$\mathcal{N} = \frac{1}{2}$ Supersymmetry In 2 Dimensions

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Outline of Talk

- Introduction to $\mathcal{N} = \frac{1}{2}$ Supersymmetry

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- Motivation

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- Undeformed $\mathcal{N} = 2$ Non-linear Sigma Model

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- Conclusion and Outlook

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Standard Superspace algebra:

$$\{\theta^A, \theta^B\} = 0$$
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$$\{\overline{Q}^i{}_{\dot{A}}, \overline{Q}^j{}_{\dot{B}}\} = -4C^{AB}\sigma^\mu{}_{A\dot{A}}\sigma^\nu{}_{B\dot{B}}\frac{\partial^2}{\partial y^\mu \partial y^\nu}$$

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$$\Longrightarrow \mathcal{N} = \frac{1}{2} \text{ SUSY}$$

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- “Crypto-Hermiticity”

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- Constant Graviphoton background
- Two-dimensional case of interest in string theories as a description of the world sheet

Superfields

Chiral Superfield:

$$\Phi\left(y^{\mu},\theta^{A},\overline{\theta}^{\dot{A}}\right)=\varphi\left(y^{\mu}\right)+\sqrt{2}\theta\psi\left(y^{\mu}\right)+\theta\theta F\left(y^{\mu}\right)$$

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Antichiral Superfield:

$$\overline{\Phi}\left(y^{\mu},\theta^{A},\overline{\theta}^{\dot{A}}\right)=\overline{\varphi}\left(y^{\mu}\right)+\sqrt{2}\overline{\theta\psi}\left(y^{\mu}\right)-2i\theta\sigma^{\nu}\overline{\theta}\partial_{\nu}\overline{\varphi}\left(y^{\mu}\right)$$
$$+\overline{\theta\theta}\left(\overline{F}\left(y^{\mu}\right)+i\sqrt{2}\theta\sigma^{\nu}\partial_{\nu}\overline{\psi}\left(y^{\mu}\right)+\theta\theta\partial^{2}\overline{\varphi}\left(y^{\mu}\right)\right)$$

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Where

$$y^{\mu}=x^{\mu}+i\theta^{A}\sigma^{\mu}_{A\dot{B}}\overline{\theta}^{\dot{B}}$$

are the chiral coordinates

$\mathcal{N} = 2$ Non-Linear σ-Model

$\mathcal{N} = 2$ Lagrangian:

$$\mathcal{L} = \int \mathrm{d}^2\theta \mathrm{d}^2\overline{\theta} K\left(\Phi, \overline{\Phi}\right)$$

where $K\left(\Phi, \overline{\Phi}\right)$ is the Kähler potential

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$$\begin{aligned}\mathcal{L} = 4\,&\partial_{\overline{j}}K(\varphi,\overline{\varphi})\partial_+\partial_-\overline{\varphi}^{\overline{j}} + \partial_i\partial_{\overline{j}}K(\varphi,\overline{\varphi})\big(2i\psi_+{}^i\partial_-\overline{\psi}_+{}^{\overline{j}} + 2i\psi_-{}^i\partial_+\overline{\psi}_-{}^{\overline{j}} + F^i\overline{F}^{\overline{j}}\big)\\ &+ 4\,\partial_{\overline{j}}\partial_{\overline{l}}K(\varphi,\overline{\varphi})\partial_+\overline{\varphi}^{\overline{j}}\partial_-\overline{\varphi}^{\overline{l}} - \partial_i\partial_k\partial_{\overline{j}}K(\phi,\overline{\varphi})\psi_+{}^i\psi_-{}^k\overline{F}^{\overline{j}}\\ &+ \partial_i\partial_{\overline{j}}\partial_{\overline{l}}K(\varphi,\overline{\varphi})\big(2i\psi_+{}^i\overline{\psi}_+{}^{\overline{j}}\partial_-\overline{\varphi}^{\overline{l}} + 2i\psi_-{}^i\overline{\psi}_-{}^{\overline{j}}\partial_+\overline{\varphi}^{\overline{l}} - F^i\overline{\psi}_+{}^{\overline{j}}\overline{\psi}_-{}^{\overline{l}}\big)\\ &+ \partial_i\partial_k\partial_{\overline{j}}\partial_{\overline{l}}K(\varphi,\overline{\varphi})\psi_+{}^i\psi_-{}^k\overline{\psi}_+{}^{\overline{j}}\overline{\psi}_-{}^{\overline{l}}\end{aligned}$$

where $K\left(\phi,\overline{\phi}\right)$ is the Kähler potential

SUSY Operators

The SUSY operators are given by:

$$Q_\pm = \frac{\partial}{\partial\theta^\pm}$$

$$\overline{Q}_\pm = -\frac{\partial}{\partial\overline{\theta}^\pm} - i\theta^\pm\frac{\partial}{\partial y^\pm}$$

