

B_s mixing phase and lepton flavor violation in supersymmetric SU(5)

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Work in progress

in collaboration with

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SUSY 2008, COEX, 2008-06-19

First evidence of new physics in $b \leftrightarrow s$ transitions

- B_s mixing phase is theoretically clean

$$\phi_s = \arg \langle B_s | \mathcal{H}_{\text{eff}}^{\Delta B=2} | \overline{B}_s \rangle$$

- SM prediction

$$\phi_s^{\text{SM}} \simeq -2\eta\lambda^2 \simeq -0.04$$

- Measurements

$$\phi_s = \begin{cases} -0.57^{+0.24+0.07}_{-0.30-0.02} & \text{DØ, 0802.2255} \\ [-1.36, -0.24] \text{ or } [-2.90, -1.78] & \text{CDF, 0712.2397} \end{cases}$$

- World average

$$\phi_s = -0.73 \pm 0.16 \text{ or } -2.42 \pm 0.14$$

3.7 σ away from ϕ_s^{SM} !

UTfit, 0803.0659

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New sources of flavor & CP violation in MSSM

- Down-type scalar quark mass term:

$$-\mathcal{L}_{\text{soft}} \ni \tilde{d}_{Ai}^* (M_{d,AB}^2)_{ij} \tilde{d}_{Bj}$$

$A, B = L, R$ and $i, j = 1, 2, 3$

- may lead to an interaction

$$\tilde{d}_{Bj} \dots \times \dots \tilde{d}_{Ai} = -i (M_{d,AB}^2)_{ij}$$

if M_d^2 is non-diagonal in the basis where m_d is diagonal and a $d-\tilde{d}-\tilde{g}$ vertex preserves flavor

- For almost diagonal M_d^2 , use a mass insertion parameter

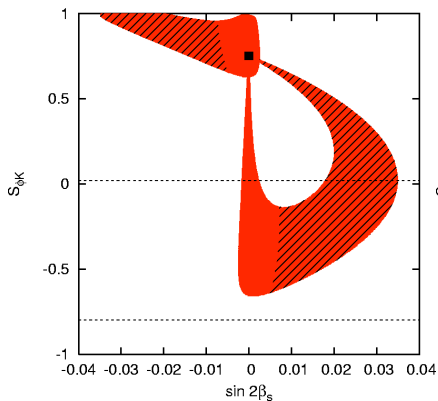
$$\begin{aligned} (\delta_{ij}^d)_{AB} &\equiv (\Delta_{ij}^d)_{AB} / \tilde{m}^2 \\ (M_{d,AB}^2)_{ij} &= \tilde{m}^2 \delta_{AB} \delta_{ij} + (\Delta_{ij}^d)_{AB} \end{aligned}$$

Focus on LL and RR mixings

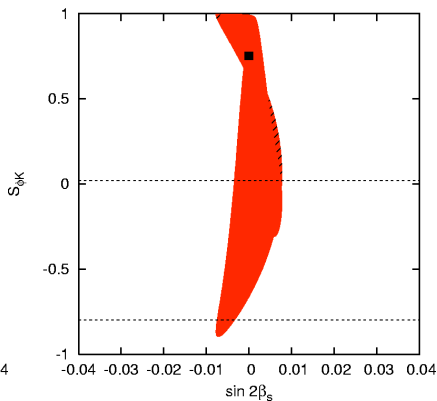
- $B \rightarrow X_s \gamma$ constraints on LR and RL insertions are too strong to make an appreciable difference in ϕ_s

With Kane, Ko, Kolda, Wang $\times 2$, PRL(2003); PRD(2004)

Ciuchini, Franco, Masiero, Silvestrini, PRD(2003)



LR



RL

SUSY GUT and flavor physics

- Superpotential of SUSY SU(5) model

$$W_{\text{GUT}} = T^T \lambda_U T H + T^T \lambda_D \bar{F} \bar{H} + N^T \lambda_N \bar{F} H + N^T M_N N$$

$$T [10] = \{Q, \bar{U}, \bar{E}\}, \quad \bar{F} [\bar{5}] = \{\bar{D}, L\}$$

- Part of W_{GUT} in terms of MSSM fields, in a particular basis

$$W_{\text{SSM}} = Q^T [V_Q^T \hat{Y}_U] \bar{U} H_u + Q^T [\hat{Y}_D] \bar{D} H_d + \bar{E}^T [\hat{Y}_D] L H_d \\ + N^T [\hat{Y}_N V_L] L H_u + N^T M_N N$$

