Grand unification and enhanced quantum gravitational effects

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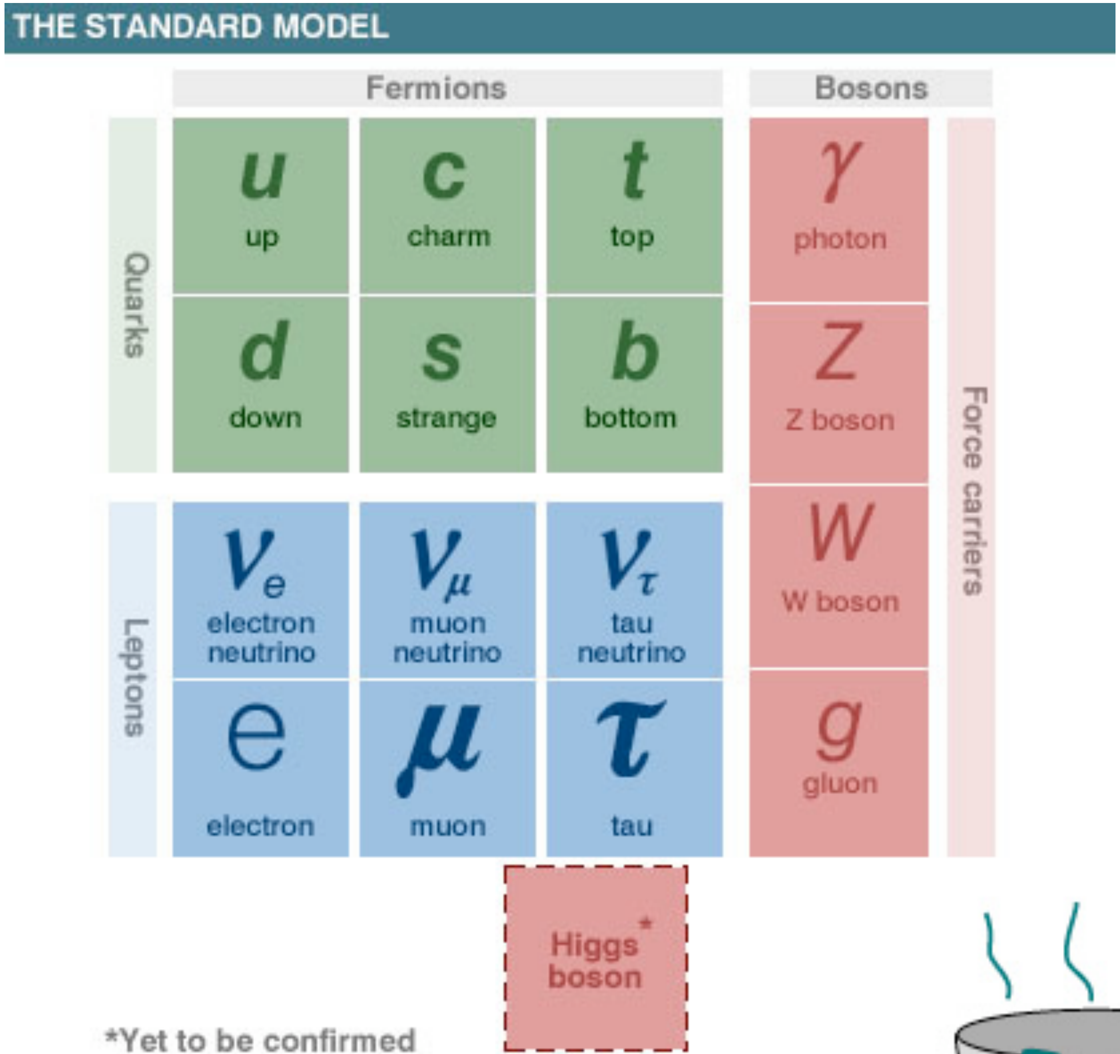
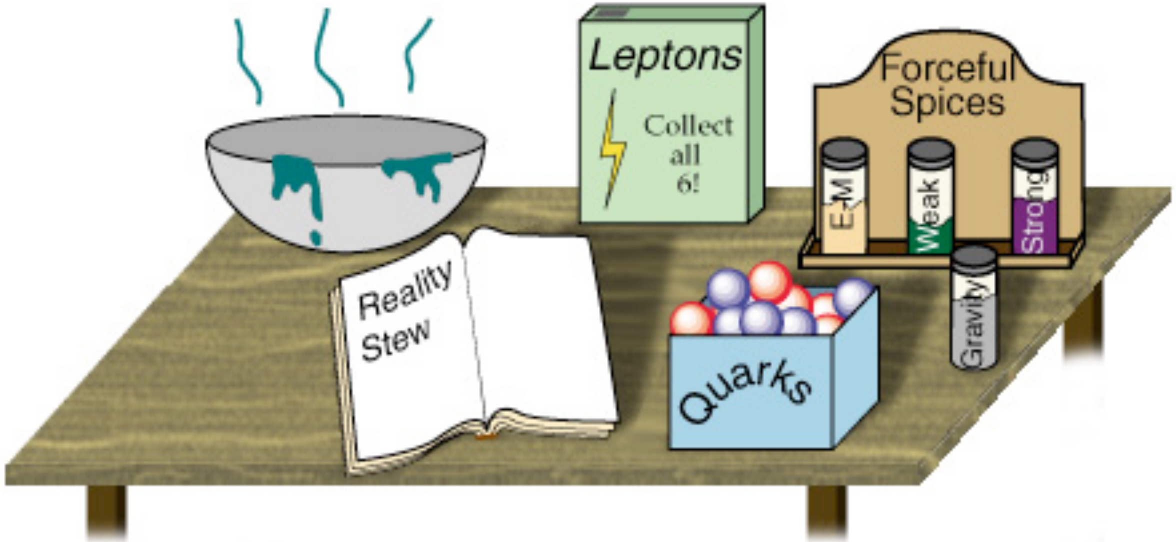
Outline

