Quark and lepton masses at the GUT scale including SUSY threshold corrections

Talk at SUSY08, Seoul, South Korea, June 19th, 2008

based on arXiv:0805.0325,
in collaboration with Martin Spinrath

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Motivation

- One important information for constructing GUT models of flavour: Yukawa couplings of quarks and leptons at $M_{GUT} \approx 2 \times 10^{16}$ GeV
- SUSY threshold corrections have been ignored in previous works which provide this information (e.g.: Fusaoka, Koide ('98); Xing, Zhang, Zhou ('07))
  ... although it is well known that they can have large impact
- Goal of our study (S.A., Spinrath ('08)):
  Possible GUT scale ranges of quark and lepton Yukawa couplings (and of their ratios) including the (tan β enhanced) SUSY threshold effects?

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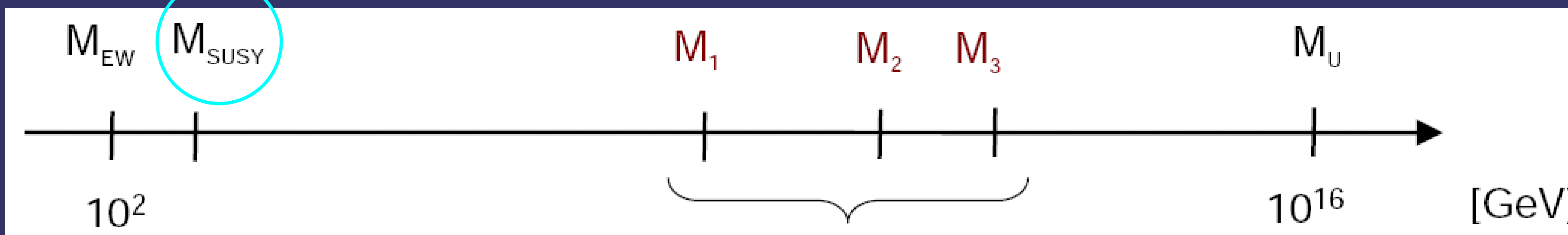
Outline

- Motivation
- Technical details (diagrams for tan β enhanced corrections, formulae, relevant SUSY parameters)
- Analysis & results (how threshold corrections modify the GUT scale values of the Yukawa couplings; new allowed ranges, consequences)
- Summary and conclusions

From low energy to M_{GUT}

- RG Running from low energies to M_{GUT}, matching at thresholds



SUSY thresholds
(here: one scale matching)

Right-handed neutrino
thresholds

GUT thresholds

In the large tanβ regime of the MSSM: tanβ enhanced threshold corrections dominate the threshold effects (large for Y_d and Y_e)

- Matching condition for down-type quarks and charged leptons @ the SUSY scale (1 TeV in most shown examples)