- Soft scalar mass terms

$$-\mathcal{L}_{\text{soft}} \supset \tilde{Q}^\dagger m_{10}^2 \tilde{Q} + \tilde{E}^\dagger m_{10}^2 \tilde{E} + \tilde{U}^\dagger m_{10}^2 \tilde{U} + \tilde{D}^\dagger m_{\bar{5}}^2 \tilde{D} + \tilde{L}^\dagger m_{\bar{5}}^2 \tilde{L}$$

- GUT flavor relations

$$(\delta_{ij}^d)_{LL} = (\delta_{ij}^l)_{RR}^*, \quad (\delta_{ij}^d)_{RR} = (\delta_{ij}^l)_{LL}^* \quad \text{at } M_{\text{GUT}}$$

Grand unification of flavor violations

Ciuchini, Masiero, Silvestrini, Vempati, Vives, PRL(2004)

- Off-diagonal entries of scalar mass matrix are not renormalized very much if between M_{GUT} and M_{SUSY} is a desert

$$(\delta_{ij}^d)_{RR} = (\delta_{ij}^l)_{LL}^* \quad \text{at } M_{\text{GUT}} \quad \Longrightarrow \quad (\delta_{ij}^d)_{RR} \approx \frac{m_L^2}{m_d^2} (\delta_{ij}^l)_{LL}^* \quad \text{at } M_{\text{SUSY}}$$

- Large Y_N may add an additional term to the weak-scale relation
- Naturally expected correlations among flavor transitions

Transitions of L		Transitions of \bar{D}
$\mu \rightarrow e$	\leftrightarrow	$s \rightarrow d$
$\tau \rightarrow e$	\leftrightarrow	$b \rightarrow d$
$\tau \rightarrow \mu$	\leftrightarrow	$b \rightarrow s$

Caveat: $Y_D \neq Y_E^T$ at M_{GUT}

- Fermion mass relations from W_{GUT} ,

$$m_e = m_d, \quad m_\mu = m_s, \quad m_\tau = m_b \quad \text{at } M_{\text{GUT}},$$

are inconsistent with data

- A solution: add non-renormalizable terms
- In the basis where Y_D is diagonal,

$$Y_U = V_Q^T \hat{Y}_U U_Q^*, \quad Y_D = \hat{Y}_D, \quad Y_E = U_L^T \hat{Y}_E U_R^*, \quad Y_N = U_L^T V_L^T \hat{Y}_N$$

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$$\delta_{LL}^l = \delta_{RR}^{d*}, \quad \delta_{RR}^l = \delta_{LL}^{d*}$$

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but don't give up yet!

$Y_D - Y_E^T$ from non-renormalizable operators

- Non-renormalizable terms account for Yukawa difference

$$W_{\text{NR}} \ni \sqrt{2} \left(h_1^{ij} \bar{H}_a \frac{\Sigma_b^a}{M_*} T_i^{bc} \bar{F}_{jc} + h_2^{ij} \bar{H}_a T_i^{ab} \frac{\Sigma_b^c}{M_*} \bar{F}_{jc} \right)$$

$$Y_D - Y_E^T = \xi h_2, \quad \xi \equiv \frac{1}{2} \frac{\langle \Sigma_1^1 \rangle}{M_*} \approx \frac{M_{\text{GUT}}}{M_*} \approx 10^{-2}$$

- In the basis where Y_D is diagonal,

$$\frac{Y_E}{[\widehat{Y}_E]_{33}} \sim \begin{pmatrix} \cos\beta & \cos\beta & \cos\beta \\ \cos\beta & \cos\beta & \cos\beta \\ \cos\beta & \cos\beta & 1 \end{pmatrix} \Rightarrow U_{L,R} \sim \left(\begin{array}{cc|c} [U_{L,R}]_{ab} & & \cos\beta \\ \cos\beta & \cos\beta & 1 \end{array} \right)$$

- 1-3 & 2-3 mixings suppressed by $\cos\beta$
- 1-2 mixing angles in $[U_{L,R}]_{ab}$ unlimited: may be zero, small, large

Baek, Goto, Okada, Okumura, PRD(2001)

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Exploit Planck-suppression of Yukawa difference

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$$(\delta_{a3}^l)_{LL} = [U_L]_{ab} (\delta_{b3}^d)_{RR}^* [U_L]_{33}^* + \mathcal{O}(\cos^2 \beta \delta_{RR}^d), \quad a, b = 1, 2$$

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- 1–2 mixing and quark–lepton FCNC correlation

