Observing dynamical SUSY breaking with lattice simulation

June 19, 2008 @ SUSY 08

KANAMORI Issaku(RIKEN)

Based on collaborations with Fumihiko Sugino and Hiroshi Suzuki:
Prog.Theo.Phys. 119 (2008) 797 (arXiv:0711.2132 [hep-lat])
Phys.Rev.D77:091502(2008) (arXiv:0711.2099 [hep-lat])
and work in progress

Introduction

Recent development of lattice SUSY: any practical usage?

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Observing dynamical SUSY breaking using lattice simulation

non-perturbative

- not broken in tree level$\Rightarrow$not broken in all orders
- Witten index: *Not* available in some models
  - 2-dim $\mathcal{N} = (2, 2)$ pure SYM (maybe broken? Hori-Tong)

Plan

1. Introduction
2. Lattice formulation: Sugino model
3. Hamiltonian as the order parameter
4. Result: SQM and SYM
5. Conclusion

Scalar Q for $\mathcal{N} \geq 2$ on lattice

SUSY on lattice: impossible?($\because\ Q \sim \sqrt{P}$)

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If $\mathcal{N} \geq 2$, it is possible!

CKKU, Sugino, Catterall, DKKN, ST,...

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- topological twist$\Rightarrow$scalar Q on a site
- simulation (check of the formulation) Catterall, Suzuki, FKST

Sugino model

target: 2-dim $\mathcal{N} = (2,2)$ SYM

nilpotent Q (Twisted) SUSY Algebra, continuum

$$Q^2 = \delta_{\phi}^{(\mathrm{gauge})} \qquad Q_0^2 = \delta_{\overline{\phi}}^{(\mathrm{gauge})} \qquad \{Q, Q_0\} = 2i\partial_0 + 2\delta_{A_0}^{(\mathrm{gauge})}$$

Q-exact Lagrangian (continuum)

$$\mathcal{L} = Q(\dots) = \frac{1}{g^2}\mathrm{tr}\Big\{\frac{1}{4}F_{01}^2 + D_\mu\phi D_\mu\overline{\phi} + \frac{1}{4}[\phi,\overline{\phi}]^2 + i\psi_\mu D_\mu\eta + \cdots - \frac{1}{4}\eta[\phi,\eta] + \cdots\Big\}$$

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nilpotent Q Lattice version   Sugino, JHEP 01(2004)067

$$Q^2 = \delta_\phi^{(\mathrm{gauge})}$$

Q-exact Lagrangian (lattice)

$$\mathcal{L} = Q(\ldots) = \mathcal{L}[U(x,\mu), \phi(x), \overline{\phi}(x), H(x) \qquad \text{bosons}$$

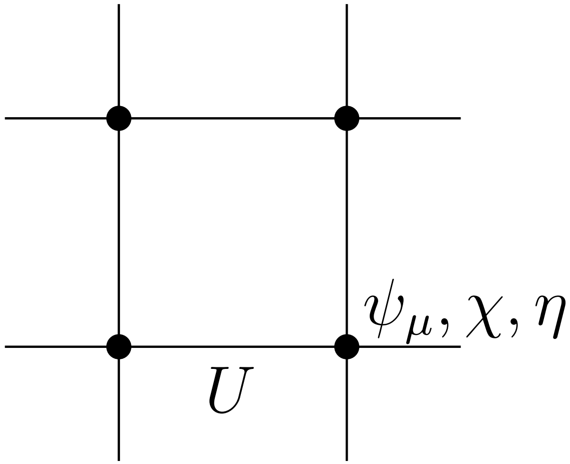
$$\eta(x), \chi(x), \psi_0(x), \psi_1(x)] \qquad \text{fermions}$$

$$QU(x,\mu) = i\psi_\mu(x)U(x,\mu)$$

$$Q\psi_\mu(x) = i\psi_\mu(x)\psi_\mu(x) - i(\phi(x) - U(x,\mu)\phi(x+\hat{\mu})U(x,\mu)^{-1})$$

$$Q\phi = 0$$

$$\vdots$$



Model

Lattice formulation with

- Nilpotent Q
- Q-exact action (= Q-invariant)

How to make Hamiltonian?