$$y_i^{\rm MSSM} = \frac{y_i^{\rm SM}}{\cos\beta\,(1+\epsilon_i \tan\beta)}$$

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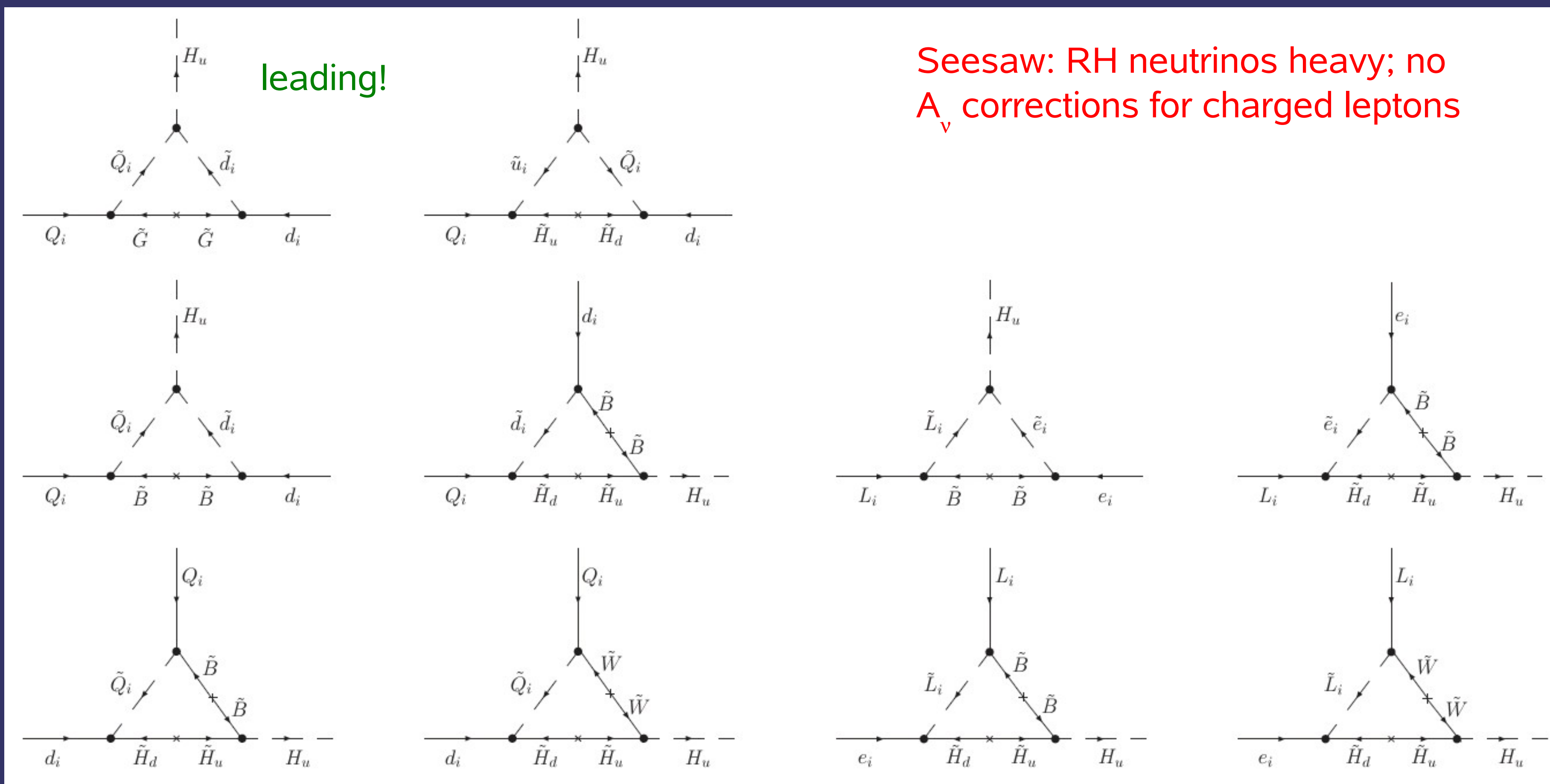
Diagrams for tanβ enhanced corrections

Down-type quarks | Charged leptons



- Coupling at 1-loop to H_u $\rightarrow$ tan β enhanced (in large tan β regime)

SUSY threshold corrections for the down-type quaks

- Formulae for down-type quarks: Decomposed $\epsilon_i = \epsilon_i^G + \epsilon_i^B + \epsilon_i^W + \epsilon^y \delta_{ib}$

$$\epsilon_i^G = -\frac{2\alpha_S}{3\pi} \frac{\mu}{M_3} H_2(u_{\tilde{Q}_i}, u_{\tilde{d}_i}) \, ,$$

$$\epsilon_i^B = \frac{1}{16\pi^2} \left[\frac{g'^2}{6} \frac{M_1}{\mu} \Big(H_2(v_{\tilde{Q}_i}, x_1) + 2H_2(v_{\tilde{d}_i}, x_1) \Big) + \frac{g'^2}{9} \frac{\mu}{M_1} H_2(w_{\tilde{Q}_i}, w_{\tilde{d}_i}) \right]$$

$$\epsilon_i^W = \frac{1}{16\pi^2} \frac{3g^2}{2} \frac{M_2}{\mu} H_2(v_{\tilde{Q}_i}, x_2) \, ,$$

$$\epsilon^y = -\frac{y_t^2}{16\pi^2} \frac{A_t}{\mu} H_2(v_{\tilde{Q}_3}, v_{\tilde{u}_3}) \, ,$$

see e.g.: Carena et al. ('99), Buras et al. ('02), Freitas et al. ('07)

$$u_{\tilde{f}} = m_{\tilde{f}}^2 / M_3^2, \; v_{\tilde{f}} = m_{\tilde{f}}^2 / \mu^2, \; w_{\tilde{f}} = m_{\tilde{f}}^2 / M_1^2, \; x_1 = M_1^2 / \mu^2 \text{ and } x_2 = M_2^2 / \mu^2$$