Transitions of \bar{D}		Transitions of L
$b \rightarrow d$	\longleftrightarrow	$\tau \rightarrow e$
$b \rightarrow s$	\longleftrightarrow	$\tau \rightarrow \mu$



$$\text{if } [U_L]_{ab} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

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- Take the sum of branching fractions

$$\begin{aligned}
 B(\tau \rightarrow (e + \mu)\gamma) &\propto |(\delta_{13}^l)_{LL}|^2 + |(\delta_{23}^l)_{LL}|^2 \\
 &\approx |(\delta_{13}^d)_{RR}|^2 + |(\delta_{23}^d)_{RR}|^2 + \mathcal{O}[\cos^2 \beta (\delta_{RR}^d)^2]
 \end{aligned}$$

Constraint on δ_{RR}^d roughly independent of U_L



- Suppose $U_L = U_R = \mathbf{1}$ for numerical analysis

Exploit Planck-suppression of Yukawa difference

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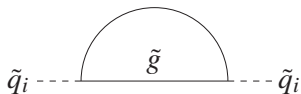
- Suppose $U_L = U_R = \mathbf{1}$ for numerical analysis

RG running of squark mass matrix below M_{GUT}

- Diagonal components

$$[m_{\tilde{q}}^2]_{ii}(M_{\text{SUSY}}) \approx m_0^2 + 6M_{1/2}^2 = (1 + 6x)m_0^2$$

$$x \equiv M_{1/2}^2/m_0^2$$

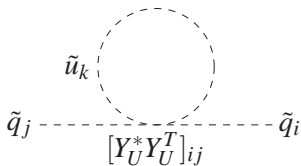


GUT scale version of $m_{\tilde{g}}^2/m_{\tilde{q}}^2$ called x at M_Z

Gabbiani, Gabrielli, Masiero, Silvestrini, NPB(1996)

- Off-diagonal components

$$\Delta[m_{\tilde{Q}}^2]_{ij} \simeq -\frac{6}{(4\pi)^2} [V_Q^\dagger \hat{Y}_U^2 V_Q]_{ij} m_0^2 \ln \frac{M_{\text{GUT}}}{M_{\text{SUSY}}}$$



- Low & high scale mass insertions related by

$$(\delta_{ij}^d)_{LL}(M_{\text{SUSY}}) \approx \frac{(\delta_{ij}^d)_{LL}(M_{\text{GUT}}) + q_{ij}}{1 + 6x}, \quad (\delta_{ij}^d)_{RR}(M_{\text{SUSY}}) \approx \frac{(\delta_{ij}^d)_{RR}(M_{\text{GUT}})}{1 + 6x}$$

$$q_{ij} \equiv \Delta_s[m_{\tilde{Q}}^2]_{ij}/m_0^2 \quad \leftarrow \text{independent of } x$$

x -dependence of B mixing

- $B_s - \overline{B}_s$ transition amplitude

$$\begin{aligned}
 & \propto \frac{(\delta_{23}^d)_{LL} (\delta_{23}^d)_{RR}}{m_S^2} \Big|_{M_{\text{SUSY}}} \\
 & \approx \frac{[(\delta_{23}^d)_{LL}(M_{\text{GUT}}) + q_{ij}] \cdot (\delta_{23}^d)_{RR}(M_{\text{GUT}})}{(1 + 6x)^2 m_S^2} \\
 & \propto \frac{x}{(1 + 6x)^3} \quad \text{for fixed } M_{1/2} \text{ \& } (\delta_{23}^d)_{AB}(M_{\text{GUT}}) \\
 & \implies \text{Maximized at } x \approx 1/12
 \end{aligned}$$

- $(\delta_{23}^d)_{LL} (\delta_{23}^d)_{LL} / m_S^2$ and $(\delta_{23}^d)_{RR} (\delta_{23}^d)_{RR} / m_S^2$ behave in the same way

RG running of slepton mass matrix below M_{GUT}

- Diagonal components

$$[m_{\tilde{l}}^2]_{ii}(M_{\text{SUSY}}) \approx m_0^2$$

- Off-diagonal components

$$\Delta[m_{\tilde{l}}^2]_{ij} \simeq -\frac{6}{(4\pi)^2} [V_L^\dagger \widehat{Y}_N^2 V_L]_{ij} m_0^2 \ln \frac{M_{\text{GUT}}}{M_R}$$