Differential Operators

Can derive differential operators that implement the SUSY transformations according to:

$$q_\pm \Phi = [Q_\pm, \Phi] \qquad \overline{q}_\pm \Phi = \left[\overline{Q}_\pm, \Phi\right]$$

Differential operators are given by:

$$q_\pm = \psi_\pm \frac{\partial}{\partial \varphi} \mp F \frac{\partial}{\partial \psi_\mp} - i\partial_\pm \overline{\varphi} \frac{\partial}{\partial \overline{\psi}_\pm} \pm i\partial_\pm \overline{\psi}_\mp \frac{\partial}{\partial \overline{F}}$$

$$\overline{q}^0_\pm = -\overline{\psi}_\pm \frac{\partial}{\partial \overline{\varphi}} \pm \overline{F} \frac{\partial}{\partial \overline{\psi}_\mp} + i\partial_\pm \varphi \frac{\partial}{\partial \psi_\pm} \mp i\partial_\pm \psi_\mp \frac{\partial}{\partial F}$$

Classical and One-Loop Action

Action may be written:

$$S = \int \mathrm{d}^2x \; q_+ q_- \overline{q}^0_+ \overline{q}^0_- K\left(\varphi, \overline{\varphi}\right)$$

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Such that:

$$S_{0B} = \int \mathrm{d}^2x \; q_+ q_- \overline{q}^0_+ \overline{q}^0_- \left(K + \frac{1}{2\pi\epsilon} \mathrm{tr}\left(ln\left(K_{i\bar{j}}\right)\right)\right)$$

is the action at one-loop

Introduce Deformation

Impose non-anticommutation of θ's

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Choose Clifford algebra to be of the form:

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M is the energy scale of the deformation

Star Product

We can implement the Non-anticommutativity by replacing products of fields with the "Star Product" given by:

$$\theta^+ \star \theta^- = \theta^+\theta^- + \frac{1}{2M}, \qquad \theta^- \star \theta^+ = -\theta^+\theta^- + \frac{1}{2M}$$

$$\theta^+ \star \theta^+\theta^- = -\frac{1}{2M}\theta^+, \qquad \theta^- \star \theta^+\theta^- = \frac{1}{2M}\theta^-$$

$$\theta^+\theta^- \star \theta^+\theta^- = \frac{1}{4M^2}$$

Deformed σ-Model

Standard $\mathcal{N} = 2$ Lagrangian:

$$
\begin{aligned}
\mathcal{L} = {} & 4\,\partial_{\bar{j}} K(\varphi,\overline{\varphi})\partial_+\partial_-\overline{\varphi}^{\bar{j}} + \partial_i\partial_{\bar{j}} K(\varphi,\overline{\varphi})\big(2i\psi_+{}^i\partial_-\overline{\psi}_+{}^{\bar{j}} + 2i\psi_-{}^i\partial_+\overline{\psi}_-{}^{\bar{j}} + F^i\overline{F}^{\bar{j}}\big) \\
& + 4\,\partial_{\bar{j}}\partial_{\bar{l}} K(\varphi,\overline{\varphi})\partial_+\overline{\varphi}^{\bar{j}}\partial_-\overline{\varphi}^{\bar{l}} - \partial_i\partial_k\partial_{\bar{j}} K(\varphi,\overline{\varphi})\psi_+{}^i\psi_-{}^k\overline{F}^{\bar{j}} \\
& + \partial_i\partial_{\bar{j}}\partial_{\bar{l}} K(\varphi,\overline{\varphi})\big(2i\psi_+{}^i\overline{\psi}_+{}^{\bar{j}}\partial_-\overline{\varphi}^{\bar{l}} + 2i\psi_-{}^i\overline{\psi}_-{}^{\bar{j}}\partial_+\overline{\varphi}^{\bar{l}} - F^i\overline{\psi}_+{}^{\bar{j}}\overline{\psi}_-{}^{\bar{l}}\big) \\
& + \partial_i\partial_k\partial_{\bar{j}}\partial_{\bar{l}} K(\varphi,\overline{\varphi})\psi_+{}^i\psi_-{}^k\overline{\psi}_+{}^{\bar{j}}\overline{\psi}_-{}^{\bar{l}}
\end{aligned}
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$$+ 4\,\partial_{\bar{j}} \partial_{\bar{l}} K_0(\varphi, F, \overline{\varphi}) \partial_+ \overline{\varphi}^{\bar{j}} \partial_- \overline{\varphi}^{\bar{l}} - \partial_i \partial_k \partial_{\bar{j}} K_0(\varphi, F, \overline{\varphi}) \psi_+{}^i \psi_-{}^k \overline{F}^{\bar{j}}$$
$$+ \partial_i \partial_{\bar{j}} \partial_{\bar{l}} K_0(\varphi, F, \overline{\varphi}) \big(2i\psi_+{}^i \overline{\psi}_+{}^{\bar{j}} \partial_- \overline{\varphi}^{\bar{l}} + 2i\psi_-{}^i \overline{\psi}_-{}^{\bar{j}} \partial_+ \overline{\varphi}^{\bar{l}} - F^i \overline{\psi}_+{}^{\bar{j}} \overline{\psi}_-{}^{\bar{l}}\big)$$
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Deformed $\mathcal{N}=2$ Lagrangian:

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& + \frac{4}{M} \partial_i \partial_{\bar{j}} K_1(\varphi, F, \overline{\varphi}) F^i \partial_+ \partial_- \overline{\varphi}^{\bar{j}} - \frac{4}{M} \partial_i \partial_k \partial_{\bar{j}} K_1(\varphi, F, \overline{\varphi}) \psi_+{}^i \psi_-{}^k \partial_+ \partial_- \overline{\varphi}^{\bar{j}} \\
& + \frac{4}{M} \partial_i \partial_{\bar{j}} \partial_{\bar{l}} K_1(\varphi, F, \overline{\varphi}) F^i \partial_+ \overline{\varphi}^{\bar{j}} \partial_- \overline{\varphi}^{\bar{l}} - \frac{4}{M} \partial_i \partial_k \partial_{\bar{j}} \partial_{\bar{l}} K_1(\varphi, F, \overline{\varphi}) \psi_+{}^i \psi_-{}^k \partial_+ \overline{\varphi}^{\bar{j}} \partial_- \overline{\varphi}^{\bar{l}}
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\end{aligned}$$

$$\textit{where}\quad K_m = \int_{-\frac{1}{2}}^{\frac{1}{2}} \xi^m K\left(\varphi + \frac{\xi}{M}F, \overline{\varphi}\right)\mathrm{d}\xi$$

One-Loop Calculation

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- Kernel - Differential Operators

Deformed Operators

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For instance:

$$\left[\overline{Q}_\pm, \Phi^n_\star \star \overline{\Phi}^m\right]_\star = \left[\overline{q}^\Phi_\pm - \frac{i}{2M}\left(\partial''_\pm q'_\mp - \partial'_\pm q''_\mp\right)\right] \Phi^n_\star \star \overline{\Phi}^m$$

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Only interested in Kähler potential - symmetrised product of fields.

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$$\times\left[K_0\left(\varphi,F,\overline{\varphi}\right) - q_+q_-K_1\left(\varphi,F,\overline{\varphi}\right)\right]$$
$$-\frac{1}{4M^2}\overline{\theta}^+\overline{\theta}^- q'_+ q'_- \partial''_+\partial''_- K_0\left(\varphi,F,\overline{\varphi}\right)$$

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$$\begin{aligned} K_\star\left[\Phi,\overline{\Phi}\right] &= \left(1+\theta^+ q_+\right)\left(1+\theta^- q_-\right)\left(1-\overline{\theta}^+\overline{q}_+^{0\prime\prime}\right)\left(1-\overline{\theta}^-\overline{q}_-^{0\prime\prime}\right) \\ &\quad \times\left[K_0\left(\varphi,F,\overline{\varphi}\right)-q_+q_-K_1\left(\varphi,F,\overline{\varphi}\right)\right] \\ &\quad -\frac{1}{4M^2}\overline{\theta}^+\overline{\theta}^- q'_+ q'_- \partial''_+\partial''_- K_0\left(\varphi,F,\overline{\varphi}\right) \end{aligned}$$