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SUSY threshold corrections for the down-type quaks

➲ Formulae for charged leptons: Decomposed as $\epsilon_i = \epsilon_i^B + \epsilon_i^W$

$$
\begin{aligned}
\epsilon_i^B &= \frac{1}{16\pi^2}\left[\frac{g'^2}{2}\frac{M_1}{\mu}\left(-H_2(v_{\tilde{L}_i}, x_1) + 2H_2(v_{\tilde{e}_i}, x_1)\right) - g'^2\frac{\mu}{M_1}H_2(w_{\tilde{L}_i}, w_{\tilde{e}_i})\right] \\
\epsilon_i^W &= \frac{1}{16\pi^2}\frac{3g^2}{2}\frac{M_2}{\mu}H_2(v_{\tilde{L}_i}, x_2)\,.
\end{aligned}
$$

$$H_2(x,y) = \frac{x\ln x}{(1-x)(x-y)} + \frac{y\ln y}{(1-y)(y-x)}$$

(negative for x, y > 0)

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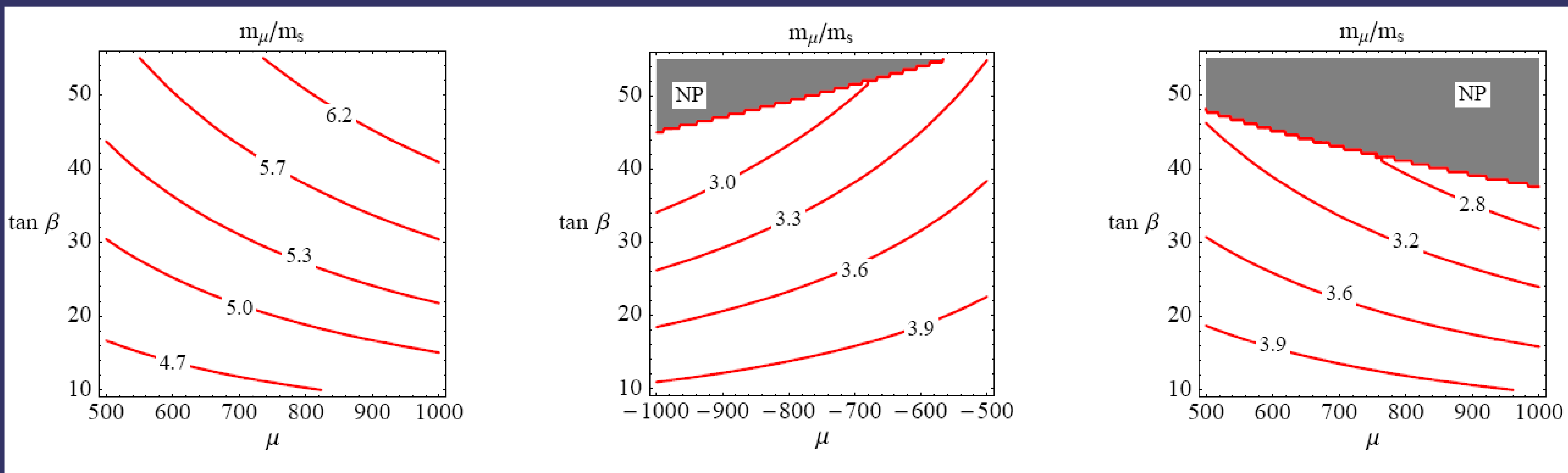
Dependence on μ and tan β: Example m_μ/m_s