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$$(\delta_{ij}^l)_{LL}(M_{\text{SUSY}}) \approx (\delta_{ij}^l)_{LL}(M_{\text{GUT}}) + l_{ij}, \quad (\delta_{ij}^l)_{RR}(M_{\text{SUSY}}) \approx (\delta_{ij}^l)_{RR}(M_{\text{GUT}})$$
$$l_{ij} \equiv \Delta[m_{\tilde{l}}^2]_{ij}/m_0^2$$

- We ignore Y_N in numerical analysis, but there are cases with large Y_N to which our result can be applied

Parameter dependence of LFV

- LFV decay amplitudes

$$A(\tau \rightarrow \mu \gamma) \propto \frac{\mu \tan \beta \cdot (\delta_{23}^l)_{LL}(M_{\text{GUT}})}{m_S^2}$$

$$A(\mu \rightarrow e \gamma) \propto \frac{(\delta_{13}^l)_{RR}(M_{\text{GUT}}) \cdot (\delta_{33}^l)_{RL}(M_{\text{GUT}}) \cdot (\delta_{32}^l)_{LL}(M_{\text{GUT}})}{m_S^2}$$
$$\approx \frac{m_\tau \mu \tan \beta}{m_0^2} \times \frac{(\delta_{13}^l)_{RR}(M_{\text{GUT}}) \cdot (\delta_{32}^l)_{LL}(M_{\text{GUT}})}{m_S^2}$$

- Monotonically decreasing functions of $m_0^2 = M_{1/2}^2/x$
- Proportional to $\tan \beta$
- $A(\mu \rightarrow e \gamma)$ is inversely proportional to $m_0^2 \times m_S^2$

LFV and B mixing as functions of $x \equiv M_{1/2}^2/m_0^2$

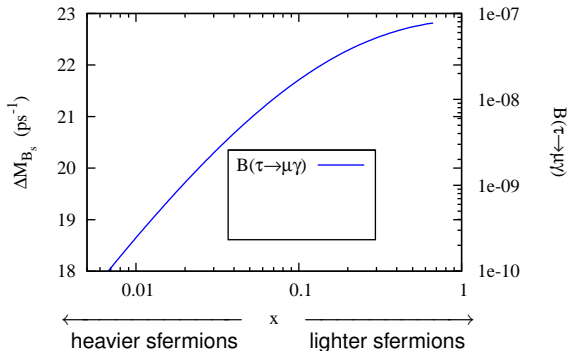
- Fix $M_{1/2} = 180$ GeV, $\tan\beta = 5$; vary m_0 from 220 GeV to ∞
- Fix $(\delta_{23}^d)_{RR} = -0.1$ and $(\delta_{ij}^d)_{LL}$ to RG-induced values at M_{GUT}

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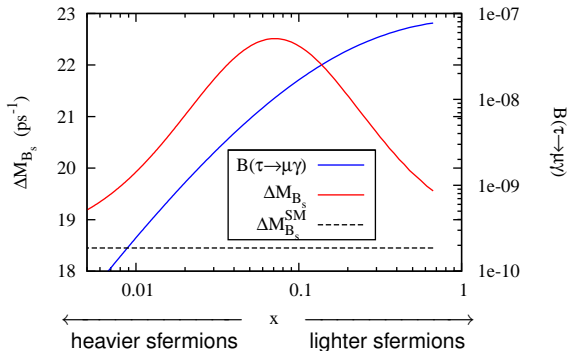
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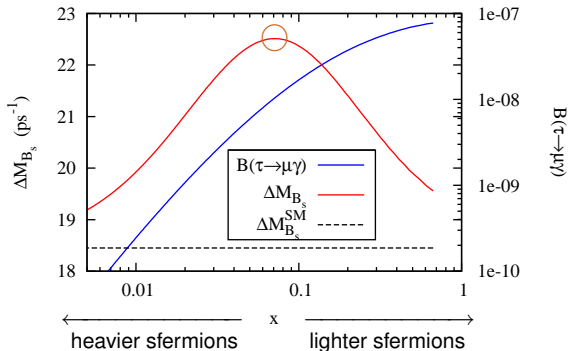
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There is a value of x that optimizes the sensitivity of B mixing to GUT scale δ 's

⇒ Important when comparing hadronic and leptonic constraints

RG running of sfermion mass matrix above M_{GUT}

- Off-diagonal components of m_T^2 — GUT version of $\Delta[m_Q^2]_{ij}$

$$\Delta m_T^2 \simeq -\frac{6}{(4\pi)^2} [3\lambda_U^* \lambda_U^T + 2\lambda_D^* \lambda_D^T] m_0^2 \ln \frac{M_*}{M_{\text{GUT}}}$$

add to $(\delta_{ij}^d)_{LL}$, $(\delta_{ij}^l)_{RR}(M_{\text{GUT}})$

\hookrightarrow relates $(\delta_{13}^d)_{RR}$ and $(\delta_{23}^d)_{RR}$ with $\mu \rightarrow e\gamma$