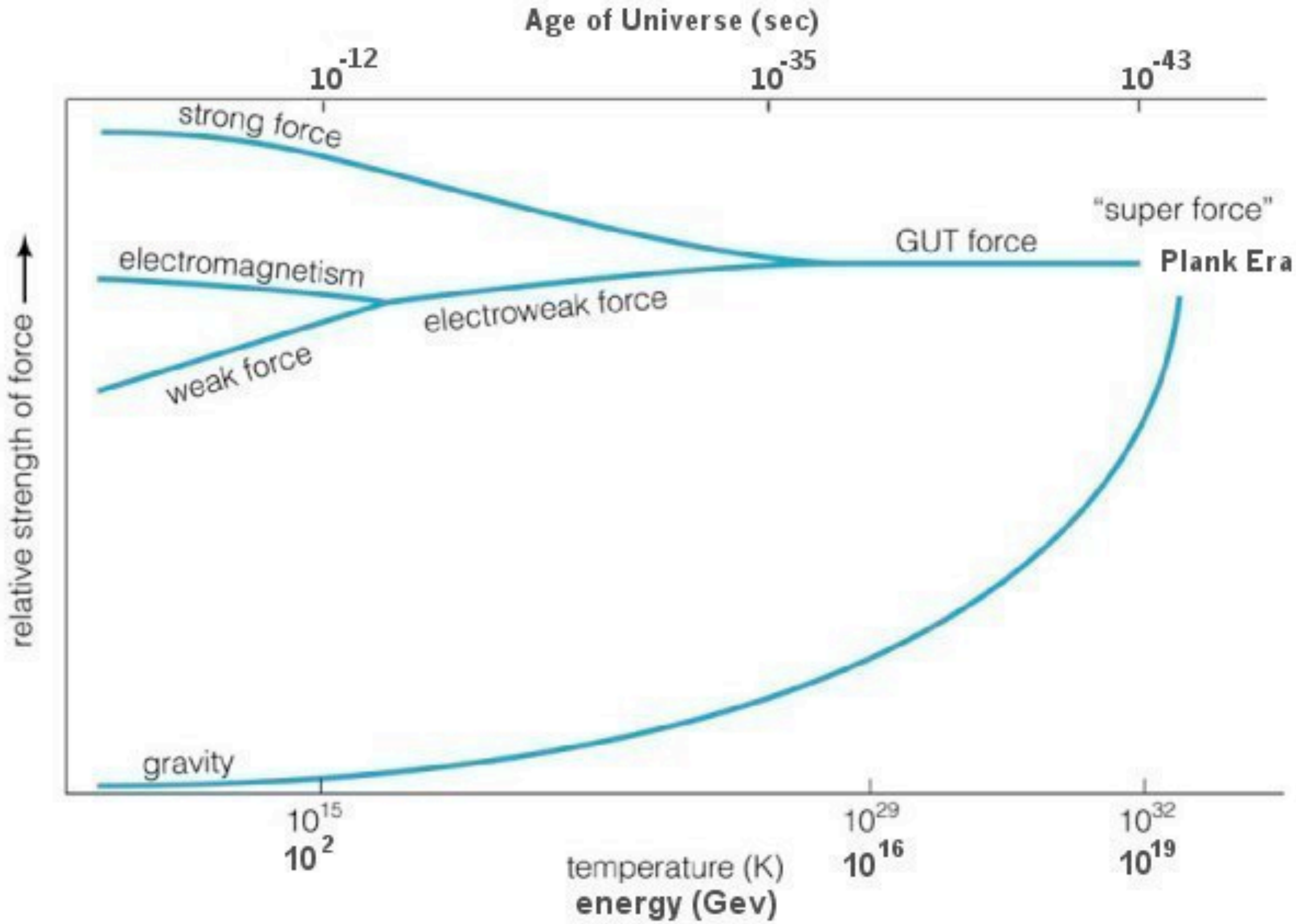
- LEP data and Unification
- Running of the Planck Mass
- Impact of Quantum Gravity on Unification
- Conclusions