Example: Soft masses ~1 TeV



$M_3 > 0, \mu > 0$ | **$M_3 > 0, \mu < 0$** | **$M_3 < 0, \mu > 0$**

(remark: potential conflict with $(g-2)_\mu$)

- Effects larger for larger tan β and larger |μ| (leading $\varepsilon_i^G \sim \mu/M_3$)
- Sign of μM_3 (sign of ε_i^G) "+" or "-": down-type quark Yukawas ↓ or ↑
- SUSY EW contributions ($\varepsilon_i^B + \varepsilon_i^W$) important: 2nd vs. 3rd diagram

Example ranges of SUSY parameters

- Gaugino masses and μ particularly important: 3 example ranges for SUSY parameters at the matching scale M_{SUSY} (taken = 1 TeV)

SUSY parameter	case g_+	case g_-	case a
$m_{\tilde{f}}$ in TeV	[0.5, 1.5]	[0.5, 1.5]	[0.5, 1.5]
M_1 in TeV	[0.5, 1]	[0.5, 1]	[1.65, 3.3]
M_2 in TeV	[1, 2]	[1, 2]	[0.5, 1]
M_3 in TeV	[3, 6]	[3, 6]	[-9, -4.5]
μ in TeV	0.5	-0.5	0.5
A_t in TeV	± 1	± 1	± 1
M_{SUSY} in TeV	1	1	1

"g_+" and "g_-" are inspired by gaugino unification: $M_1 : M_2 : M_3 \approx 1 : 2 : 6$

"a" is inspired by anomaly mediated SUSY breaking: $M_1 : M_2 : M_3 \approx 3.3 : 1 : -9$

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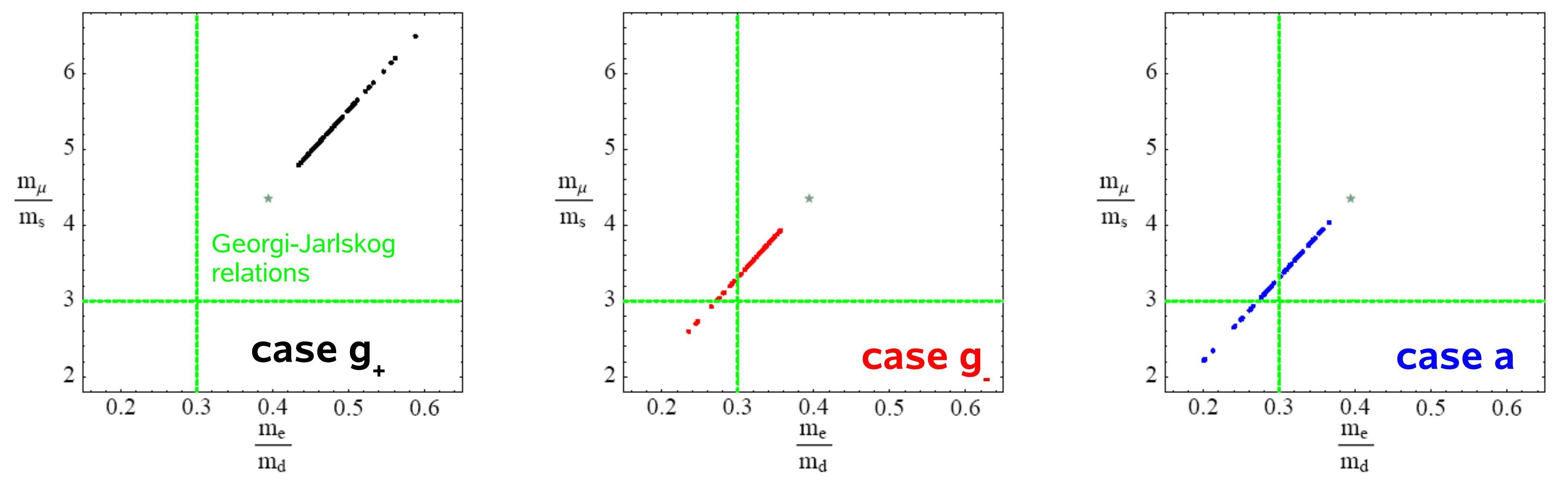
SUSY threshold effects: m_μ/m_s & m_e/m_d

experimental quark errors not yet included; no phen. constraints applied (requires complete model)



scatter plot in order to illustrate the ranges; green star: without SUSY threshold corrections

