# Modulated inflation

**Tomohiro Matsuda**
(Saitama Institute of Technology)

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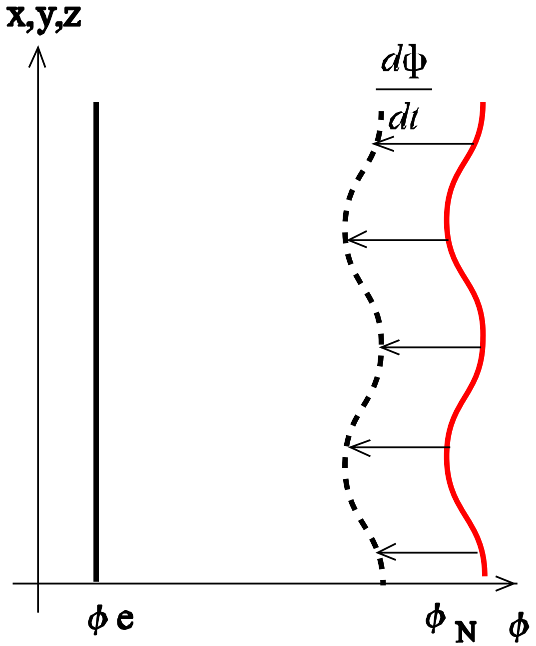
## Abstract

We consider cosmological perturbations caused by modulated inflaton velocity. During inflation, the inflaton is damped and the velocity is determined by the parameters such as couplings or masses that depend on the moduli. The number of e-foldings is different in different patches if there are spatial fluctuations of such parameters. Based on this simple idea, we consider "modulated inflation" in which the curvature perturbation is generated by the fluctuation of the inflaton velocity.

Let me first discuss common ideas for cosmological perturbations in terms of the $\delta N$ formalism. If $H$ is a constant, $\delta N$ is given by $\delta N \simeq H\delta t$.

## Traditional Scenario

According to the traditional inflationary scenario, the curvature perturbation is generated at the horizon crossing, which is indicated by the red line below ($\sim \delta\phi_N$)

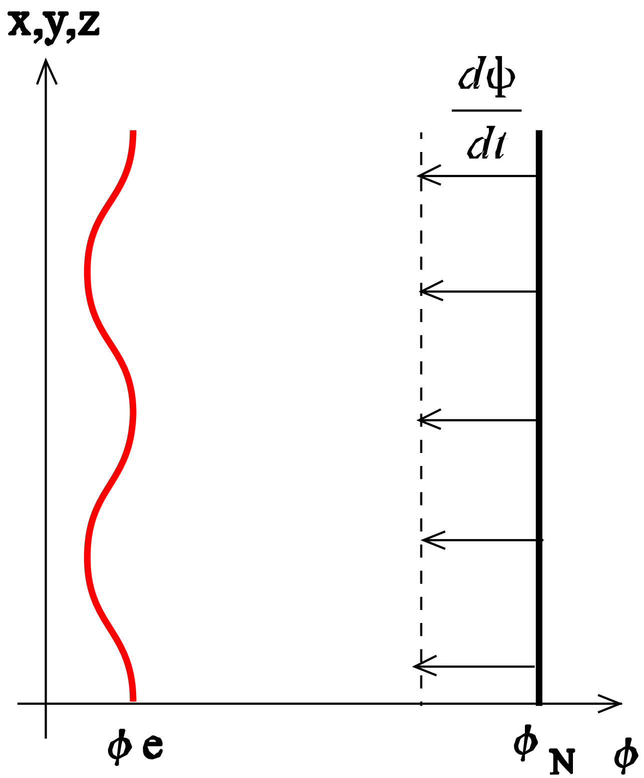



In terms of the $\delta N$-formalism, the fluctuation of the time elapsed during inflation $\delta t \simeq \delta\phi_N/\dot{\phi}$ is caused by the fluctuation of the start-line.