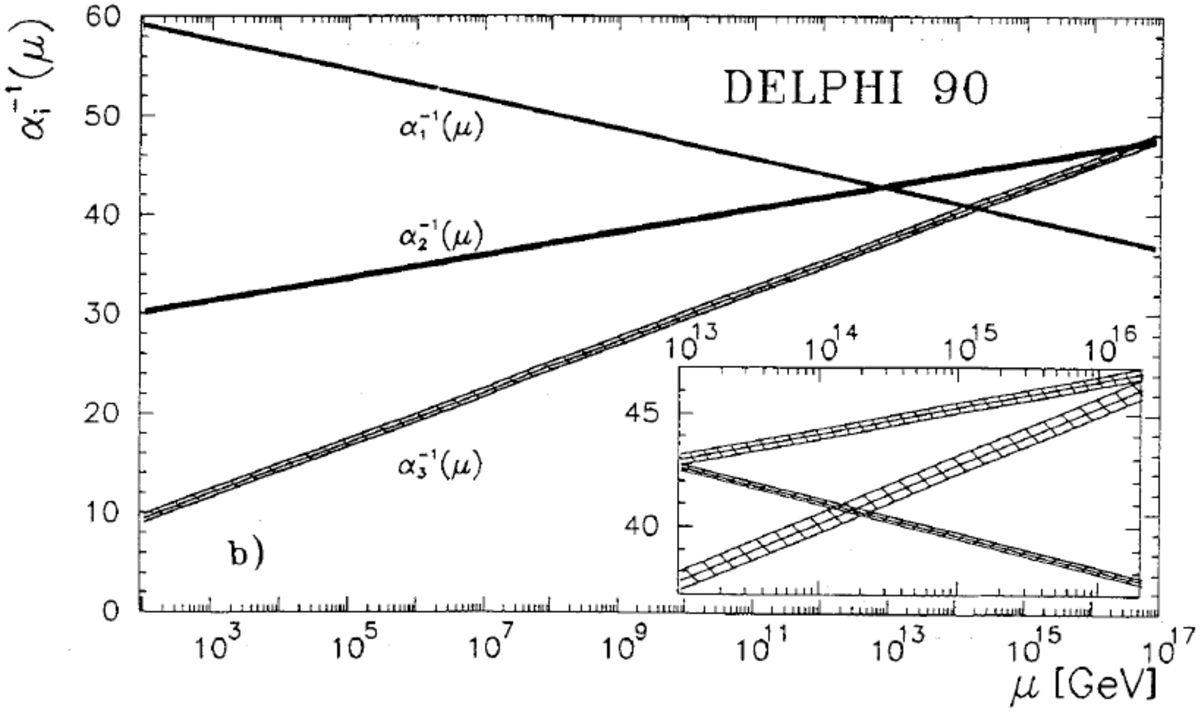
Standard Model cries for unification!