- SUSY threshold corrections can have large effect on GUT scale values of the ratios
- Georgi-Jarlskog relations $m_\mu = 3\, m_s$ and $m_e = 1/3\, m_d$ (Georgi, Jarlskog ('79)) achievable for g_- and a, but disfavoured for case g_+ (see also: Ross, Serna ('07))
- Alternative GUT scale relations can be realised

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Formulae for simple analytical estimates of GUT scale ratios

- Approximation: Use "0th-order" for Yukawas + Thresholds

$$\frac{m_e(M_{\rm GUT})}{m_d(M_{\rm GUT})} \approx \frac{\hat{m}_e(M_{\rm GUT})}{\hat{m}_d(M_{\rm GUT})} \frac{1+\epsilon_d \tan\beta}{1+\epsilon_e \tan\beta} = \frac{\hat{m}_e(M_{\rm GUT})}{\hat{m}_d(M_{\rm GUT})} (1 + (\epsilon_d - \epsilon_e)\tan\beta) + \mathcal{O}(\epsilon_e^2 \tan^2\beta)$$

GUT scale Yukawa without SUSY thresholds:

	$\hat{y}_e$ in 10^{-4}	$\hat{y}_\mu$ in 10^{-2}	y_τ
$\tan\beta = 30$	0.62	1.31	0.23
$\tan\beta = 40$	0.88	1.85	0.34
$\tan\beta = 50$	1.21	2.55	0.51

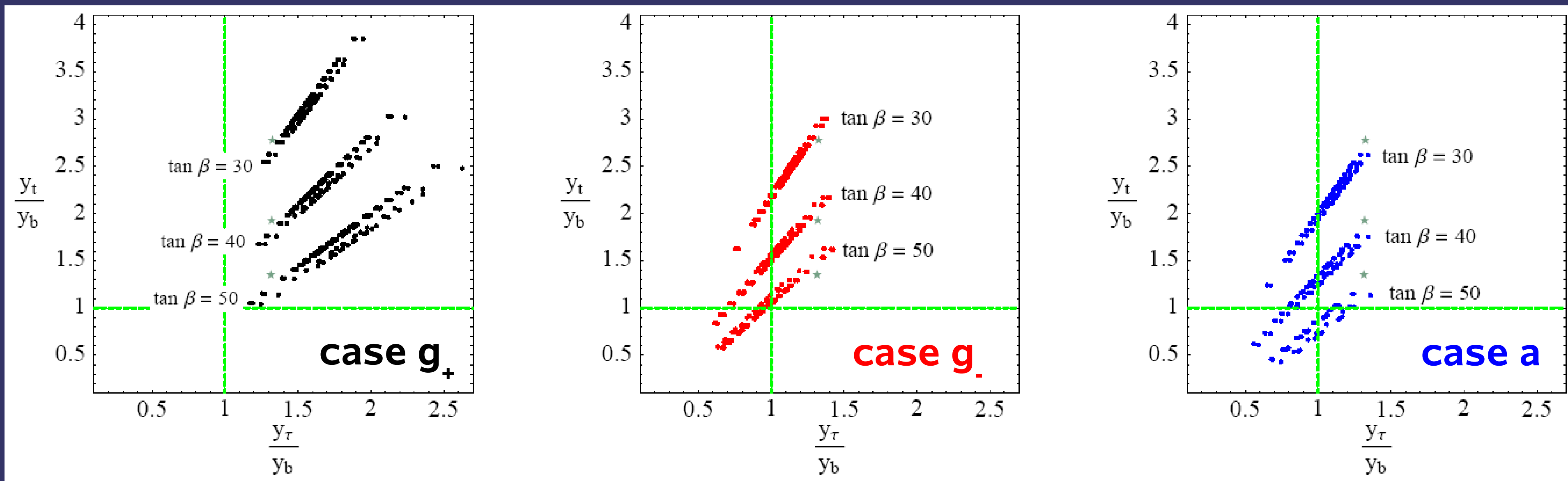
	$\hat{y}_d$ in 10^{-4}	$\hat{y}_s$ in 10^{-2}	$\hat{y}_b$
$\tan\beta = 30$	1.57	0.30	0.18
$\tan\beta = 40$	2.22	0.43	0.26
$\tan\beta = 50$	3.06	0.59	0.39

