Inflation by non-minimal coupling

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I am the inflaton.
I will roll down slowly.

Weird shape!
Often, it is not easy to get this potential out of simple set-up or simple assumptions.
In this talk

- I want to show you that the non-minimal coupling ($\sim K(\Phi) R$) actually provides a very simple way of getting the ‘weird’ potential.

- You will get very intriguing bonus here:
  Inflaton = Higgs! / Inflaton = Dark Matter!

Q. By the way, why do I think of the non-minimal coupling term after all??
(at least) Three good reasons

- $(a \phi^2)R$ is the same order of $M^2R$. So there is no good reason why there is no such term.
- Conformal symmetry
- Supersymmetry
The simplest scalar theory

- In flat space, without gravity, the massless, real scalar field theory is conformally invariant. \((s=1-D/2)\)

\[
S = \int d^4x \frac{1}{2} \eta^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi
\]

\[
\eta_{\mu\nu} \rightarrow \Omega^2 \eta_{\mu\nu}, \varphi \rightarrow \Omega^s \varphi
\]

- However, when gravity comes in, it is not invariant.

\[
S = \int d^4x \sqrt{-g} \left\{ \frac{M^2}{2} R + \frac{1}{2} g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi \right\}
\]

\[
g_{\mu\nu} \rightarrow \Omega^2 g_{\mu\nu}, \varphi \rightarrow \Omega^s \varphi
\]

\[
R \rightarrow \Omega^{-2}(R - 2(D-1)\nabla^2 \ln \Omega - (D-2)(D-1)(\partial \ln \Omega)^2)
\]

- There is a simple cure of this. It is nothing but the non-minimal coupling term.

\[
S = \int d^4x \sqrt{-g} \left\{ \frac{M^2 + a\varphi^2}{2} R + \frac{1}{2} g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi \right\}
\]

\[
a = -\frac{D-2}{4(D-1)}
\]
SUGRA + chiral super fields

\[ S = \int d^4x \left\{ L_{\text{SUGRA}} + \frac{1}{2} [K(\Phi, \Phi^*)]_D + 2 \text{Re}[W(\Phi)]_F \right\} \]

\[ L_{\text{bosonic}} = \sqrt{-g} \left\{ \frac{M^2 - K(\varphi, \varphi^*)}{3} R + g_{mn} \partial_\varphi_m \partial_\varphi^*_n + \ldots \right\} \]

Again, non-minimal coupling term arises. It is nothing but the Kahler potential.
A theory with the Lagrangian in the form of

$$ S = \int d^4x \sqrt{g} \left[ \frac{M^2 + K(\phi)}{2} R + \frac{1}{2} (\partial \phi)^2 - V(\phi) \right] $$

can provide the flat potential (for inflation) at the large field value limit if

$$ \lim_{\phi \to \infty} \frac{V(\phi)}{K(\phi)^2} = Const. $$

[Note]
(1) This Lagrangian is actually generic in CFT (String theory) and generic SUGRA.
(2) We assumed $K, V \to \infty$ at $\phi \to \infty$. 

[SCP & S. Yamaguchi (2008)]
Schematics of the proof

Start from the theory with the non-minimal coupling

\[ S = \int d^4x \sqrt{g} \left[ \frac{M^2 + K(\phi)}{2} R + \frac{1}{2}(\partial \phi)^2 - V(\phi) \right] \]

Move to the Einstein Frame by conformal transformation

\[ S_{EH} = \int d^4x \sqrt{g_E} \left[ \frac{M_{pl}^2}{2} R_E + \frac{1}{2}(\partial h)^2 - U(h) \right] \]

See the ‘physical’ Potential \( U(h) \) and read out the condition for ‘flatness’ at high energy.

\[ \lim_{\phi \to \infty} \frac{V(\phi)}{K(\phi)^2} = \text{Const.} \]
Proof: Step 1

- go to the Einstein Frame:

Take conformation transformation

\[
\begin{align*}
    g_{\mu\nu} &= e^{-2\omega} g_{\mu\nu}^E, \\
    e^{2\omega} &\equiv \frac{M^2 + K(\varphi)}{M_{Pl}^2}
\end{align*}
\]

\[
\begin{align*}
\sqrt{-g} &\to e^{-4\omega} \sqrt{-g_E} \\
R &\to e^{-2\omega} (R - 2(D - 1)\nabla^2 \omega - (D - 2)(D - 1)(\partial \omega)^2) = R_E
\end{align*}
\]

We get

\[
\int d^4x \sqrt{-g_E} \left( \frac{M_{Pl}^2}{2} R_E + \frac{3}{4} \frac{K'(\varphi)^2}{e^{4\omega} M_{Pl}^2} (\partial \varphi)^2 + \frac{1}{2} e^{-2\omega} (\partial \varphi)^2 - e^{-4\omega} V(\varphi) \right)
\]

- Einstein action
- Non-canonical scalar kinetic term
- Scalar potential
Find canonically normalized field ‘h’

\[
S_{EH} = \int d^4 x \sqrt{-g_E} \left[ \frac{M_{Pl}^2}{2} R_E + \frac{1}{2} (\partial h)^2 - U(h) \right]
\]

where \( U(h) = e^{-4w}V(\varphi) \)

\[
\frac{dh}{d\phi} = \sqrt{\frac{M_{Pl}^2}{M^2 + K(\varphi)} + \frac{3 M_{Pl}^2}{2 (M^2 + K(\varphi))^2} K'(\varphi)^2}.
\]
Proof: Step-3

