Improvement of the Determination of the WIMP Mass from Direct Dark Matter Detection Data

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in collaboration with M. Drees

based on JCAP 0806, 012 (arXiv:0803.4477 [hep-ph])

Determining the WIMP mass model-independently

Correcting the systematic deviation of the reconstructed WIMP mass

Summary

Determining the WIMP mass model-independently

- Differential event rate for elastic WIMP-nucleus scattering

$$\frac{dR}{dQ} = \mathcal{A} F^2(Q) \int_{v_{\rm min}}^{v_{\rm esc}} \left[\frac{f_1(v)}{v}\right] dv$$

Here

$$v_{\rm min} = \alpha\sqrt{Q}$$

is the minimal incoming velocity of incident WIMPs that can deposit the recoil energy Q in the detector.

$$\mathcal{A} \equiv \frac{\rho_0 \sigma_0}{2 m_\chi m_{\rm r,N}^2} \qquad \alpha \equiv \sqrt{\frac{m_{\rm N}}{2 m_{\rm r,N}^2}} \qquad m_{\rm r,N} = \frac{m_\chi m_{\rm N}}{m_\chi + m_{\rm N}}$$

ρ_0: WIMP density near the Earth
σ_0: total cross section ignoring the form factor suppression
$F(Q)$: elastic nuclear form factor
$f_1(v)$: one-dimensional velocity distribution of halo WIMPs

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Astrophysics

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Particle Physics

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Determining the WIMP mass model-independently

- Determining the moments of the WIMP velocity distribution

$$\langle v^n \rangle = \alpha^n \left[\frac{2Q_{\rm thre}^{1/2} r_{\rm thre}}{F^2(Q_{\rm thre})} + I_0 \right]^{-1} \left[\frac{2Q_{\rm thre}^{(n+1)/2} r_{\rm thre}}{F^2(Q_{\rm thre})} + (n+1) I_n \right]$$

$$I_n = \sum_a \frac{Q_a^{(n-1)/2}}{F^2(Q_a)} \qquad\qquad r_{\rm thre} = \left(\frac{dR}{dQ} \right)_{Q=Q_{\rm thre}}$$

[M. Drees and CLS, JCAP 0706, 011]

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[M. Drees and CLS, JCAP 0706, 011]

- Determining the WIMP mass

$$m_\chi = \frac{\sqrt{m_X m_Y} - m_X \mathcal{R}_n}{\mathcal{R}_n - \sqrt{m_X/m_Y}}$$

$$\mathcal{R}_n \equiv \frac{\alpha_Y}{\alpha_X}$$

$$= \left[\frac{2Q_{{\rm thre},X}^{(n+1)/2} r_{{\rm thre},X} / F_X^2(Q_{{\rm thre},X}) + (n+1) I_{n,X}}{2Q_{{\rm thre},X}^{1/2} r_{{\rm thre},X} / F_X^2(Q_{{\rm thre},X}) + I_{0,X}} \right]^{1/n} \Big(X \longrightarrow Y \Big)^{-1} \qquad (n \neq 0)$$

[CLS and M. Drees, arXiv:0710.4296]

Determining the WIMP mass model-independently

- Spin-independent (SI) WIMP-nucleus cross section (neutralino)

$$\sigma_0^{\rm SI} \simeq \left(\frac{4}{\pi}\right) m_{\rm r,N}^2 A^2 |f_{\rm p}|^2 \qquad f_{\rm p}: \text{ effective WIMP} - \text{proton coupling}$$

- Determining the WIMP mass

$$m_\chi^{\rm SI} = \frac{(m_X/m_Y)^{5/2}\, m_Y - m_X \mathcal{R}_\sigma}{\mathcal{R}_\sigma - (m_X/m_Y)^{5/2}}$$

$$\mathcal{R}_\sigma = \frac{\mathcal{E}_Y}{\mathcal{E}_X} \left[\frac{2 Q_{{\rm thre},X}^{1/2} r_{{\rm thre},X} / F_X^2(Q_{{\rm thre},X}) + I_{0,X}}{2 Q_{{\rm thre},Y}^{1/2} r_{{\rm thre},Y} / F_Y^2(Q_{{\rm thre},Y}) + I_{0,Y}}\right]$$