Order parameter for SUSY breaking: Hamiltonian

$\mathcal{H} = 0$: SUSY $\mathcal{H} > 0$: ~~SUSY~~

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$\Downarrow$

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Order parameter for SUSY breaking: Hamiltonian

$\mathcal{H} = 0$: SUSY $\quad \mathcal{H} > 0$: ~~SUSY~~

How to choose the origin?

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Use the algebra! $\{Q, Q_0\} = 2i\partial_0$

$Q\mathcal{J}_0^{(0)} = 2\mathcal{H}$

Discretized "Noether current" for Q_0:

$$\mathcal{J}_0^{(0)}(x) = \frac{1}{a^4 g^2} \mathrm{tr}\Big\{ \eta(x)[\phi(x), \overline{\phi}(x)]^2 + 2\chi(x)H(x) - 2i\psi_0(x)\left(\overline{\phi}(x) - U(x,0)\overline{\phi}(x+a\hat{0})U(x,0)^{-1}\right) + 2i\psi_1(x)\left(\overline{\phi}(x) - U(x,1)\overline{\phi}(x+a\hat{1})U(x,1)^{-1}\right)\Big\}$$

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The conjugate applied field: temperature $Z = \mathrm{tr}\, e^{-\beta H}$

$\Rightarrow$ *anti*-periodic boundary condition for fermions

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- ~~SUSY~~ by the boundary condition(or temperature)
- We need $\beta \to \infty$

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$$(\text{Witten index}) = Z_{\rm PBC} = (\operatorname{tr} e^{-\beta H})_{\rm PBC}$$

$$\langle H \rangle_{\rm PBC} = \frac{-\frac{\partial}{\partial\beta}(\text{Witten index})}{Z_{\rm PBC}} = \frac{0}{Z_{\rm PBC}} \xrightarrow{\text{simulation}} 0$$

	SUSY	~~SUSY~~
correct $\langle H \rangle_{\rm PBC}$	$= 0$	> 0
simulation of "$\langle H \rangle_{\rm PBC}$"	$= 0$	$= 0$

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Our hamiltonian: $\langle Q\text{-exact} \rangle_{\mathrm{PBC}} = 0 \propto -\frac{\partial}{\partial \beta}(\text{Witten index})$

We should measure

the ground state energy:

$$\mathcal{E}_0 \equiv \lim_{\beta\to\infty} \langle \mathcal{H} \rangle_{\mathrm{aPBC}} = \lim_{\beta\to\infty} \langle Q \mathcal{J}_0^{(0)}/2 \rangle_{\mathrm{aPBC}} \begin{cases} = 0 & \text{SUSY} \\ > 0 & \text{~~SUSY~~} \end{cases}$$

Using

- Nilpotent Q
- Q-exact action (= Q-invariant)