- Off-diagonal components of m_F^2 — GUT version of $\Delta[m_l^2]_{ij}$

$$\Delta m_F^2 \simeq -\frac{6}{(4\pi)^2} [4\lambda_D^\dagger \lambda_D + \lambda_N^\dagger \lambda_N] m_0^2 \ln \frac{M_*}{M_{\text{GUT}}}$$

add to $(\delta_{ij}^l)_{LL}$, $(\delta_{ij}^l)_{RR}(M_{\text{GUT}})$ if λ_N large

\hookrightarrow popular when $S_{CP}^{B \rightarrow \phi K}$ showed a discrepancy

Constraints on $(\delta_{23}^d)_{RR}$ in SUSY GUT

- Linked to $\tau \rightarrow \mu \gamma$

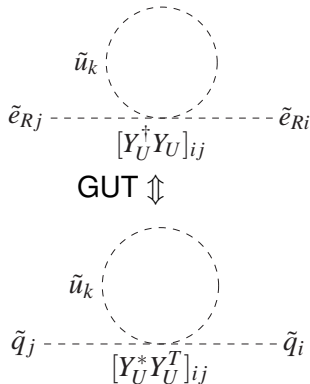
Hisano, Shimizu, PLB(2003)

Constraints on $(\delta_{23}^d)_{RR}$ in SUSY GUT

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Hisano, Shimizu, PLB(2003)

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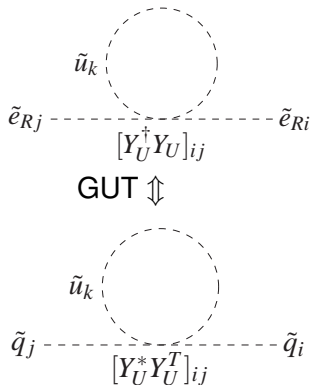
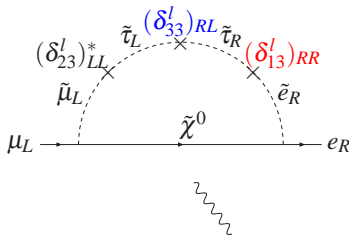


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Hisano, Shimizu, PLB(2003)

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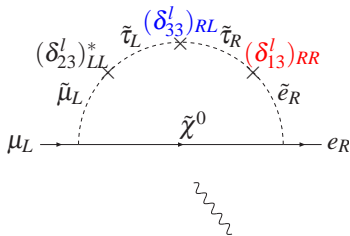


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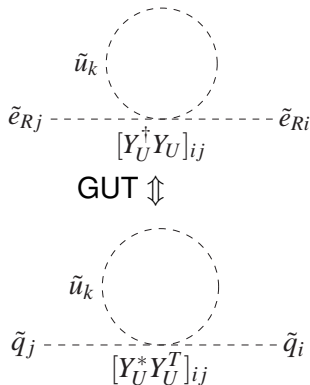
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Enhanced by a factor m_τ/m_μ

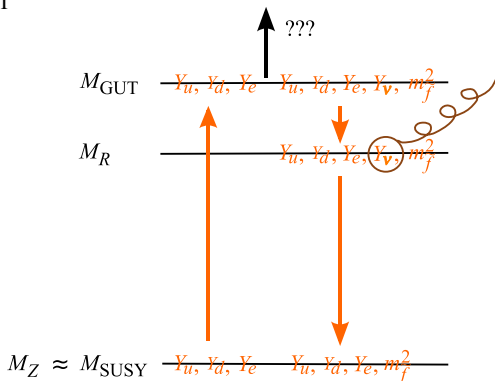
Hisano, Moroi, Tobe, Yamaguchi, PRD(1996)

Baek, Goto, Okada, Okumura, PRD(2001)



Procedure of numerical analysis

- 1 Run Y_u, Y_d, Y_e from M_Z to M_{GUT}
- 2 Go to the basis where Y_d and Y_e are diagonal
- 3 Set m_f^2 and $M_{1/2}$ at M_{GUT} **by hand**;
Assume $A(M_{\text{GUT}}) = 0$
- 4 Run down to M_R
- 5 Remove N
- 6 Run down to M_Z
- 7 Fix μ from EWSB condition
- 8 Compute flavor violations