- Neglecting $Q_{\rm thre}$

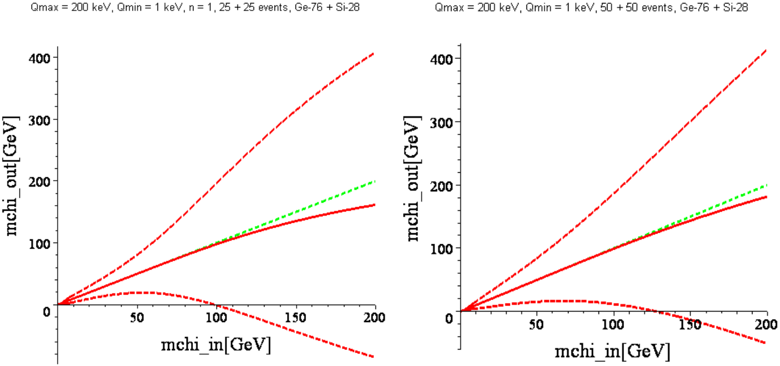
$$\mathcal{R}_\sigma = \frac{\mathcal{E}_Y}{\mathcal{E}_X} \left(\frac{I_{0,X}}{I_{0,Y}}\right)$$

$$\sigma\left(m_\chi^{\rm SI}\right) = \frac{\mathcal{R}_\sigma (m_X/m_Y)^{5/2} |m_X - m_Y|}{\left[\mathcal{R}_\sigma - (m_X/m_Y)^{5/2}\right]^2} \left[\frac{\sigma^2(I_{0,X})}{I_{0,X}^2} + \frac{\sigma^2(I_{0,Y})}{I_{0,Y}^2}\right]^{1/2}$$

[M. Drees and CLS, JCAP 0806, 012]

Determining the WIMP mass model-independently

- Reconstructed WIMP mass $m_\chi(n = 1)$ and $m_\chi^{\rm SI}$ (1 – 200 keV, ^{76}Ge + ^{28}Si, 25 + 25 / 50 + 50 events)



[CLS and M. Drees, arXiv:0710.4296]

➢ A smaller deviation, but a larger statistical error!

Correcting the systematic deviation of the reconstructed WIMP mass

- Matching $Q_{\rm max}$

$$Q_{\rm max,Si} = \left(\frac{\alpha_{\rm Ge}}{\alpha_{\rm Si}}\right)^2 Q_{\rm max,Ge} \qquad \Longleftarrow \qquad v_{\rm max} = \alpha\sqrt{Q_{\rm max}}$$

- Algorithmic χ^2 fit

$$\chi^2 = \sum_{i,j} \left(f_{i,X} - f_{i,Y}\right) \mathcal{C}_{ij}^{-1} \left(f_{j,X} - f_{j,Y}\right)$$

where

$$f_{i,X} = \alpha_X^i \left[\frac{2Q_{{\rm min},X}^{(i+1)/2} r_X(Q_{{\rm min},X})/F_X^2(Q_{{\rm min},X}) + (i+1) I_{i,X}}{2Q_{{\rm min},X}^{1/2} r_X(Q_{{\rm min},X})/F_X^2(Q_{{\rm min},X}) + I_{0,X}}\right] \left(\frac{1}{300~{\rm km/s}}\right)^i$$

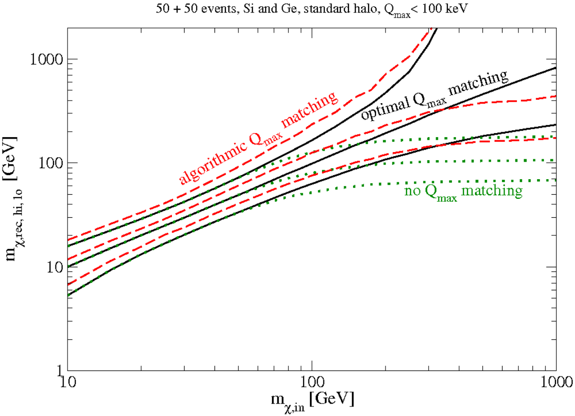
$$f_{n_{\rm max}+1,X} = \mathcal{E}_X \left[\frac{A_X^2}{2Q_{{\rm min},X}^{1/2} r_X(Q_{{\rm min},X})/F_X^2(Q_{{\rm min},X}) + I_{0,X}}\right] \left(\frac{\sqrt{m_X}}{m_\chi + m_X}\right)$$

$$\mathcal{C}_{ij} = {\rm cov}\left(f_{i,X}, f_{j,X}\right) + {\rm cov}\left(f_{i,Y}, f_{j,Y}\right)$$