	$\hat{y}_u$ in 10^{-6}	$\hat{y}_c$ in 10^{-4}	$\hat{y}_t$
$\tan\beta = 30$	2.73	1.33	0.49
$\tan\beta = 40$	2.75	1.34	0.50
$\tan\beta = 50$	2.77	1.35	0.52

Note: For the presented results, we have of course done a full numerical analysis!

SUSY threshold effects: y_t/y_b & y_τ/y_b

exp. quark errors not yet included



scatter plot in order to illustrate the ranges; green star: 3rd family Yukawa unification

- Again, the SUSY threshold corrections are seen to have large effect on the GUT scale ratios
- 3rd family Yukawa unification seems difficult for g_+, but achievable for the other two cases g_- and a

(see also: Carena et al. ('94), Hempfling ('94), Bagger et al. ('96), King, Oliveira ('00), Blazek et al. ('02), Altmannshofer et al. ('08))

- Alternative GUT scale relations can be realised

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Final results: i) Ranges for GUT scale ratios

Case 0: no thresholds; In all cases: quark errors included

	ratio	case 0	case g_+	case g_-	case a
$\tan\beta = 30$	m_e/m_d	$[0.28,\ 0.67]$	$[0.30,\ 0.86]$	$[0.21,\ 0.61]$	$[0.20,\ 0.62]$
	m_μ/m_s	$[3.39,\ 6.07]$	$[3.73,\ 7.79]$	$[2.54,\ 5.49]$	$[2.40,\ 5.63]$
	y_τ/y_b	$[1.27,\ 1.38]$	$[1.20,\ 2.02]$	$[0.71,\ 1.43]$	$[0.60,\ 1.39]$
	y_t/y_b	$[2.56,\ 3.02]$	$[2.36,\ 4.19]$	$[1.50,\ 3.28]$	$[1.14,\ 2.87]$
$\tan\beta = 40$	m_e/m_d	$[0.28,\ 0.67]$	$[0.31,\ 0.93]$	$[0.19,\ 0.59]$	$[0.17,\ 0.60]$
	m_μ/m_s	$[3.39,\ 6.07]$	$[3.85,\ 8.41]$	$[2.28,\ 5.30]$	$[2.07,\ 5.47]$
	y_τ/y_b	$[1.26,\ 1.38]$	$[1.16,\ 2.32]$	≤ 1.46	≤ 1.41
	y_t/y_b	$[1.77,\ 2.11]$	$[1.55,\ 3.31]$	≤ 2.38	≤ 1.94
$\tan\beta = 50$	m_e/m_d	$[0.28,\ 0.67]$	$[0.32,\ 1.00]$	$[0.16,\ 0.57]$	$[0.14,\ 0.59]$
	m_μ/m_s	$[3.39,\ 6.07]$	$[3.98,\ 9.06]$	$[2.02,\ 5.12]$	$[1.72,\ 5.31]$
	y_τ/y_b	$[1.25,\ 1.38]$	$[1.08,\ 2.73]$	≤ 1.49	≤ 1.43
	y_t/y_b	$[1.22,\ 1.50]$	$[0.94,\ 2.74]$	≤ 1.81	≤ 1.31

➲ Enlarged and shifted ranges for ratios: alternative GUT relations possible due to threshold effects !

