Black hole entropy
from strongly coupled gauge theory
--- direct confirmation by Monte Carlo simulation

SUSY 2008 COEX, Seoul, Korea

19 June 2008

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References

PRL 99 ('07) 161602 [arXiv:0706.1647]

PRL 100 ('08) 021601 [arXiv:0707.4454]

Based on collaboration with

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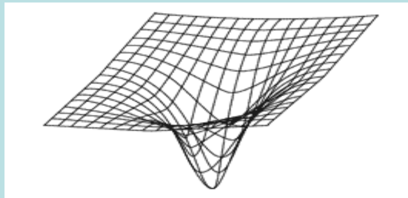
Introduction

The challenge :

string theory → black hole physics

Bekenstein-Hawking entropy :

Bekenstein and Hawking (early 70's)



$$S = \frac{k_{\mathrm{B}} c^3}{4 G \hbar} A$$

- Complete derivation requires
  the counting of quantum states of black hole.
- What is the microscopic origin of the entropy?
- gauge/gravity correspondence (conjecture)
  provides a new approach.

  Maldacena '97

  Itzhaki-Maldacena-Sonnenschein-Yankielowicz '98

In this study

We confirm the gauge/gravity correspondence
from first principle by using MC simulation.

- The black hole :

  classical black D0-brane

- The SYM :

  strongly coupled 1d SYM at finite temperature

  (representing the dynamics of open strings attached to D-brane)

From these

the open strings attached to D0-brane → microscopic d.o.f of the black D0-brane

c.f.) Strominger-Vafa ('96)

The dual SUGRA and what we do

$$ds^2 = \alpha' \left(-\frac{U^{7/2}}{4\pi^2\sqrt{15\pi}} dt^2 + \frac{4\pi^2\sqrt{15\pi}}{U^{7/2}} dU^2 + \frac{4\pi^2\sqrt{15\pi}}{U^{3/2}} d\Omega^2_{(8)} \right)$$

$$e^{\phi} = 4\pi^2 \frac{1}{N} \left(\frac{240\pi^5}{U^7} \right)^{3/4}$$

← dilaton

curvature radius

1. The temperature region, where the correspondence holds

the decoupling limit

$\alpha' \to 0$ large - N limit

⇒

$$N^{-10/21} \ll \frac{T}{\lambda^{1/3}} \ll 1$$

curvature $R \sim 1/\alpha' \ll 1$

dilaton $e^{\phi} \sim 1/N \ll 1$

classical solution for the SUGRA is valid

2. The energy of the black hole

$$\frac{E}{N^2} = C\, T^{14/5} \quad C = 7.407\cdots$$

? ⇔ the energy on gauge theory side (we calculate)

The model : 1d SYM with 16 supercharges at finite temp.

$$S = \frac{1}{g^2}\int_0^\beta dt\,\mathrm{tr}\left\{\frac{1}{2}(D_t X_i)^2 - \frac{1}{4}\left([X_i, X_j]\right)^2 + \frac{1}{2}\psi^T D_t \psi + \frac{i}{2}\psi^T \gamma_i [X^i, \psi]\right\}$$