Result for SQM

(known): form of the potential $\Rightarrow$ broken or not

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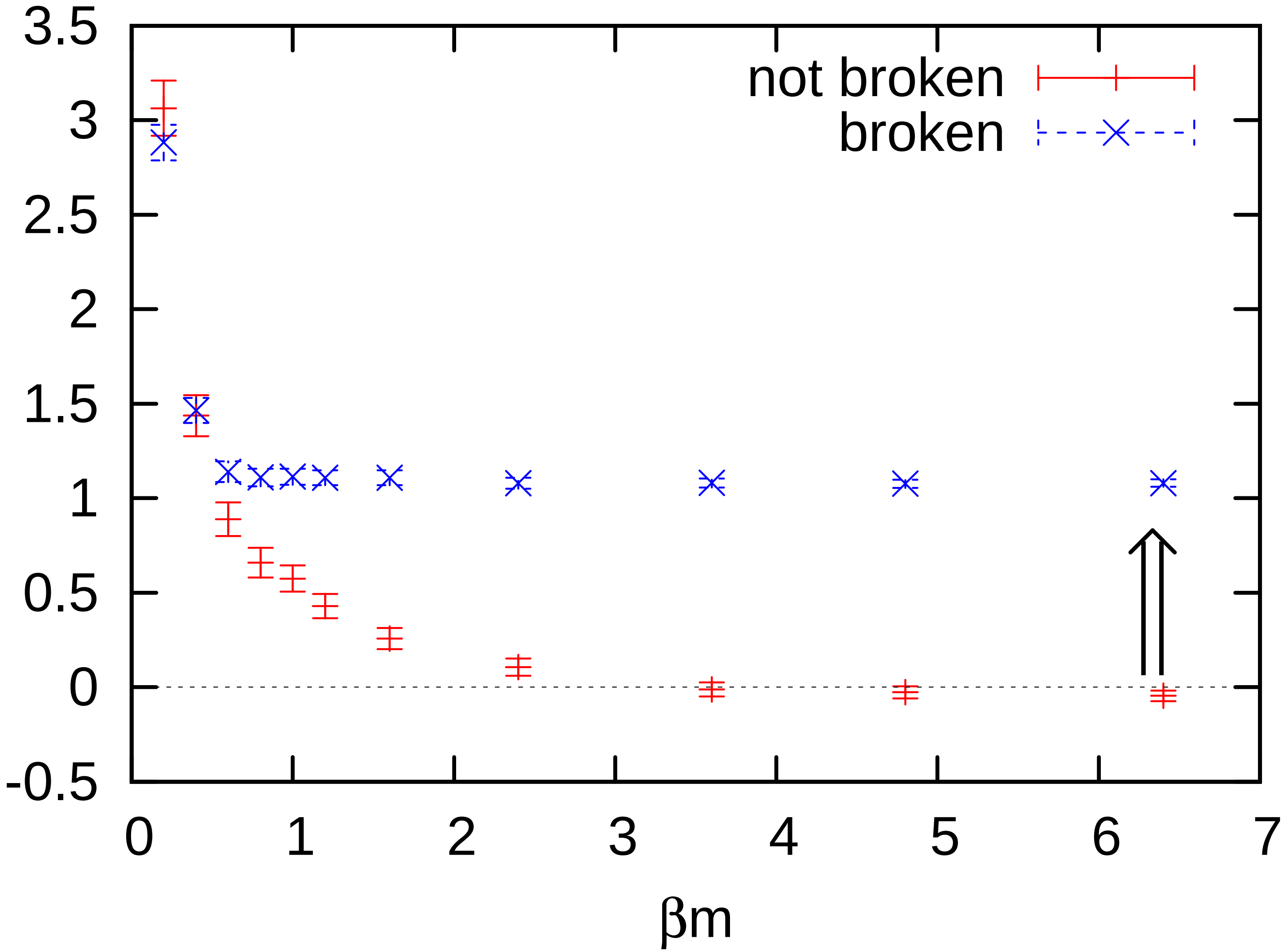
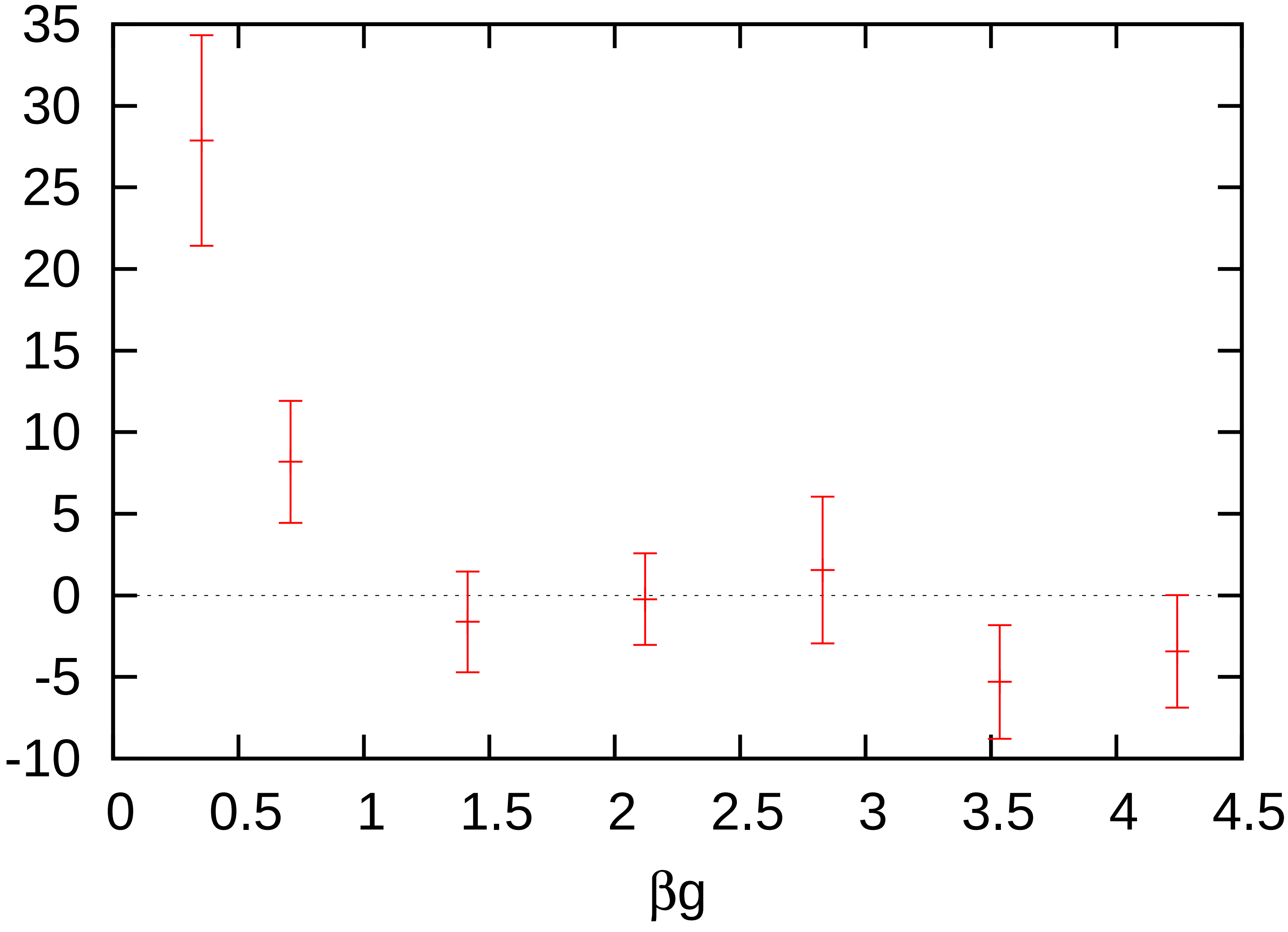


