Doubly Lopsided Models from SUSY SU(N)

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THE BASIC IDEA: The Standard Model Group and the unified groups $SU(5)$ and $SO(10)$ do not distinguish the families from each other.

$\Rightarrow$ "flavor symmetries" must be introduced to distinguish the families in $SU(5)$ or $SO(10)$ models

HOWEVER: in $SU(N)$, with $N > 5$, the families do not look the same under $SU(N)$, in general.

$\Rightarrow$ No flavor symmetries are needed. The "horizontal" family hierarchy can come from the "vertical" grand unified symmetry.

Doubly Lopsided Models: An idea for explaining quark and lepton masses and mixings.

Use $SU(5)$ language for convenience:

$$\mathbf{10}_i = (\ell^+, Q, u^c)_i \qquad \overline{\mathbf{5}}_i = (L, d^c)_i$$

Suppose that the mixing of $\overline{\mathbf{5}}_1$, $\overline{\mathbf{5}}_2$, and $\overline{\mathbf{5}}_3$ with each other is $O(1)$, but that the mixing of $\mathbf{10}_1$, $\mathbf{10}_1$, and $\mathbf{10}_3$ is small:

$$\mathbf{10}_1 \overbrace{\underbrace{\quad\quad}_{\delta} \mathbf{10}_2 \underbrace{\quad\quad}_{\epsilon}}^{\delta\epsilon} \mathbf{10}_3$$

$$u_i(M_U)_{ij}u^c_j \subset (\mathbf{10}_1\mathbf{10}_2\mathbf{10}_3)\begin{pmatrix}\delta^2\epsilon^2 & \delta\epsilon^2 & \delta\epsilon \\ \delta\epsilon^2 & \epsilon^2 & \epsilon \\ \delta\epsilon & \epsilon & 1\end{pmatrix} m \begin{pmatrix}\mathbf{10}_1 \\ \mathbf{10}_2 \\ \mathbf{10}_3\end{pmatrix}$$

$$d_i(M_D)_{ij}d^c_j \subset (\mathbf{10}_1\mathbf{10}_2\mathbf{10}_3)\begin{pmatrix}\delta\epsilon & \delta\epsilon & \delta\epsilon \\ \epsilon & \epsilon & \epsilon \\ 1 & 1 & 1\end{pmatrix} m' \begin{pmatrix}\overline{\mathbf{5}}_1 \\ \overline{\mathbf{5}}_2 \\ \overline{\mathbf{5}}_3\end{pmatrix}$$

$$\ell^-_i(M_L)_{ij}\ell^+_j \subset (\overline{\mathbf{5}}_1\overline{\mathbf{5}}_2\overline{\mathbf{5}}_3)\begin{pmatrix}\delta\epsilon & \epsilon & 1 \\ \delta\epsilon & \epsilon & 1 \\ \delta\epsilon & \epsilon & 1\end{pmatrix} m' \begin{pmatrix}\mathbf{10}_1 \\ \mathbf{10}_2 \\ \mathbf{10}_3\end{pmatrix}$$

$$\nu_i(M_\nu)_{ij}\nu_j \subset (\overline{\mathbf{5}}_1\overline{\mathbf{5}}_2\overline{\mathbf{5}}_3)\begin{pmatrix}1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1\end{pmatrix} m_\nu \begin{pmatrix}\overline{\mathbf{5}}_1 \\ \overline{\mathbf{5}}_2 \\ \overline{\mathbf{5}}_3\end{pmatrix}$$

This explains that the family hierarchy is steeper for u,c,t than for d,s,b and e,μ,τ, and least steep for ν_1,ν_2,ν_3:

$$\frac{m_u}{m_c} \sim \delta^2 \simeq \frac{1}{400}, \qquad \frac{m_c}{m_t} \sim \epsilon^2 \simeq \frac{1}{400},$$

$$\frac{m_d}{m_s} \sim \delta \simeq \frac{1}{20}, \qquad \frac{m_s}{m_b} \sim \epsilon \simeq \frac{1}{50},$$

$$\frac{m_e}{m_\mu} \sim \delta \simeq \frac{1}{200}, \qquad \frac{m_\mu}{m_\tau} \sim \epsilon \simeq \frac{1}{20},$$

$$\frac{m_1}{m_2} \sim 1 \sim ?, \qquad \frac{m_2}{m_3} \sim 1 \simeq \frac{1}{5},$$