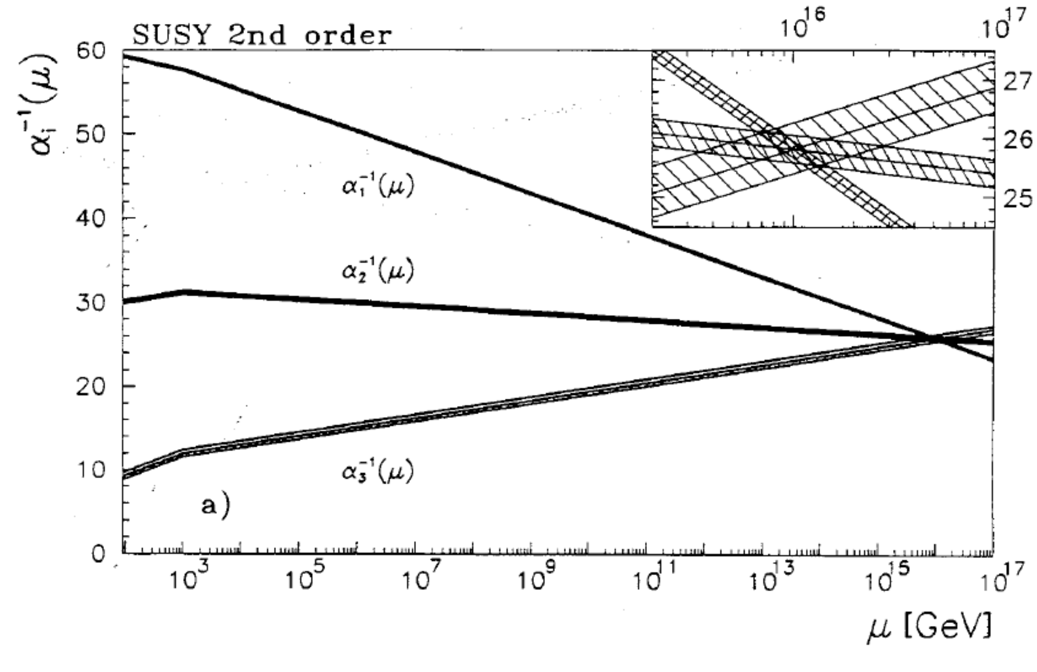
A grand unification?