Result for 2-dim $\mathcal{N} = (2,2)$ SYM

$SU(2)$, seems not broken within the error



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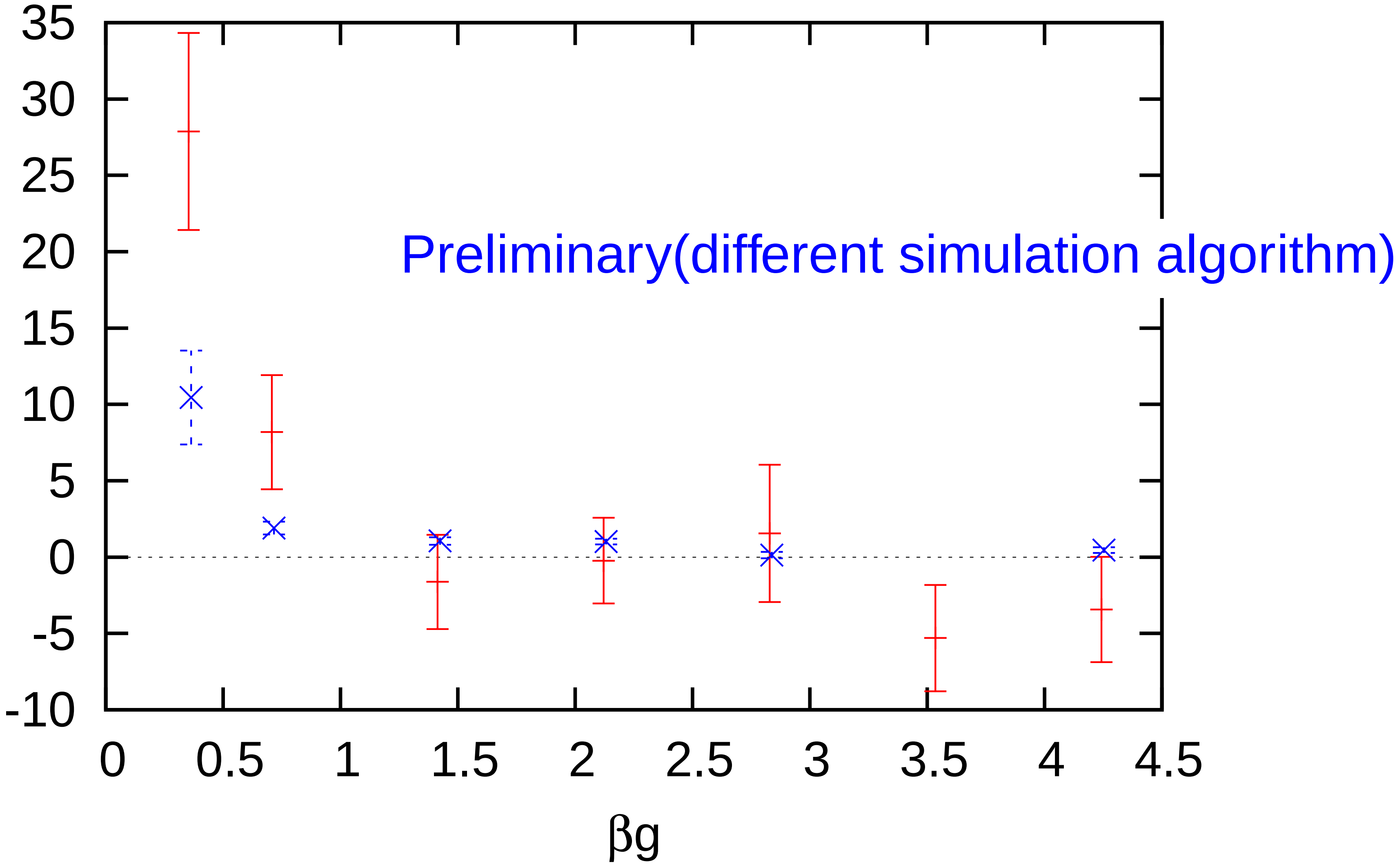
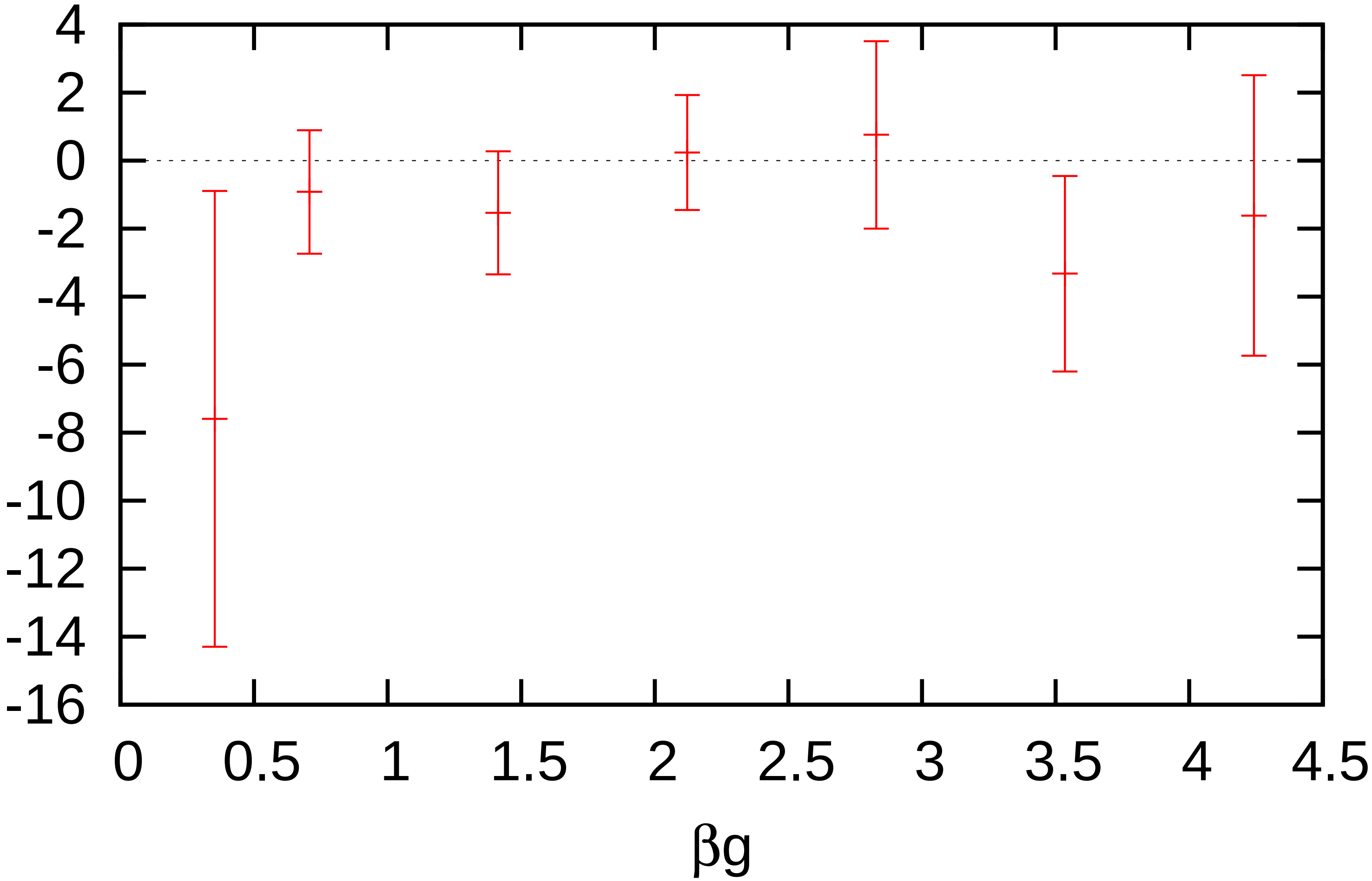