It also explains that the CKM angles are small and the MNS angles are $O(1)$:

$$\sin\theta^q_{12} \sim \delta \simeq \tfrac{1}{5}, \quad \sin\theta^q_{23} \sim \epsilon \simeq \tfrac{1}{25}, \quad \sin\theta^q_{13} \sim \delta\epsilon \simeq \tfrac{1}{300}$$

EMBEDDING FAMILIES IN SU(N):

Anomaly-free sets of anti-symmetric tensors will yield chiral families $(\mathbf{10}+\overline{\mathbf{5}})$ plus vector-like pairs ($\mathbf{5}+\overline{\mathbf{5}}$ and $\mathbf{10}+\overline{\mathbf{10}}$) that get superlarge mass. Economical sets tend to contain a few larger tensors and many anti-fundamental reps.

An SU(7) Example:

$$
\begin{array}{lcl}
\psi^{ABC} & \rightarrow & \psi^{\alpha\beta\gamma}(\overline{\mathbf{10}}) + \psi^{\alpha\beta 6}, \psi^{\alpha\beta 7}(2(\mathbf{10})) + \psi^{\alpha 67}(\mathbf{5}) \\
2(\psi^{AB}) & \rightarrow & \psi^{\alpha\beta}_{(a)}(2(\mathbf{10})) + \psi^{\alpha 6}_{(a)}, \psi^{\alpha 7}_{(a)}(4(\mathbf{5})) + \psi^{67}_{(a)}(\mathbf{1}) \\
8(\psi_A) & \rightarrow & \psi_{(m)\alpha}(8(\overline{\mathbf{5}})) + \psi_{(m)6}, \psi_{(m)7}(16(\mathbf{1}))
\end{array}
$$

$a=1,2 \qquad m=1,\cdots,8$

The light 10's: $\psi^{\alpha\beta 6}$ $\psi^{\alpha\beta 7}$ $\psi^{\alpha\beta}_{(a)}$ different under SU(7)

The light $\overline{5}$'s: $\psi_{(m)\alpha}$ $\psi_{(m')\alpha}$ $\psi_{(m'')\alpha}$ NOT different

Let $\mathbf{10}_1 \equiv \psi^{\alpha\beta 7}$, $\mathbf{10}_2 \equiv \psi^{\alpha\beta 6}$, $\mathbf{10}_3 \equiv \psi^{\alpha\beta}$,
and $\overline{\mathbf{5}}_1 \equiv \psi_{(1)\alpha}$, $\overline{\mathbf{5}}_2 \equiv \psi_{(2)\alpha}$, $\overline{\mathbf{5}}_3 \equiv \psi_{(3)\alpha}$

Suppose this hierarchy of VEVs:

$$\underbrace{\langle H^{67}\rangle, \langle \overline{H}_{67}\rangle}_{\sim M_{P\ell}} \gg \underbrace{\langle H^{7}\rangle, \langle \overline{H}_{6}\rangle}_{\sim \epsilon M_{P\ell}} \gg \underbrace{\langle H^{6}\rangle, \langle \overline{H}_{7}\rangle}_{\sim \delta\epsilon M_{P\ell}}$$

$$\underbrace{\langle H^{2}\rangle, \langle \overline{H}_{2}\rangle}_{\sim M_{W}} \gg \underbrace{\langle H^{27}\rangle, \langle \overline{H}_{26}\rangle}_{\sim \epsilon M_{W}} \gg \underbrace{\langle H^{26}\rangle, \langle \overline{H}_{27}\rangle}_{\sim \delta\epsilon M_{W}}$$