Another idea for cosmological perturbation is

**"At the end" Scenario**

The curvature perturbation is generated at the end. The fluctuation of the goal-line is caused by the fluctuation of a light field ($\mathcal{M}$), $\phi_e(\mathcal{M}) \to \delta\phi_e \simeq \phi_e' \delta\mathcal{M}$.




$\delta N \simeq H\delta t$ is caused by the fluctuation of the goal-line, $\delta\phi_e$

It is very natural to consider the fluctuation of the distance ($\delta\phi$) to obtain $\delta t$ that causes the fluctuation of the number of e-foldings $\delta N$.

However, we know that the time elapsed after horizon crossing $\sim |t_N - t_e|$ depends not only on the distance $\sim |\phi_N - \phi_e|$ but also on the inflaton velocity $\dot{\phi}$.

$$\downarrow$$

If the inflaton velocity depends on a light field(moduli) $\mathcal{M}$, the fluctuation $\delta\mathcal{M}$ may lead to the fluctuation of the inflaton velocity $\delta\dot{\phi} \simeq (\dot{\phi})'\delta\mathcal{M}$, which eventually causes $\delta t$ and $\delta N$.

The effect would be obvious if

(1) there are many massless degrees of freedom that may affect the inflaton velocity;

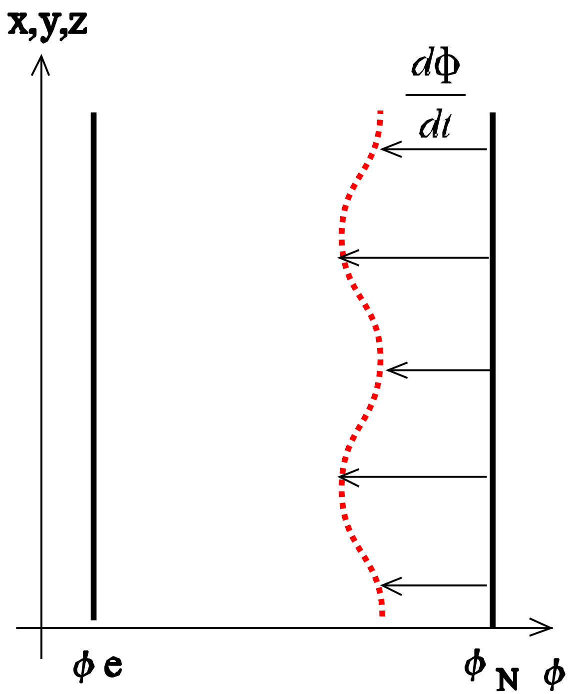
(near ESP, there would be significant non-gaussinity from $\delta\dot{\phi}(\mathcal{M}_i)$)

(2) the inflaton mass is slightly larger than the Hubble parameter and there is no significant perturbation from $\delta\phi_N$. $\ast$ Inflation is fast-rolling but not oscillatory.

Therefore, in this talk we focus on

Modulated inflaton velocity

Spatial fluctuation of the time elapsed during inflation $\delta t$ is caused by the fluctuation of the inflaton velocity. $\ast$ See the red dotted line.




$\delta N \simeq H\delta t$ is generated by the fluctuation of the inflaton velocity $\delta\dot{\phi}(\mathcal{M}_i)$, even if the start and the end-line is completely flat.

Moreover, if the fundamental parameters are determined by the moduli in the underlying (string) theory, it would be natural to think that;

**Planck scale may depend on moduli; $M_p(\mathcal{M})$**

The Planck scale $M_p$ may depend on moduli that may have fluctuations during inflation. Thus, it is important to ask what is the consequence of $\delta M_p$ that is caused by $\delta\mathcal{M}$.

We can see that the perturbation related to $\delta M_p(\mathcal{M})$ is explained in terms of $\delta\dot{\phi}$.

Let us see more details of this scenario, considering some simple examples.