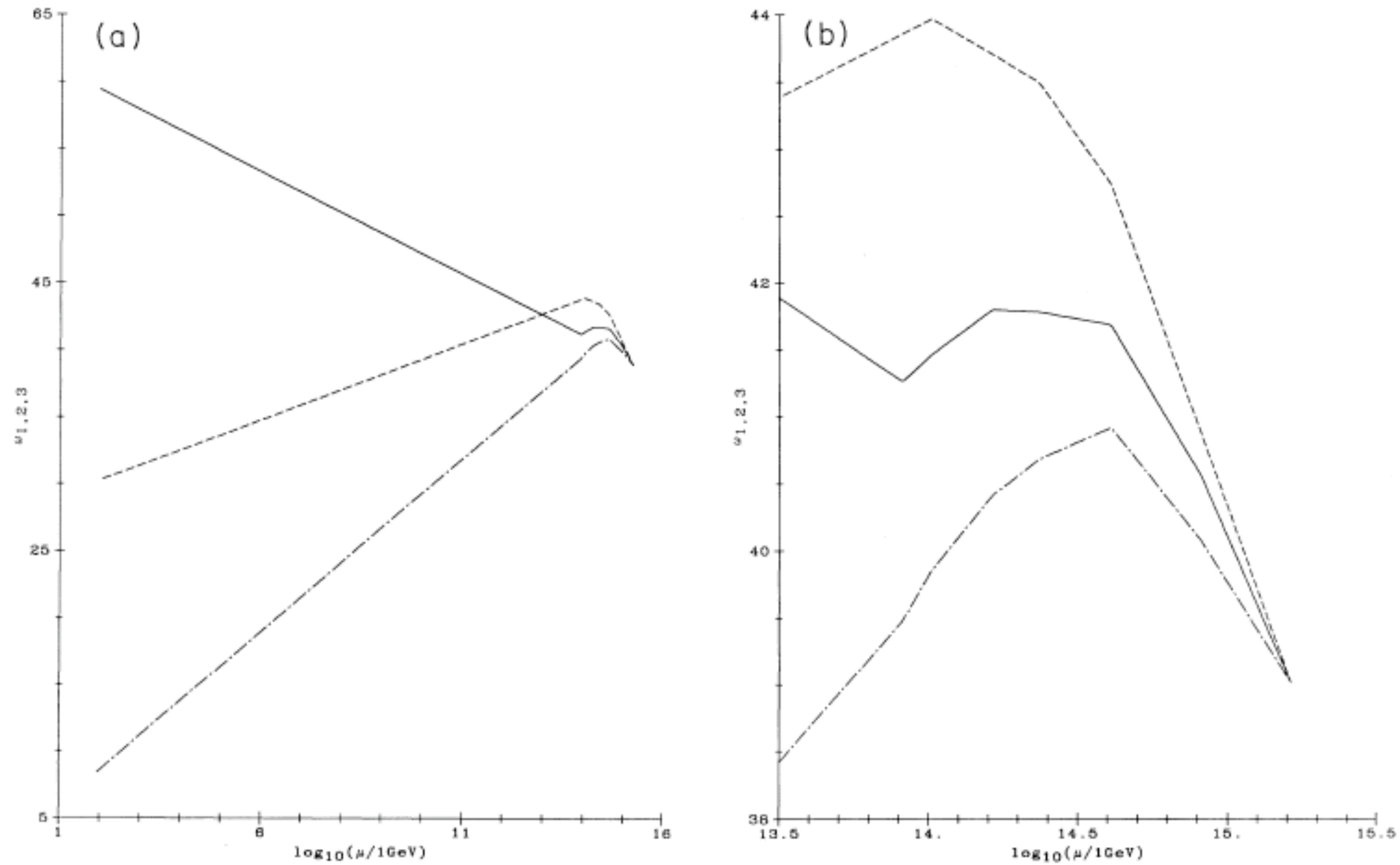
What can we learn from LEP data on unification?



standard answer:
supersymmetry is needed to
unify the couplings!



But this is not unique! E.g. Lavoura & Wolfenstein PRD 48, 264 (1993)



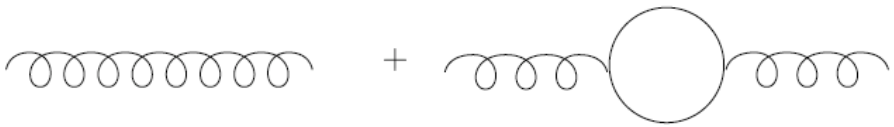
SO(10) with 210, 126, 10: one can lower the mass of some Higgses to get unification but not too much proton decay

Running of Newton's constant

- Consider GR with a scalar field

$$S = \int d^4x \sqrt{-g} \left(-\frac{1}{16\pi G} R + \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi \right)$$

- Newton's constant gets renormalized by fluctuations of quantum fields



$$\frac{1}{G_{\rm ren}} = \frac{1}{G_{\rm bare}} + c\Lambda^2$$

- Renormalization group equation:

$$\frac{1}{16\pi} \frac{1}{G(\mu)} = \frac{1}{16\pi} \left(\frac{1}{G(0)} - N \frac{\mu^2}{12\pi} \right)$$

Running of the Planck Mass

- With spin 0, spin 1/2 (Weyl) and spin 1 fields:

$$M(\mu)^2 \;=\; M(0)^2 - \frac{\mu^2}{12\pi}\left(N_0 + N_{1/2} - 4N_1\right)$$

- Gravity becomes strong at

$$M(\mu_*) \sim \mu_*$$

- Some definitions:

$$\mu_* = M_{\rm Pl}/\eta \qquad \eta = \sqrt{1+\frac{N}{12\pi}} \qquad N \equiv N_0 + N_{1/2} - 4N_1$$

- In SUSY models:

$$N = 3N_C - 3N_V$$

How big can N be?