$\frac{1}{2M}$-terms have vanished due to symmetrisation

Kähler Potential

K may be expressed as:

$$K_\star\left[\Phi,\overline{\Phi}\right] = \left(1+\theta^+ q_+\right)\left(1+\theta^- q_-\right)\left(1-\overline{\theta}^+\overline{q}_+^{0\prime\prime}\right)\left(1-\overline{\theta}^-\overline{q}_-^{0\prime\prime}\right)$$
$$\times\left[K_0\left(\varphi,F,\overline{\varphi}\right) - q_+ q_- K_1\left(\varphi,F,\overline{\varphi}\right)\right]$$
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$\frac{1}{2M}$-terms have vanished due to symmetrisation

$\frac{1}{4M^2}$ “cross-term” remains

Kähler Potential Operators

Differential operators now take the form:

$$\overline{q}_{\pm} = -\frac{i}{4M^2}\left(\partial''_{\pm} q'_{+} q'_{-} q''_{\mp} \pm \partial'_{\pm} q'_{\mp} q''_{+} q''_{-}\right) + \overline{q}^{\Phi}_{\pm}$$

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$$-\frac{i}{2M}\partial_{\pm}q_{\mp} \pm i(-q'_{+}q'_{-}[\partial'_{\pm}q'_{\mp}]\widetilde{\mathcal{O}} + \partial'_{\pm}q'_{\mp}\mathcal{O} + [\partial'_{\pm}q'_{\mp}]\mathcal{O})$$

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Require $\mathcal{O}$, $\widetilde{\mathcal{O}}$ to satisfy:

$$\mathcal{O} K_0^{(n)} = K_1^{(n)}$$
$$\mathcal{O} K_1^{(n)} = K_2^{(n)} - \widetilde{\mathcal{O}} K_0^{(n)}$$

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$$\text{where } \mathcal{O} = \sum_{r=1}^{\infty} a_r \left(\frac{1}{M^2}\right)^r \left(F\frac{\partial}{\partial\varphi}\right)^{2r-1}$$

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a_r's satisfy:

$$\sum_{r=1}^{n-1} \frac{a_{n-r}}{2^{2r}(2r+1)!} = \frac{1}{2^{2n}(2n+1)(2n-1)!} \quad \forall n \geqslant 1$$

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Compute a_r's iteratively:

$$a_1 = \frac{1}{12}, \qquad a_2 = -\frac{1}{720}, \qquad a_3 = \frac{1}{2^5\cdot 3^3\cdot 5\cdot 7}, \ldots$$

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In analogy with the undeformed case, we can use the differential operators to write the action as:

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&= \int \mathrm{d}^2x \; q_+ q_- \overline{q}_+ \overline{q}_- \left(K_0 - q_+ q_- K_1\right) \\
&= \int \mathrm{d}^2x \; q_+ q_- \overline{q}_+ \overline{q}_- K_0
\end{aligned}
$$

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So the deformation can be seen as "smearing" out the background geometry of the σ-model

One-Loop Deformed Action

To summarise:

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	Undeformed	Deformed
Classical	$S = \int \mathrm{d}^2x\, q_+ q_- \overline{q}^0_+ \overline{q}^0_-\, K$	$S = \int \mathrm{d}^2x\, q_+ q_- \overline{q}_+ \overline{q}_-\, K_0$
One-Loop	$S = \int \mathrm{d}^2x\, q_+ q_- \overline{q}^0_+ \overline{q}^0_- \left(K + \frac{1}{2\pi\epsilon} \mathrm{tr} ln \left(K_{i\overline{j}} \right) \right)$	?

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One-Loop	$S = \int d^2x\, q_+ q_- \overline{q}^0_+ \overline{q}^0_- \left(K + \frac{1}{2\pi\epsilon} \mathrm{tr} ln \left(K_{i\bar{j}} \right) \right)$	?

Can we write one-loop deformed action in a similar form?

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However, when we write down the one-loop diagrams to lowest order in M, we find they are invariant under $q_\pm$ but not $\overline{q}_\pm$

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May be able to write:

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But cannot write:

$$\text{“}One - Loop\text{”} = q_+ q_- \overline{q}_+ \overline{q}_- (X)$$

One-Loop Kernel

Restricting ourselves to first order in $1/M^2$, can write one-loop contributions in the form:

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Part of kernel may be written concisely as:

$$\mathcal{K}_B^{(1)\prime} = \frac{\partial^2 L_{M^2}}{\partial F^i \partial \overline{\varphi}^{\bar{j}}} K^{i\bar{j}} - \frac{\partial^2 L_{M^2}}{\partial F^i \partial F^j} K^{i\bar{k}} K^{j\bar{l}} \left(K_{\overline{kl}m} F^m - K_{\overline{kl}mn} \psi^m \psi^n \right)$$

where L_{M^2} is the M^2 term of the action. However, the remainder cannot be written so neatly.