The first example is

Standard kinetic term and conventional interaction term in $V(\phi, \mathcal{M})$.

**Basic formula**

The definition of the number of e-foldings for constant $H$ is

$$N = \int H dt = \int H \frac{\dot{\phi} d\phi + \dot{\mathcal{M}} d\mathcal{M}}{\dot{\phi}^2 + \dot{\mathcal{M}}^2}. \quad (1)$$

If there is no bend in the trajectory($\dot{\mathcal{M}} \simeq 0$), the perturbation related to the inflaton velocity is expanded as

$$\delta N \simeq -\int_{\phi_e}^{\phi_N} \frac{H}{\dot{\phi}^2} \left( \delta\dot{\phi} - \dot{\phi} A \right) d\phi, \quad (2)$$

where we consider linear scalar perturbations of the metric,

$$ds^2 = -(1+2A)dt^2 + 2aB_i dx^i dt + a^2 \left[ (1-2\psi)\delta_{ij} + 2E_{ij} \right] dx^i dx^j. \quad (3)$$

From the conventional energy and momuntum constraints we find

$$\dot{\phi}\left(\dot{\delta\phi}-\dot{\phi}A\right)-\ddot{\phi}\delta\phi \simeq \dot{\phi}\left(\dot{\delta\phi}-\dot{\phi}A\right) \propto \frac{k^2}{a^2}, \tag{4}$$

which suggests that the factor appearing in the delta-N formula decays after horizon crossing. Therefore, $e^{-2Ht}$ factor must be included in the calculation, even if the perturbation $\dot{\phi}\delta\dot{\phi}$ itself is supposed to be a constant.

Of course, $e^{-2Ht}$ in the integral does not lead to an exponential suppression after the integration. It is easy to see that there is no exponential suppression for large $N(\sim Ht)$;

$$\int_0^{t_e} \delta C H e^{-2Ht} dt \simeq \frac{1}{2}\delta C. \tag{5}$$

The actual suppression factor is not significant for the modulated velocity.

## In fact,

Correction from the terms proportional to $k^2/a^2$ has been disregarded in previous studies, since $k^2/a^2$ is obviously small at a distance. However, if they appear in the equation of $\dot{\mathcal{R}}$, these terms may yield significant correction to $\mathcal{R}$ after integration, as we will see later in this talk.

Next, we consider moduli-dependent kinetic term.

Inflaton kinetic term $\sim \frac{1}{2}\omega(\mathcal{M})g^{ab}\phi_{,a}\phi_{,b}$

The number of e-foldings is

$$N = \int H dt \simeq \int \frac{H}{\dot{\phi}} d\phi \simeq \int \frac{3H^2}{V_\phi}\omega d\phi. \tag{6}$$

Again, we assume no bend in the trajectory.

$\delta N$ that is related to the moduli perturbation ($\delta\mathcal{M}$) is

$$\delta N^{(\delta\mathcal{M})} \simeq \int \frac{3H^2}{V_\phi}\omega'\delta\mathcal{M} d\phi \simeq \frac{\omega'\delta\mathcal{M}}{\omega}\int \frac{3H^2}{V_\phi}\omega d\phi \simeq \frac{\omega'\delta\mathcal{M}}{\omega} N \tag{7}$$

What is important here is $\rightarrow$

**Note**

Unlike the perturbation caused by the potential, the constraints from the energy and momuntum does not yield $\frac{k^2}{a^2}$ factor for the perturbation related to the kinetic term.

Let us see what happens if the boundary perturbations are flat while the modulated velocity is significant. We consider ↓

**Fast-roll inflation with hybrid-type potential**

Inflaton may have large mass ($m_\phi \simeq O(1)H$) due to the "$\eta$-problem". Then $\delta\phi$ may either decay or cannot cross the horizon during inflation. Non-oscillationary (fast-roll) inflation is possible if the friction is significant.