- Typical GUT model involves a lot of scalar fields to reproduce SM in the low energy regime.
- Also to get a decent fit to mass spectrum.
- Let us look at some concrete examples:
  - Minimal SUSY SU(5) (with 3 families + Higgses **24**, **5**, **<u>5</u>**) has:

$$N = 165 \qquad \eta = 2.3$$

  - SUSY-SO(10)
    - Dutta, Mimura and Mohapatra (PRD 72, 075009, 2005)

$$\mathbf{126}, \overline{\mathbf{126}}, \mathbf{210} \text{ and } \mathbf{10} \qquad N = 1425 \qquad \eta = 6.2$$

    - Parida and Cajee (Eur. Phys. J. C 44, 447, 2005)

$$\mathbf{10}, \mathbf{16}, \overline{\mathbf{16}} \text{ and } \mathbf{45} \qquad N = 270 \qquad \eta = 2.9$$

  - E8 × E8 (Slansky (Phys. Rept 79, 1, 1981)

$$\mathbf{248} \text{ and } \mathbf{3875}$$

Why does this matter?

- Let us look again at operators discussed already by Hill (1984); Shafi and Wetterich (1984); Hall and Sarid (1991).

$$\frac{c}{\hat{\mu}_*}\mathrm{Tr}\left(G_{\mu\nu}G^{\mu\nu}H\right)$$

$$\hat{\mu}_* = \mu_*/\sqrt{8\pi} = \hat{M}_{\mathrm{Pl}}/\eta \text{ with } \hat{M}_{\mathrm{Pl}} = 2.43\times 10^{18}\,\mathrm{GeV}$$

- New effect: running of the Planck mass.
- *H*: Higgs field in the adjoint of GUT group.
- Let us look at a toy model to make our point: SUSY SU(5)

$$\langle H\rangle = M_X\,(2,2,2,-3,-3)\,/\sqrt{50\pi\alpha_G}$$

- The kinetic terms of SU(3)×SU(2)×U(1) are modified:

$$-\frac{1}{4}(1+\epsilon_1)F_{\mu\nu}F^{\mu\nu}_{\mathrm{U}(1)} - \frac{1}{2}(1+\epsilon_2)\,\mathrm{Tr}\left(F_{\mu\nu}F^{\mu\nu}_{\mathrm{SU}(2)}\right)$$
$$-\frac{1}{2}(1+\epsilon_3)\,\mathrm{Tr}\left(F_{\mu\nu}F^{\mu\nu}_{\mathrm{SU}(3)}\right)$$

- with:

$$\epsilon_1 = \frac{\epsilon_2}{3} = -\frac{\epsilon_3}{2} = \frac{\sqrt{2}}{5\sqrt{\pi}}\frac{c\eta}{\sqrt{\alpha_G}}\frac{M_X}{\hat{M}_{\mathrm{Pl}}}$$

- After field and coupling constants redefinitions

$$A^i_\mu \to (1+\epsilon_i)^{1/2} A^i_\mu \qquad\qquad g_i \to (1+\epsilon_i)^{-1/2} g_i$$

- One obtains the new unification condition:

$$\alpha_G = (1+\epsilon_1)\,\alpha_1(M_X) = (1+\epsilon_2)\,\alpha_2(M_X)$$
$$= (1+\epsilon_3)\,\alpha_3(M_X)\ .$$