bosonic fields : $A(t)$ $X_i(t)$ $(i = 1, \cdots, 9)$ $N \times N$ hermitian

fermionic fields : $\psi_\alpha(t)$ $(\alpha = 1, \cdots, 16)$ 〃

temperature

compactified with period β $Z(\beta) = \mathrm{tr}\ e^{-\beta\mathcal{H}}$

p.b.c. → bosonic fields

a.p.b.c. → fermionic fields

→ $\beta^{-1} = T$

the effective coupling

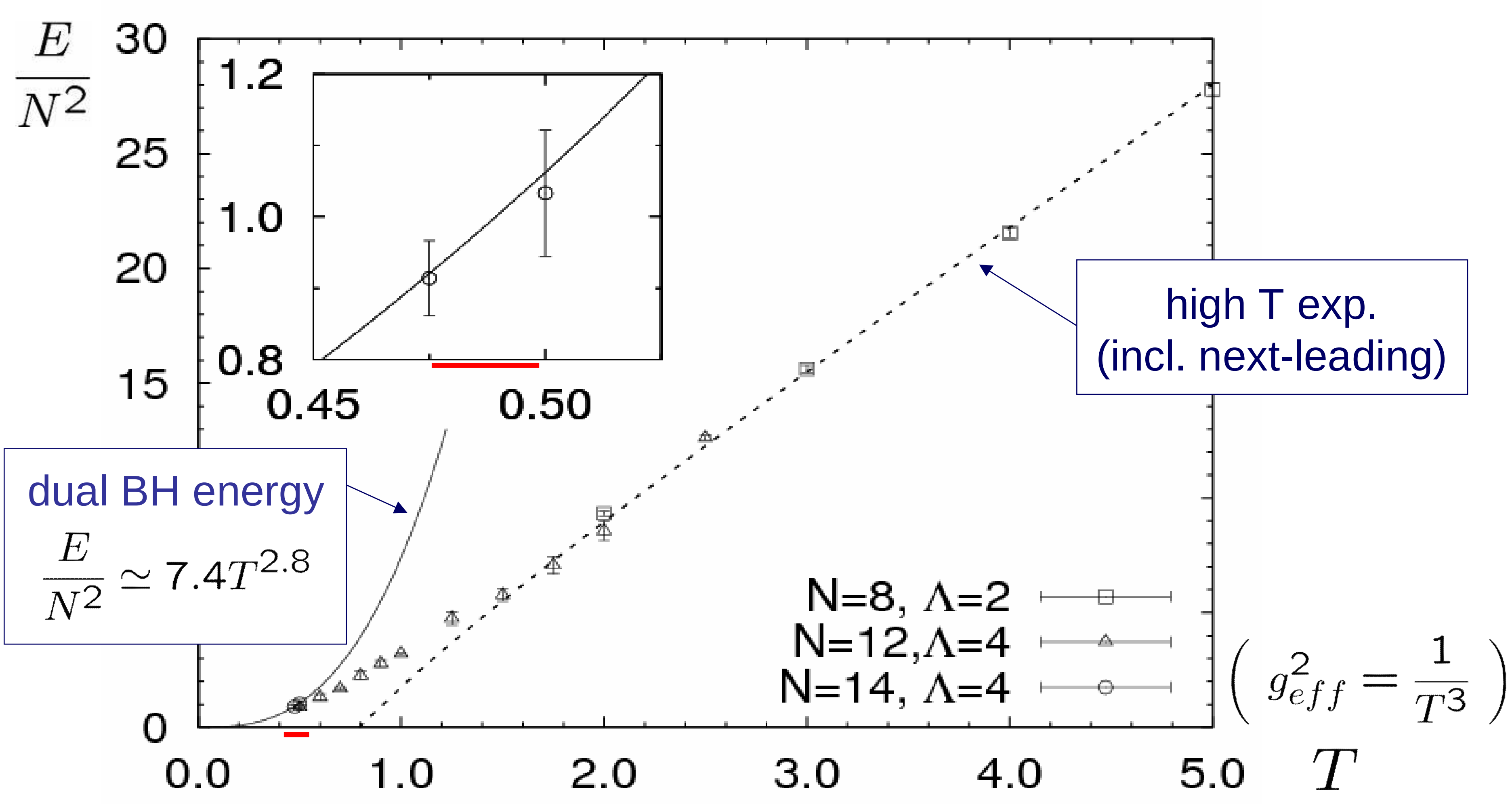
$g^2 = (energy)^3$ → $g_{eff}^2 = \frac{\lambda}{T^3}$ $\lambda \equiv g^2 N = 1$

('t Hooft coupling constant)

high temp. ⟺ weakly coupled

low temp. ⟺ strongly coupled

Monte Carlo results of the energy for SYM



the temperature range
where the correspondence exists

$$N^{-10/21} \ll T \ll 1$$

Summary

1d SYM with 16 supercharges at finite temp.

- the first Monte Carlo results

  low temp. (strong coupling region) ⇐ Monte Carlo simulation

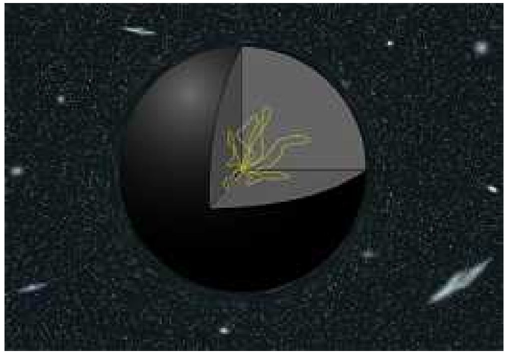
  ➢ $E/N^2 \simeq 7.4\,T^{2.8}$ (the energy of the black D0-brane)

- confirmation of gauge/gravity correspondence from first principles

  (1d SYM ⇔ black D0-brane)

⇒ Microscopic d.o.f. of the black D0-brane can be explained from the open strings attached to D0-brane.

the inner structure
of black D0-brane
KEK Press Release (Jan. '08)



Future prospects

- IR instability in SYM and quantum instability of the black hole

- simulating M-theory using Matrix Theory conjecture

  Banks-Fischler-Shenker-Susskind ('97)

- confirming the gauge/gravity correspondence
  for the Wilson loop operators

  ⇒ Identifying Schwarzschild radius in gauge theory

  c.f) Kabat-Lifschytz-Lowe ('01)

  Maldacena ('98), Rey-Yee('98), Alday- Maldacena ('98)

  in progress

  (Hanada-Miwa-Nishimura-S.T.)