One-Loop Kernel

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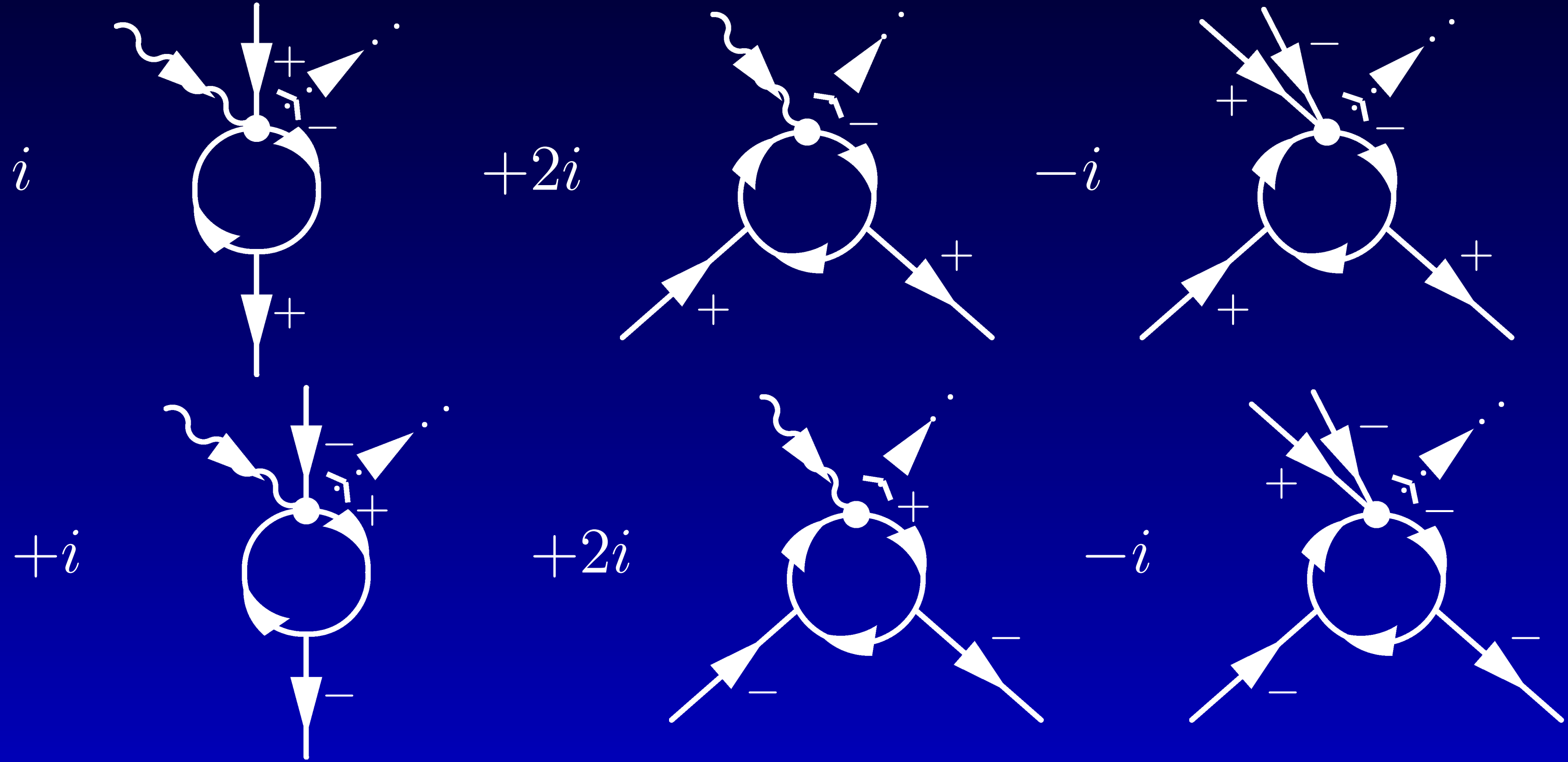
Full Kernel:

$$\mathcal{K}_B^{(1)} = \frac{\partial^2 L_{M^2}}{\partial F^i \partial \overline{\varphi}^{\bar{j}}} K^{i\bar{j}} - \frac{\partial^2 L_{M^2}}{\partial F^i \partial F^j} K^{i\bar{k}} K^{j\bar{l}} \left(K_{\overline{kl}m} F^m - K_{\overline{kl}mn} \psi^m \psi^n \right) + \frac{1}{24M^2} \left(A_1 + A_2 + 2A_3 + 2A_4 \right)$$

The A_i's represent $\mathcal{N} = \frac{1}{2}$ invariant sections of the kernel. When written diagrammatically, some patterns emerge.