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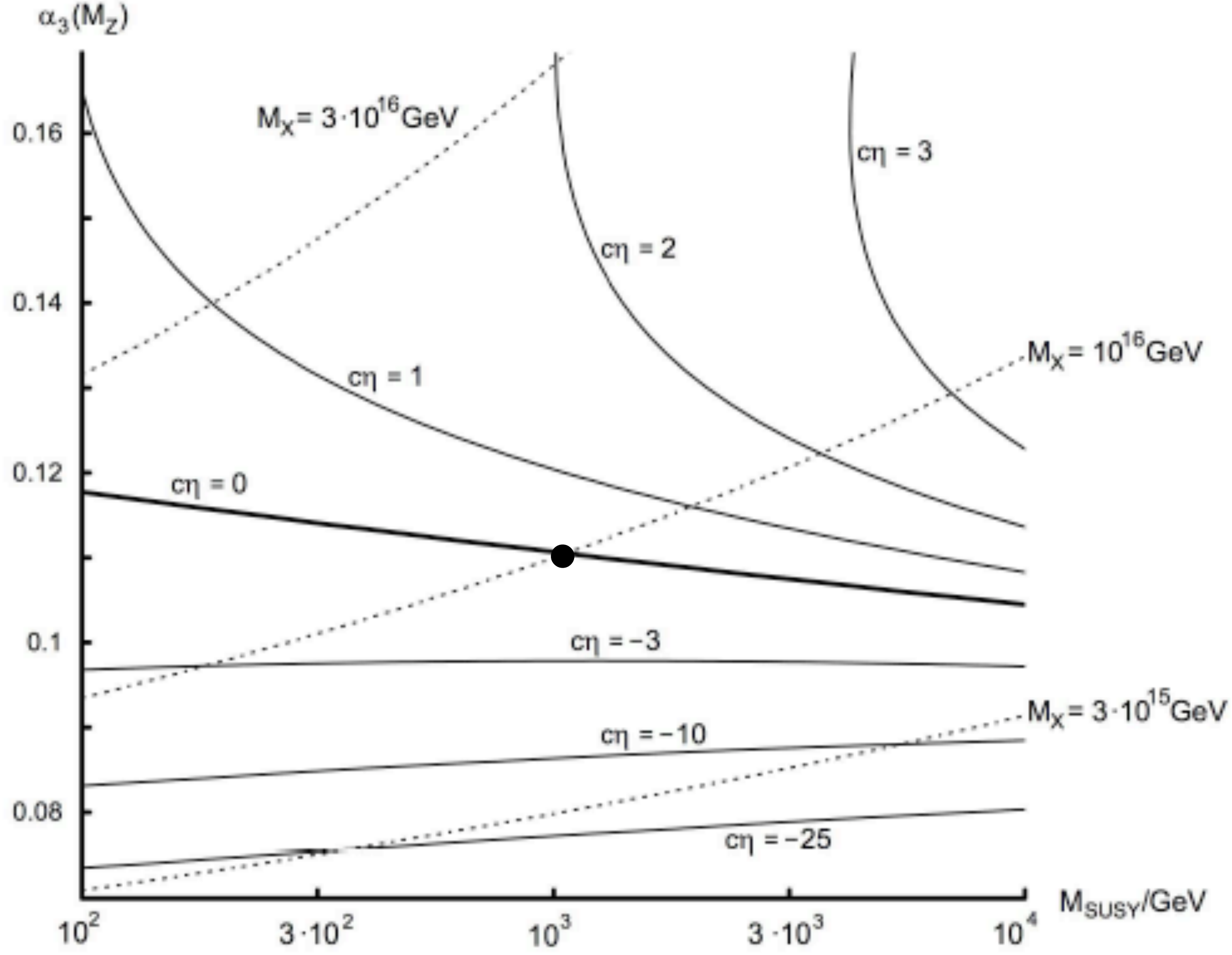
Usual solution: $\alpha_3(M_Z)$=0.117, M_{SUSY}=1 TeV

$$\eta = \sqrt{1 + \frac{N}{12\pi}}$$

Each point on this picture satisfies unification



LEP does not favor supersymmetric unification!!!

Uncertainty due to new operator is bigger than two loop effects!!!

i	1	2	3
$\alpha_i^1(M_X)$	0.03815	0.03767	0.03814
$\alpha_i^2(M_X)$	0.03897	0.03899	0.03868
$\delta\alpha_i = \alpha_i^2 - \alpha_i^1$	8.2×10^{-4}	13.2×10^{-4}	5.4×10^{-4}
$\delta\alpha_i/\alpha_i^1$	+2.1%	+3.5%	+1.4%
$\epsilon_i(c\eta = -5)$	−0.0167	−0.0503	+0.0335
$\alpha_G(M_X)$	0.0389	0.0389	0.0389
$\alpha_{Gi} = \alpha_G/(1+\epsilon_i)$	0.0396	0.0410	0.0376
$\delta_i = \alpha_G - \alpha_{Gi}$	-6.6×10^{-4}	-20.6×10^{-4}	12.6×10^{-4}
δ_i/α_G	−1.7%	−5.3%	+3.2%

TABLE I: The upper half of the table shows shifts in the predictions for the values of the coupling constants at $M_X = 10^{16}$ GeV due to inclusion of two-loop running. These shifts are comparable in size or even smaller than the necessary splittings between the α_{Gi} due to (8) in the case $\eta = 5$, $c = -1$ (lower half).