Boundary conditions at M_{GUT}

- Scalar mass matrices

$$m_q^2 = m_0^2 \begin{bmatrix} 1 & (\delta_{12}^d)_{LL} & (\delta_{13}^d)_{LL} \\ (\delta_{12}^d)_{LL}^* & 1 & (\delta_{23}^d)_{LL} \\ (\delta_{13}^d)_{LL}^* & (\delta_{23}^d)_{LL}^* & 1 \end{bmatrix} = m_e^{2*}$$
$$m_d^2 = m_0^2 \begin{bmatrix} 1 & 0 & (\delta_{13}^d)_{RR} \\ 0 & 1 & (\delta_{23}^d)_{RR} \\ (\delta_{13}^d)_{RR}^* & (\delta_{23}^d)_{RR}^* & 1 \end{bmatrix} = m_l^{2*}$$

- Three cases to study

$ (\delta_{12}^d)_{LL} $	$ (\delta_{13}^d)_{LL} $	$ (\delta_{23}^d)_{LL} $	$ (\delta_{23}^d)_{RR} $
4.8×10^{-5}	1.5×10^{-3}	free	0
4.8×10^{-5}	1.5×10^{-3}	7.4×10^{-3}	free
4.8×10^{-5}	1.5×10^{-3}	free	= free

- 'Default value' of a mass insertion is that from RGE above M_{GUT} , not zero

How high should the cutoff be?

- M_* might be low $\sim M_{\text{GUT}}$
- In this case, RG running of sfermion masses above M_{GUT} is negligible
- Is RG-induced mass insertion generic enough to include in a model-independent analysis?
- Yes
- If $M_* \sim M_{\text{GUT}}$, uncontrollable non-renormalizable operators spoil quark–lepton flavor relations completely; recall

$$Y_D - Y_E^T = \xi h_2, \quad \xi \approx \frac{M_{\text{GUT}}}{M_*} \sim \mathcal{O}(1)$$

- Nonsense to relate quark and lepton flavors unless $M_* \gg M_{\text{GUT}}$

Hadronic constraints

Observable	Measured value	Imposed constraint
ΔM_s	$17.77 \pm 0.12 \text{ ps}^{-1}$	$17.77 \text{ ps}^{-1} \pm 30\%$
ϕ_s	-0.73 ± 0.16 or -2.42 ± 0.14	$[-1.10, -0.36] \cup$ $[-2.77, -2.07]$ at 95% CL
$B(B \rightarrow X_s \gamma)$	$(355 \pm 24_{-10}^{+9} \pm 3) \times 10^{-6}$	2σ
$S_{\phi K}$	0.39 ± 0.18	2σ
$ d_n $	$< 6.3 \times 10^{-26} \text{ e cm}$	

Leptonic constraints

- Upper bounds used in the numerical analysis

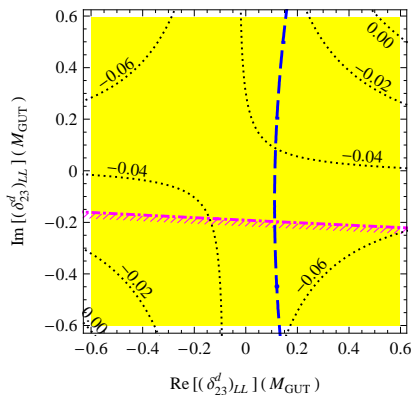
Process	Present upper bound	Future upper bound
$B(\mu \rightarrow e\gamma)$	1.2×10^{-11} MEGA	1×10^{-13} MEG
$B(\tau \rightarrow e\gamma)$	3.1×10^{-7} BABAR	1×10^{-8}
$B(\tau \rightarrow \mu\gamma)$	6.8×10^{-8} Belle	1×10^{-8}

- CDR of super B factory in Italy expects

$$B(\tau \rightarrow e\gamma) < 2 \times 10^{-9} \text{ and } B(\tau \rightarrow \mu\gamma) < 2 \times 10^{-9}$$