- Read the condition for the flatness at the high energy:

\[ U(h) = e^{-4w}V(\varphi) \quad e^{2w} = \frac{M^2 + K(\varphi)}{M_{pl}^2} \]

\[ U(h) = \left( \frac{M^2 + K(\varphi)}{M_{pl}^2} \right)^{-2}V(\varphi) \]

For the increasing (e.g. monomial) function of K and V,

\[ U(h) \approx M_{pl}^4 \frac{V(\varphi)}{K(\varphi)^2} \]

Then we get the flat potential at high energy.

\[ \lim_{\varphi \to \infty} \frac{V(\varphi)}{K(\varphi)^2} = \text{Const.} \]
Let’s take a closer look

At small field value limit:

\[ K \ll M^2 \approx M^2_{pl}, \quad \frac{dh}{d\varphi} \approx 1, h \approx \varphi, U \approx V \]

At large field value limit:

\[ K \gg M^2, U \approx M^4_{pl} \frac{V}{K^2} \approx \text{Const.} \]

- This is exactly what we want to have for the ‘slow-roll’ inflation!!
There are obvious advantages of scenario with non-minimal coupling

Low energy physics is given by $V(\phi)$

- Take $V$ for your low energy model

Inflation takes place when $V/K^2 = \text{const}$.

- Take a proper $K$ for having inflation
[Ex1] The SM Higgs as the Inflaton

\[ K = \alpha |\varphi|^2, \]
\[ V = \lambda (|\varphi|^2 - v^2)^2 \]

At low energy, the inflaton is responsible for the EWSB.
At high energy, the Higgs is responsible for the inflation.
To fit the density fluctuation:

\[ \sqrt{\frac{\lambda}{\alpha^2}} \sim 10^{-5} \]
[Ex2] Monomial Potential

\[ K = a\phi^m, \quad V = \frac{\lambda}{2m}\phi^{2m} \]

\[ V / K^2 \to \text{Const.} \]

\[ U = \frac{M^4_{\text{Pl}}\lambda}{2ma^2} \left( 1 + \frac{M^2}{a}\phi^{-m} \right)^{-2} \]

[Note]
This potential is exactly same with the potential in \textit{brane inflation} where the inflaton is the radion field.
Another observable is the amplitude of the scalar perturbation.

$$\delta_H = \frac{\delta \rho}{\rho} \approx \frac{1}{5 \sqrt{3} H M_{Pl} U'} U^{3/2} = 1.91 \times 10^{-5}.$$ 

This gives a constraint for the parameters

$$\frac{U}{\epsilon} = (0.027 M_{Pl})^4.$$  

Small ratio is commonly Required.

$$\sqrt{\frac{\lambda_0}{a_0}} \approx 2.3 \times 10^{-5}, \quad (m = 1)$$

$$\sqrt{\frac{\lambda_0}{a_0^2 (1+1/(6a_0))}} \approx 2.1 \times 10^{-5}, \quad (m = 2)$$

$$\sqrt{\frac{\lambda_0}{a_0^2}} \approx 1.5 \times 10^{-5} \sqrt{m}, \quad (m \geq 3).$$
WMAP 5yrs data

Hybrid Inflation

Chaotic Inflation-like
$1 < \bar{\phi}$

Transition
$\frac{2}{3} < \bar{\phi} < 1$

Not Allowed

Flat Potential
$\bar{\phi} = 2/3$

Non-minimal Coupling

SCP & S. Yamaguchi arXiv0801:0722

$m = 2$
$m \geq 3$

$r = 1/s$
If $\sigma \neq 0$, the inflaton field can be thermalized with the SM. The relic density of the inflaton is controlled by it.
Smallness problem

Why is it small?

\[ \sqrt{\frac{\lambda}{a^2}} \sim 10^{-5} \]

Higher Dimensional embedding:

The starting \((4 + n)\)-dimensional action is written as follows:

\[
S = \int d^{4+n} \sqrt{-g} \left[ \left( -\frac{M^{2+n} + a\phi^2}{2} \right) R + \frac{1}{2} \partial_M \phi \partial^M \phi - \frac{\lambda}{4} (\phi^2 - v^2)^2 \right]
\]

Introduce dimensionless parameters:

\[
a = a_0, \\
\lambda = \frac{\lambda_0}{M^n}.
\]

\([\varphi] = 1 + n/2 \]
\([R] = 2 \]
For order of unity $\lambda_0$, $a_0$,

$$\frac{\delta \rho}{\rho} \approx 2.1 \times 10^{-5} \approx \sqrt{\frac{\lambda_0}{a_0^2}} (M L)^{-n/2}$$

Even though we started from the theory with $O(1)$ parameters, We can get the small ratio due to large volume!
We can get the “flat” potential in a simple way with $K(\phi)R$ if $V/K^2 \rightarrow \text{const.}$

This class is theoretically well motivated. (conformal theory, generic SUGRA etc.)

Predictions are in good agreement with the real data (e.g. $n_s$, $r$, no non-Gaussianity etc.)

This class of model has a big room for model builders who want to connect the low energy physics (LHC) and Cosmology!

Many works to be done.