What about SO(10) models?

- Breaking of gauge symmetry is affected by

$$\xi_A^{ij}\ (\mathbf{45}) \qquad \xi_A^{ijk}\ (\mathbf{120}) \qquad \xi_A^{ijklm}\ (\mathbf{126}) \quad \xi_A^{ijkl}\ (\mathbf{210}) \qquad \xi_S^{ij}\ (\mathbf{54})$$

- However contraction $\mathrm{Tr}(G_{\mu\nu}G^{\mu\nu}\ 45)=0$ vanishes.

- But, $\mathrm{Tr}(G_{\mu\nu}G^{\mu\nu}\ 54)$ or $\mathrm{Tr}(G_{\mu\nu}G^{\mu\nu}\ 210)$ do not!

- Analysis similar to SU(5) case:

$$\epsilon_i \sim c\eta\alpha_G^{-1/2} M_X/\hat{M}_{\rm Pl}$$

Conclusions

- Quantum gravity spoils predictions done using low energy data.
- LEP does not favor SUSY unification.
- Extrapolation from low energy data is too naïve.
- Impossible to make any prediction without knowing the full details of the unification group and symmetry breaking pattern.
- Without observing proton decay it will not be possible to claim that $SU(3)\times SU(2)\times U(1)$ unifies.
- To maintain calculability of a GUT model: avoid certain Higgses and keep number of fields as small as possible.
- Thanks for your attention.

Back up slides

Derivation of the renormalization group equation (see e.g. Larsen & Wilczek (1995))

- One loop effective action of a scalar field coupled to gravity:

$$
\begin{aligned}
e^{-W} &= \int \mathcal{D}\phi \; e^{-\frac{1}{8\pi}\int \phi(-\Delta+m^2)\phi} \\
&= [\det(-\Delta+m^2)]^{-\frac{1}{2}}.
\end{aligned}
$$

- The hear kernel is defined as:

$$H(\tau) \equiv \mathrm{Tr} e^{-\tau\Lambda} = \sum_i e^{-\tau\lambda_i}$$

where λ_i are the eigenvalues of $\Lambda = -\Delta + m^2$

- The integration over τ is divergent: introduce an ultra-violet cutoff ε^2:

$$W = \frac{1}{2}\ln\det\Lambda = \frac{1}{2}\sum_i \ln\lambda_i = -\frac{1}{2}\int_{\epsilon^2}^{\infty} d\tau \frac{H(\tau)}{\tau}.$$

- One can define:

$$H(\tau) = \int dx\, G(x, x, \tau)$$

- Where the Green's function satisfies:

$$\begin{aligned}(\frac{\partial}{\partial\tau} - \Delta_x)G(x, x', \tau) &= 0 \quad ; \\ G(x, x', 0) &= \delta(x - x')\end{aligned}$$

- In flat space one would have:

$$G_0(x, x', \tau) = \left(\frac{1}{4\pi\tau}\right)^2 \exp\left(-\frac{1}{4\tau}(x - x')^2\right)$$

- Expansion for small curvature yields:

$$H(\tau) = \frac{1}{(4\pi\tau)^2}\Big(\int d^4x\,\sqrt{-g} + \frac{\tau}{6}\int d^4x\,\sqrt{-g}\,R + \mathcal{O}(\tau^{\frac{3}{2}})\Big)$$

- One thus finds:

$$\frac{1}{G_{\rm ren}} = \frac{1}{G_{\rm bare}} + \frac{1}{12\pi\epsilon^2}$$

- This was old-fashion perturbation theory. Wilsonian approach: let us integrate out modes with |k|>μ and consider physics at energies below μ:

$$W = -\frac{1}{2}\int_{\epsilon^2}^{\mu^{-2}} d\tau\,\frac{H(\tau)}{\tau}$$

- And thus:

$$\frac{1}{G(\mu)} = \frac{1}{G(0)} - \frac{\mu^2}{12\pi}$$