop-quark mixing
  - Effects due to  $A_t \neq 0$