Check for PBC case

$\langle \mathcal{H} \rangle_{\rm PBC} = 0$ within error.

$\Rightarrow$consistent with $\langle \mathcal{H} \rangle_{\rm PBC} \propto \frac{\partial}{\partial \beta}$(Witten index)



Simulation detail(SYM)

- Computer: RIKEN Super Combined Cluster
- quench + reweight: $S = S_b + S_f, \langle \mathcal{O} \rangle_S = \dfrac{\langle \mathcal{O} \mathrm{Pf}(D) \rangle_{S_b}}{\langle \mathrm{Pf}(D) \rangle_{S_b}}$

$$Z = \int \mathcal{D}f\, \mathcal{D}b\, e^{-S_b - S_f} = \int \mathcal{D}b\, \mathrm{Pf}(D) e^{-S_b}$$

- fixed spatial physical length: $gL = 1.414$
- lattice size: 3×6 – 36×12
- lattice spacing: ag = 0.2357 – 0.0707
- 9900–99900 independent configurations for each parameter

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fermion effect: indirect

⇓

Rational Hybrid Monte Carlo(RHMC) less errors

$$Z = \int \mathcal{D}F\, \mathcal{D}b\, e^{-S_b - F^\dagger (D^\dagger D)^{-1/4} F}$$

fermion effect: direct

Conclusion

Method for observing dynamical SUSY breaking using lattice simulation

- Q-exact action with $Q^2 = 0$
- algebraic construction of $\mathcal{H}$
- measure the ground state energy:

  finite temperature $\rightarrow$ zero temperature limit

First physical application of recent development of SUSY on lattice

- Two-dimensional $\mathcal{N} = (2, 2)$ pure SYM: SUSY may *not* be broken
- Application to other system is straightforward

  cf. Sugino's talk (SQCD)

Now we can use the lattice formulation for supersymmetric theories.

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Thank you.

Taking the continuum limit