[M. Drees and CLS, JCAP 0806, 012]

Correcting the systematic deviation of the reconstructed WIMP mass

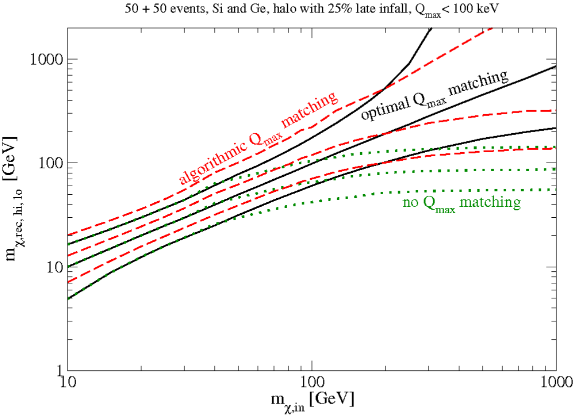
- Reconstructed WIMP mass
  ($Q_{\rm max} < 100$ keV, ^{76}Ge + ^{28}Si, 50 + 50 events)



[M. Drees and CLS, JCAP 0806, 012]

Correcting the systematic deviation of the reconstructed WIMP mass

- Reconstructed WIMP mass
  ($Q_{\rm max} < 100$ keV, ^{76}Ge + ^{28}Si, $50 + 50$ events, late infall compo.)

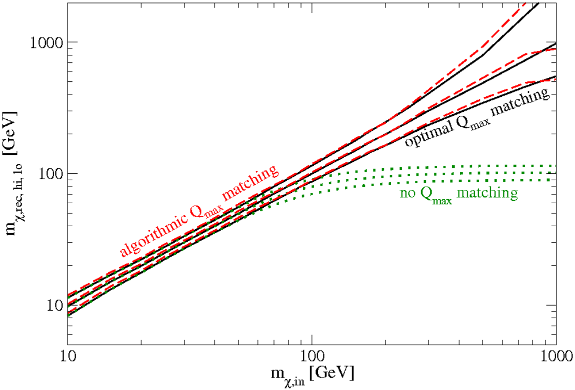


[M. Drees and CLS, JCAP 0806, 012]

Correcting the systematic deviation of the reconstructed WIMP mass

- Reconstructed WIMP mass
  ($Q_{\rm max} < 100$ keV, ^{76}Ge + ^{28}Si, 500 + 500 events)

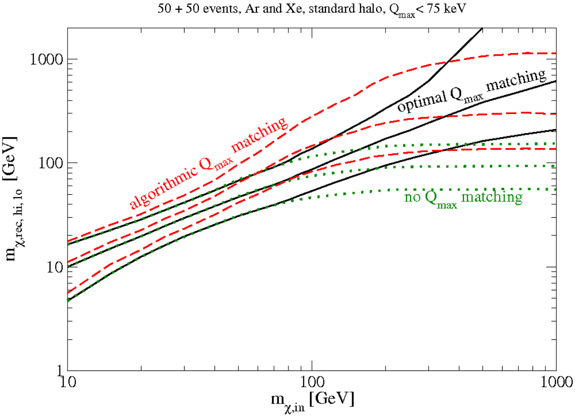
500 + 500 events, Si and Ge, standard halo, $Q_{\rm max}$ < 100 keV



[M. Drees and CLS, JCAP 0806, 012]

Correcting the systematic deviation of the reconstructed WIMP mass

- Reconstructed WIMP mass
  ($Q_{\rm max} < 75$ keV, ^{40}Ar + ^{136}Xe, 50 + 50 events)



[M. Drees and CLS, JCAP 0806, 012]

Summary

- Once two or more experiments with different detector materials obtain positive WIMP signals, we can determine the WIMP mass.

- Our methods are model-independent and require only measured recoil energies.

- The deviation of the reconstructed WIMP mass from the true one can be corrected by matching more suitable cut-off energies.

- With $\sim$ 100 keV maximal measuring energies and 2×50 events, a WIMP mass of 50 GeV can be estimated with an error of $\sim 35\%$.