Example

A_3 is given by:



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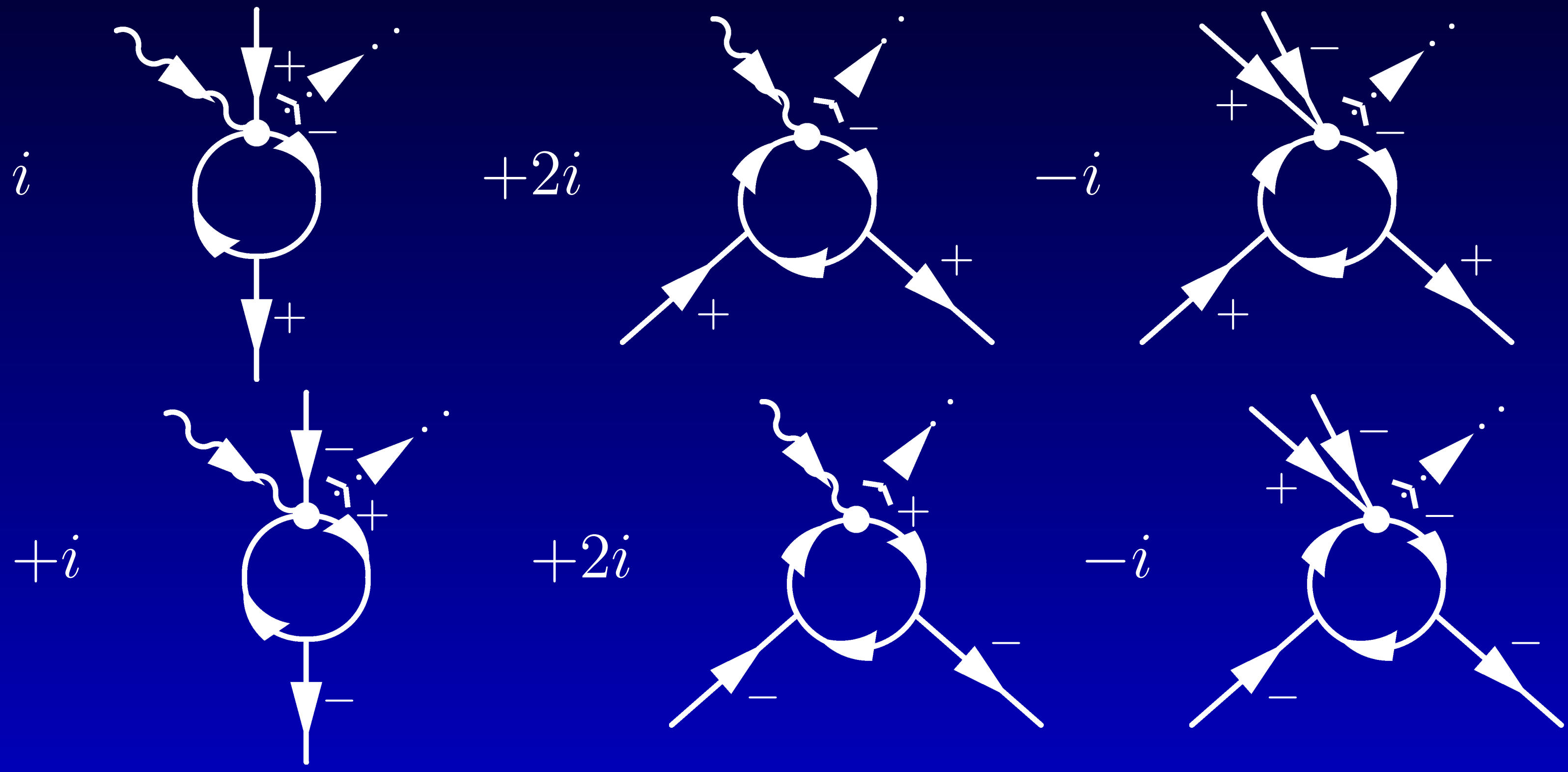

A_i coming from $\partial_+\partial_-\overline{\varphi}$-type terms much larger

Conclusion & Outlook

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- Smearing invalid at quantum level
- Now studying 4-dimensional model

Thank You

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