Hybrid inflation has the effective potential

$$V(\phi,\sigma) = \lambda\left(\sigma^2 - v^2\right)^2 + \frac{1}{2}g^2\phi^2\sigma^2 + V(\phi), \tag{8}$$

where $\phi$ is the inflaton and $\sigma$ is the trigger field. Here the end of inflation expansion occurs at

$$\phi_e = \frac{\sqrt{\lambda}v}{g}, \tag{9}$$

and the number of e-foldings is given by

$$N = \frac{1}{F_\phi} \log \frac{\phi_N}{\phi_e}, \tag{10}$$

where $F \equiv \frac{3}{2}\left(1 - \sqrt{1 - 4m^2/9H^2}\right)$. For $m \simeq H$, modulated mass leads to the fluctuation ($k^2/a^2$ is considered)

$$\delta N_{\mathcal{M}} \propto m' \delta \mathcal{M}, \tag{11}$$

where $m'$ is the derivative of $m$ with respect to $\mathcal{M}$. For example, for

$$m^2(\mathcal{M}) \equiv m_0^2 \left[1 + \beta \log(\mathcal{M}/M_*)\right], \tag{12}$$

we find $\delta N$ given by

$$\delta N_{\mathcal{M}} \simeq \beta \left(\frac{\delta \mathcal{M}}{\mathcal{M}}\right). \tag{13}$$

Since the mass of $\mathcal{M}$ must be less than $H_I$ during inflation, we find the condition

$$m_{\mathcal{M}}^2 \simeq \beta m_0^2 \left(\frac{\phi_N}{\mathcal{M}}\right)^2 < H_I^2. \tag{14}$$

The non-gaussinity parameter is

$$f_{nl} = -\frac{5}{6}\frac{N''}{(N')^2} \propto \frac{1}{\beta}, \tag{15}$$

which can be large and may take either sign.

Next, we consider the case in which modulated velocity adds significant non-gaussinity to the standard perturbation.

**Our question is**

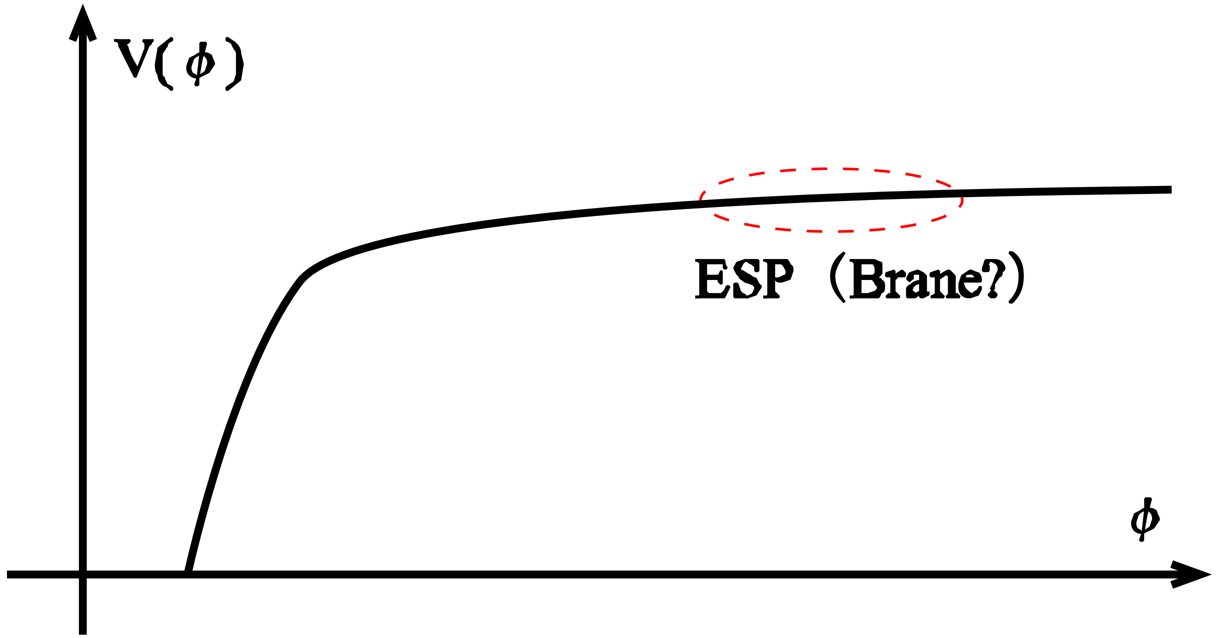
"Is it possible for the modulated velocity to add significant non-gaussinity to the standard inflationary perturbation after horizon crossing?"