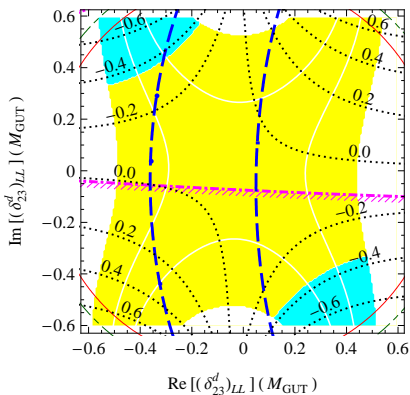
Bona et al, 0709.0451

LL case



$$m_0 = 220 \text{ GeV}$$

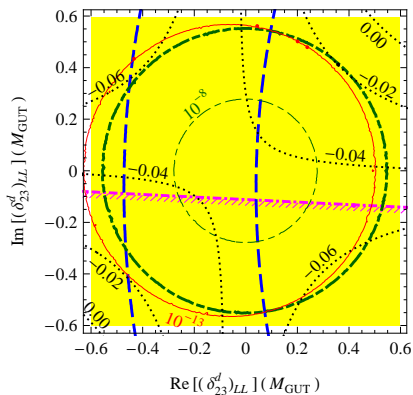
$$M_{1/2} = 180 \text{ GeV}, \quad \tan\beta = 5$$



$$m_0 = 600 \text{ GeV}$$

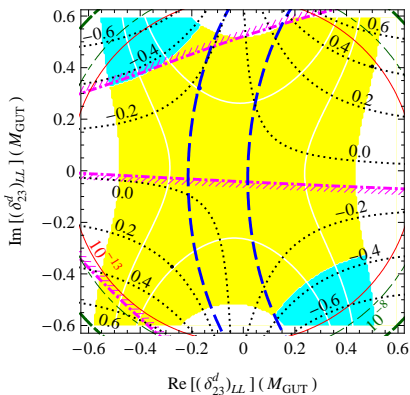
- For lower m_0 , change in ϕ_s too small
- For higher m_0 , tiny corners satisfy ϕ_s , $B \rightarrow X_s \gamma$, $S_{CP}^{\phi K}$
- For higher $\tan\beta$, ϕ_s conflicts with tighter $B \rightarrow X_s \gamma$ & $S_{CP}^{\phi K}$

LL case



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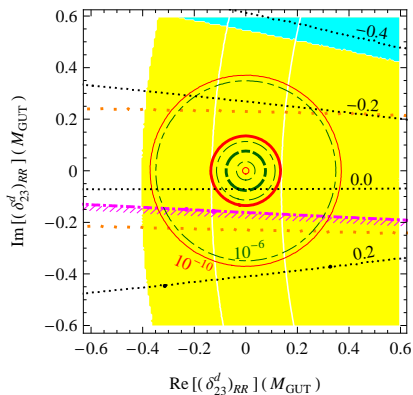
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$$m_0 = 600 \text{ GeV}$$

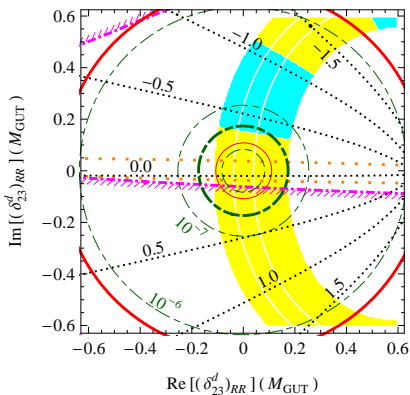
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RR case



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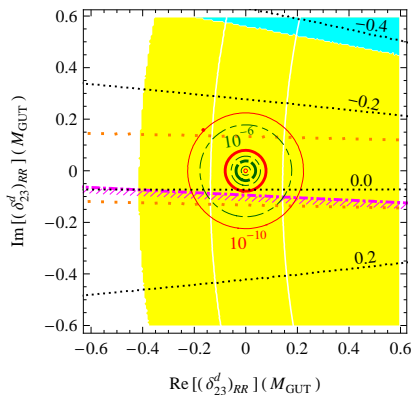
$$M_{1/2} = 180 \text{ GeV}, \quad \tan\beta = 5$$



$$m_0 = 600 \text{ GeV}$$

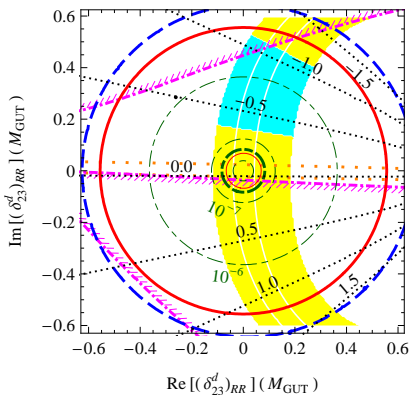
- For lower m_0 , LFV conflicts with ϕ_s
- For higher m_0 , LFV marginally ok, but d_n problematic
- Higher $\tan\beta$ worsens compatibility of ϕ_s with LFV & d_n