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Final results: ii) Ranges for GUT scale Yukawa couplings

	Yukawa	case 0	case g_+	case g_-	case a
$\tan\beta = 30$	y_e in 10^{-4}	0.62	[0.62, 0.67]	[0.58, 0.66]	[0.63, 0.79]
	y_μ in 10^{-2}	[1.30, 1.32]	[1.32, 1.41]	[1.22, 1.40]	[1.34, 1.66]
	y_τ	0.23	[0.23, 0.25]	[0.22, 0.25]	[0.24, 0.30]
	y_d in 10^{-4}	[0.92, 2.26]	[0.75, 2.14]	[0.98, 3.11]	[1.06, 3.89]
	y_s in 10^{-2}	[0.21, 0.39]	[0.17, 0.37]	[0.23, 0.53]	[0.25, 0.67]
	y_b	[0.17, 0.18]	[0.12, 0.20]	[0.16, 0.34]	[0.18, 0.48]
	y_u in 10^{-6}	[1.79, 3.88]	[1.78, 3.89]	[1.78, 3.93]	[1.79, 3.98]
	y_c in 10^{-3}	[1.14, 1.54]	[1.13, 1.54]	[1.14, 1.56]	[1.14, 1.58]
	y_t	[0.46, 0.51]	[0.46, 0.52]	[0.46, 0.54]	[0.46, 0.57]
$\tan\beta = 40$	y_e in 10^{-4}	[0.87, 0.88]	[0.86, 0.99]	[0.79, 1.62]	[0.91, 1.77]
	y_μ in 10^{-2}	[1.83, 1.87]	[1.82, 2.08]	[1.67, 3.43]	[1.93, 3.74]
	y_τ	[0.34, 0.35]	[0.34, 0.39]	[0.30, 0.67]	[0.36, 0.76]
	y_d in 10^{-4}	[1.30, 3.21]	[0.97, 3.05]	[1.40, 8.41]	[1.59, 9.81]
	y_s in 10^{-2}	[0.30, 0.55]	[0.23, 0.52]	[0.33, 1.44]	[0.37, 1.69]
	y_b	[0.25, 0.27]	[0.15, 0.33]	≥ 0.22	≥ 0.27
	y_u in 10^{-6}	[1.79, 3.91]	[1.78, 3.92]	[1.79, 4.07]	[1.80, 4.15]
	y_c in 10^{-3}	[1.14, 1.55]	[1.14, 1.56]	[1.14, 1.62]	[1.14, 1.65]
	y_t	[0.47, 0.53]	[0.46, 0.54]	[0.47, 0.65]	[0.48, 0.72]
$\tan\beta = 50$	y_e in 10^{-4}	[1.18, 1.23]	[1.13, 1.53]	[1.04, 2.31]	[1.30, 3.80]
	y_μ in 10^{-2}	[2.50, 2.60]	[2.39, 3.24]	[2.19, 4.89]	[2.74, 8.04]
	y_τ	[0.50, 0.52]	[0.47, 0.69]	[0.42, 1.07]	[0.56, 2.20]
	y_d in 10^{-4}	[1.77, 4.46]	[1.20, 4.56]	[1.91, 12.22]	[2.36, 22.68]
	y_s in 10^{-2}	[0.41, 0.77]	[0.28, 0.78]	[0.44, 2.10]	[0.55, 3.90]
	y_b	[0.36, 0.42]	[0.19, 0.60]	≥ 0.30	≥ 0.43
	y_u in 10^{-6}	[1.81, 3.95]	[1.79, 4.00]	[1.80, 4.18]	[1.81, 4.21]
	y_c in 10^{-3}	[1.15, 1.57]	[1.14, 1.59]	[1.15, 1.66]	[1.16, 1.67]
	y_t	[0.49, 0.56]	[0.46, 0.59]	[0.48, 0.78]	[0.50, 0.86]

➲ ... enlaged (& shifted) ranges for Yukawa couplings: input for GUT model building

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Summary and Conclusions

- SUSY threshold corrections (in large tan β regime) can have a large impact (have to be included, e.g., in GUT model fits to data ...)
- SUSY EW threshold corrections (Binos, Winos) are also important
- Possible GUT scale ranges of the Yukawa couplings are enlarged (with SUSY parameters unknown) and/or shifted (for certain SUSY scenarios) due to these effects. Consequences for GUT models:
  - Proposed GUT scale relations (e.g. Georgi Jarlskog relation) can be realised under certain conditions for SUSY parameters
  - New GUT scale relations become allowed, which may lead to new possibilities for GUT model building ...
- For 3 example ranges of SUSY parameters, we have have provided the GUT scale ranges for the Yukawa couplings and their ratios, including the SUSY threshold effects ...