The elements of M_U arise in the following way:

$$\psi^{\alpha\beta}\psi^{\gamma\delta}H^{267} \rightarrow (M_U)_{33} \sim M_W$$
$$\psi^{\alpha\beta}\psi^{\gamma\delta 6}H^{27} \rightarrow (M_U)_{32} \sim \epsilon M_W$$
$$\psi^{\alpha\beta}\psi^{\gamma\delta 7}H^{26} \rightarrow (M_U)_{31} \sim \delta\epsilon M_W$$
$$\psi^{\alpha\beta 6}\psi^{\gamma\delta 6}H^{27}\overline{H}_{6}/M_{P\ell} \rightarrow (M_U)_{22} \sim \epsilon^2 M_W \quad \text{etc.}$$

$$\Longrightarrow M_U \sim \begin{pmatrix} \delta^2\epsilon^2 & \delta\epsilon^2 & \delta\epsilon \\ \delta\epsilon^2 & \epsilon^2 & \epsilon \\ \delta\epsilon & \epsilon & 1 \end{pmatrix} M_W$$

With $\mathbf{10}_1 \equiv \psi^{\alpha\beta 7}$, $\mathbf{10}_2 \equiv \psi^{\alpha\beta 6}$, $\mathbf{10}_3 \equiv \psi^{\alpha\beta}$,
and $\overline{\mathbf{5}}_1 \equiv \psi_{(1)\alpha}$, $\overline{\mathbf{5}}_2 \equiv \psi_{(2)\alpha}$, $\overline{\mathbf{5}}_3 \equiv \psi_{(3)\alpha}$

And the hierarchy of VEVs:

$$\underbrace{\langle H^{67}\rangle, \langle \overline{H}_{67}\rangle}_{\sim M_{P\ell}} \gg \underbrace{\langle H^{7}\rangle, \langle \overline{H}_{6}\rangle}_{\sim \epsilon M_{P\ell}} \gg \underbrace{\langle H^{6}\rangle, \langle \overline{H}_{7}\rangle}_{\sim \delta\epsilon M_{P\ell}}$$

$$\underbrace{\langle H^{2}\rangle, \langle \overline{H}_{2}\rangle}_{\sim M_{W}} \gg \underbrace{\langle H^{27}\rangle, \langle \overline{H}_{26}\rangle}_{\sim \epsilon M_{W}} \gg \underbrace{\langle H^{26}\rangle, \langle \overline{H}_{27}\rangle}_{\sim \delta\epsilon M_{W}}$$

The elements of M_D and M_L arise in the following way:

$$\psi^{\alpha 2}\psi_{(m)\alpha}H_2 \longrightarrow (M_D)_{3m} \sim M_W$$
$$\psi^{\alpha 26}\psi_{(m)\alpha}H^{26} \longrightarrow (M_D)_{2m} \sim \epsilon M_W$$
$$\psi^{\alpha 27}\psi_{(m)\alpha}H^{27} \longrightarrow (M_D)_{1m} \sim \delta\epsilon M_W$$

$$\Longrightarrow M_D \sim \begin{pmatrix} \delta\epsilon & \delta\epsilon & \delta\epsilon \\ \epsilon & \epsilon & \epsilon \\ 1 & 1 & 1 \end{pmatrix} M_W$$

Why this hierarchy of VEVs is consistent:

Consider the minimization that gives the superlarge VEVs that break SU(7) down to SU(5). The basic idea can be seen by considering the the terms that couple Higgs multiplets of different rank.

$$W \supset M H^A \overline{H}_A + H^{AB} \overline{H}_A \overline{H}_B, \quad M \sim M_{P\ell}.$$

Setting $\partial W / \partial \overline{H}_A = 0$ gives

$$M H^A + H^{AB} \overline{H}_B = 0.$$

Or

$$\langle H^6 \rangle \sim \frac{\langle H^{67} \rangle}{M_{P\ell}} \langle \overline{H}_7 \rangle \sim \langle \overline{H}_7 \rangle, \quad \langle H^7 \rangle \sim \frac{\langle H^{76} \rangle}{M_{P\ell}} \langle \overline{H}_6 \rangle \sim \langle \overline{H}_6 \rangle.$$