Our answer is "Yes" $\longrightarrow$

## Non-gaussianity at ESP

We consider a simple mechanism to "add" large non-gaussianity to the standard inflationary perturbation. We consider hybrid inflation with standard($\sim g^2\phi^2\mathcal{M}^2$) interaction. During inflation, hybrid inflation has the effective potential

$$V(\phi,\mathcal{M}) = V_0 + \frac{1}{2}m_\phi^2\phi^2 + g^2(\phi-\phi_{ESP})^2\mathcal{M}_i^2. \qquad (16)$$



The perturbation of the inflaton velocity caused by the number of "n" massless excitation is

$$\delta\dot{\phi} \simeq \frac{ng^2(\phi - \phi_{ESP})\delta\mathcal{M}^2}{3H} \simeq ng^2\dot{\phi} \times \left(\frac{\phi - \phi_{ESP}}{\phi}\right)\frac{\delta\mathcal{M}^2}{m_\phi^2} \quad (17)$$

where the first order perturbation vanishes because $< \mathcal{M} >_0 = 0$. The second order perturbation at ESP adds significant non-gaussianity

$$\hat{f}_{NL} \simeq \eta_\phi \times \eta_\mathcal{M} \times n\left(\frac{\phi}{\phi - \phi_{ESP}}\right). \quad (18)$$

which is the so-called "uncorrelated non-gaussianity". The usual $f_{NL}$ is $f_{NL} \sim (\hat{f}_{NL}/1300)^3$. $\rightarrow$ We can "add" significant non-gaussianity to the standard perturbation.

Finally, we consider modulated Planck scale in terms of the delta-N formalism.

**$\delta M_p$ from $M_p(\mathcal{M})$**

We consider a light scalar field $\hat{\mathcal{M}}$ coupled to gravity. The theory can be given by

$$S = \int d^4x\sqrt{-g}\left[f(\hat{\mathcal{M}})R - g(\hat{\mathcal{M}})\left(\nabla\hat{\mathcal{M}}\right)^2 - \frac{1}{2}\left(\nabla\phi\right)^2 - V(\phi)\right]. \quad (19)$$

Here the Planck scale is replaced by a function of the moduli.

Considering (for simplicity) Jordan-Brans-Dicke theory, the action in the Einstein frame is given by the action with the moduli-dependent kinetic term $\sim \frac{1}{2}\omega(\mathcal{M})g^{ab}\phi_{,a}\phi_{,b}$, where

$$\omega(\mathcal{M}) = \exp\left(-\beta\kappa\mathcal{M}\right) \tag{20}$$

and the potential

$$V = \omega^2 W(\phi). \tag{21}$$

Note that there are both sources, from the potential and the kinetic term.

The perturbation caused by the potential has the factor $k^2/a^2$, while the one from the kinetic term does not have such factor.

The story in the Einstein frame is completely the same as the one that has been discussed in this talk.

## Summary

We considered cosmological perturbations caused by modulated inflaton velocity.

Important results are;

1. A small deviation from the standard perturbation may be explained by the modulated velocity.

2. The curvature perturbation can be generated during inflation even if the boundaries are completely flat.

3. It is possible to add significant non-gaussianity after horizon crossing.