RR case



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Universal boundary condition + large Y_N scenario

- Suppose that soft terms are flavor blind at M_* and RR mixings arise solely from Y_N

Hisano, Shimizu, 0805.3327

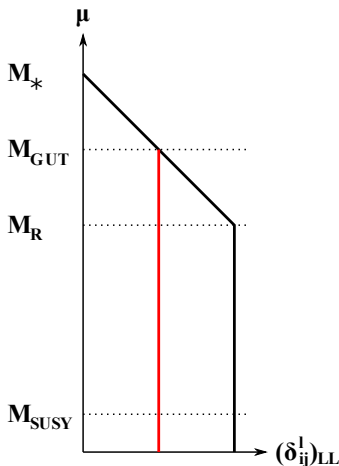
- Can convert a previous RR plot to one for this case, shrinking an LFV circle by $\alpha/(1+\alpha)$ where

$$\alpha \equiv \frac{\ln(M_*/M_{\text{GUT}})}{\ln(M_{\text{GUT}}/M_{\text{R}})}$$

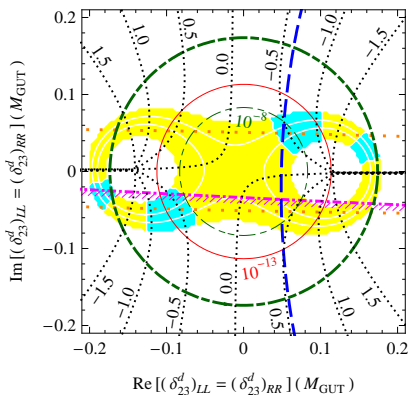
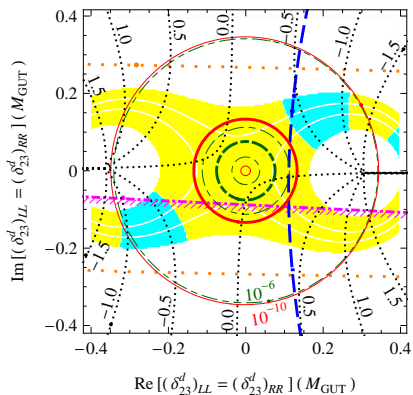
- Recall that

$$(\delta_{ij}^d)_{RR}(M_{\text{GUT}}) = (\delta_{ij}^l)_{LL}^*(M_{\text{GUT}})$$

$$(\delta_{ij}^l)_{LL}(M_{\text{GUT}}) \approx \frac{\alpha}{1+\alpha} (\delta_{ij}^l)_{LL}(M_{\text{SUSY}})$$

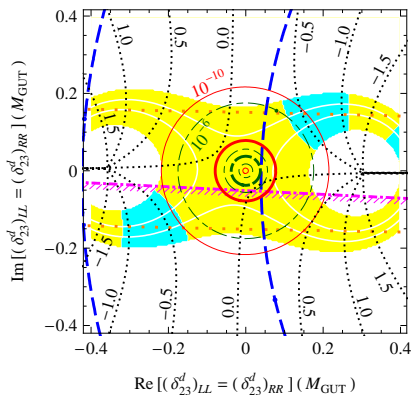


LL = RR case



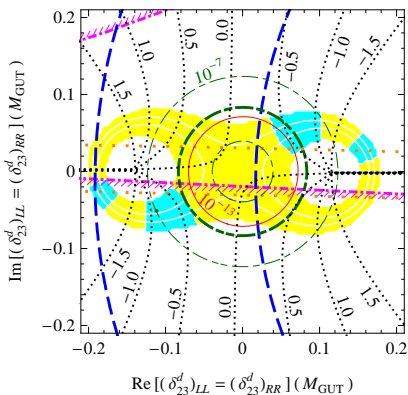
- For lower m_0 , LFV conflicts with ϕ_s , but less serious than RR case
- For higher m_0 , there are regions satisfying all the constraints
- Even higher $\tan\beta$ leaves viable corners for higher m_0

LL = RR case



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Summary

SUSY GUT + large $\phi_s = \text{LFV}$

- Hadronic constraints by themselves disfavor LL -only case
- With nonzero RR insertion, LFV rates are detectable or already excessive

Easy to reconcile ϕ_s with LFV and other constraints if

- $x \equiv M_{1/2}^2/m_0^2 \approx 1/12$
- there are both LL and RR mass insertions
- $\tan\beta$ is low

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