Custodial bulk Randall-Sundrum Model and B physics

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JS, Chang, Park, PRD71
JS, Kim, Chang, JHEP02
JS, Kim, Chang, PRD77

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1 Unsatisfactory SM

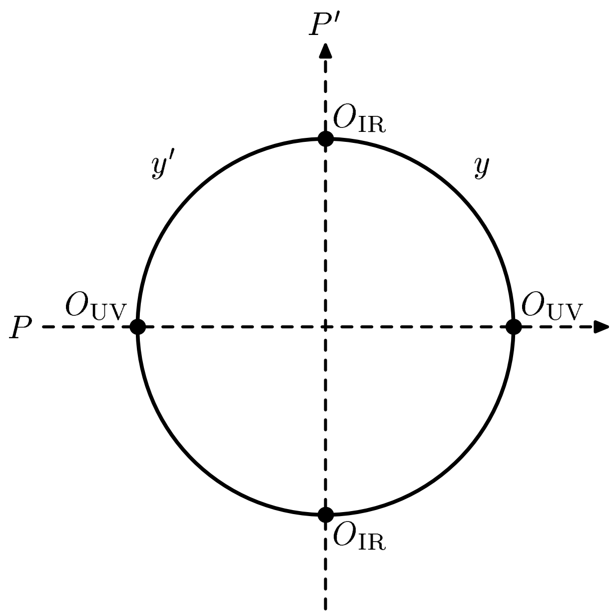
- **Two kinds of hierarchy:**
  1. **Gauge hierarchy: Why $m_H \ll M_{pl}$?**
  2. **Fermion mass hierarchy: Why $\frac{m_e}{m_t} \sim 10^{-6}$?**
- **In the SM, hierarchical parameters are attributed.**
- **New physics model for both hierarchy problems?**

Not many!

2 Bulk RS Model with custodial symmetry

- **Rare candidate which can explain both hierarchies.**
- **Five-dimensional theory**
  $$\mathbf{ds_5^2 = e^{-2k|y|}(dt^2 - d\tilde{x}^{\,2}) - dy^2} = \frac{1}{(kz)^2}\left(dt^2 - d\vec{x}^{\,2} - dz^2\right)$$
- $\mathbf{0 < y < L}$ **or** $\frac{1}{k} \le z \le \frac{1}{T}$
- **The generic curvature** k **is around the** $M_{\rm pl}$ **scale.**
- **Warped geometry** $\Longrightarrow k_{EW} = e^{-kL}k \sim$ TeV**:**
  Gauge hierarchy problem is answered with $kL \approx 35$**.**

5th Dim. space: compactified on the $S^1/\mathbb{Z}_2 \times \mathbb{Z}_2'$ orbifold



- **Two reflection symmetries**
  $\mathbb{Z}_2 : y \to -y$ **and** $\mathbb{Z}_2' : y' \to -y' \Longrightarrow$ **Two fixed points or branes.**
- $\mathbb{Z}_2 \times \mathbb{Z}_2'$ **parity :** $(++)\ \ (+-)\ \ (-+)\ \ (--)$
- **Only the bulk field with** $(++)$ **parity: zero mode.**

Hierarchical SM fermion mass $\Longleftarrow$ if fermion fields are in the bulk!

- **In 4D SM, $\mathrm{m_e/m_t} \sim 10^{-6}$.**
- **If fermions in the bulk but the Higgs field on our brane, the SM fermion mass $\propto$ the overlapping probability of the fermion mode function with the confined Higgs field.**
- **SM fermion mode function $\Longleftarrow$ bulk Dirac mass parameter c.**
- **$\mathrm{m_e/m_t} \sim 10^{-6}$ can be generated with order one c's.**

Just placing the SM fields in the bulk? NOT ENOUGH!

- **NO isospin symmetry**
- **Isospin symmetry in the SM: accidental.**
- **Too large contribution to ρ parameter[*].**

$$\Longrightarrow \quad k_{\rm EW} \gtrsim 20\,{\rm TeV}.$$

[*] JS,Kim,Kim,PRD67; Hewett et.al. JHEP09(2002)030

Bulk RS model with custodial symmetry

- **Additional bulk gauge symmetry***

$$\mathbf{SU(3)}_c \times \mathbf{SU(2)}_L \times \mathbf{SU(2)}_R \times \mathbf{U(1)}_{B-L}$$

- **$\mathbf{SU(2)}_R$: breaken by orbifold boundary condition on the Planck brane.**

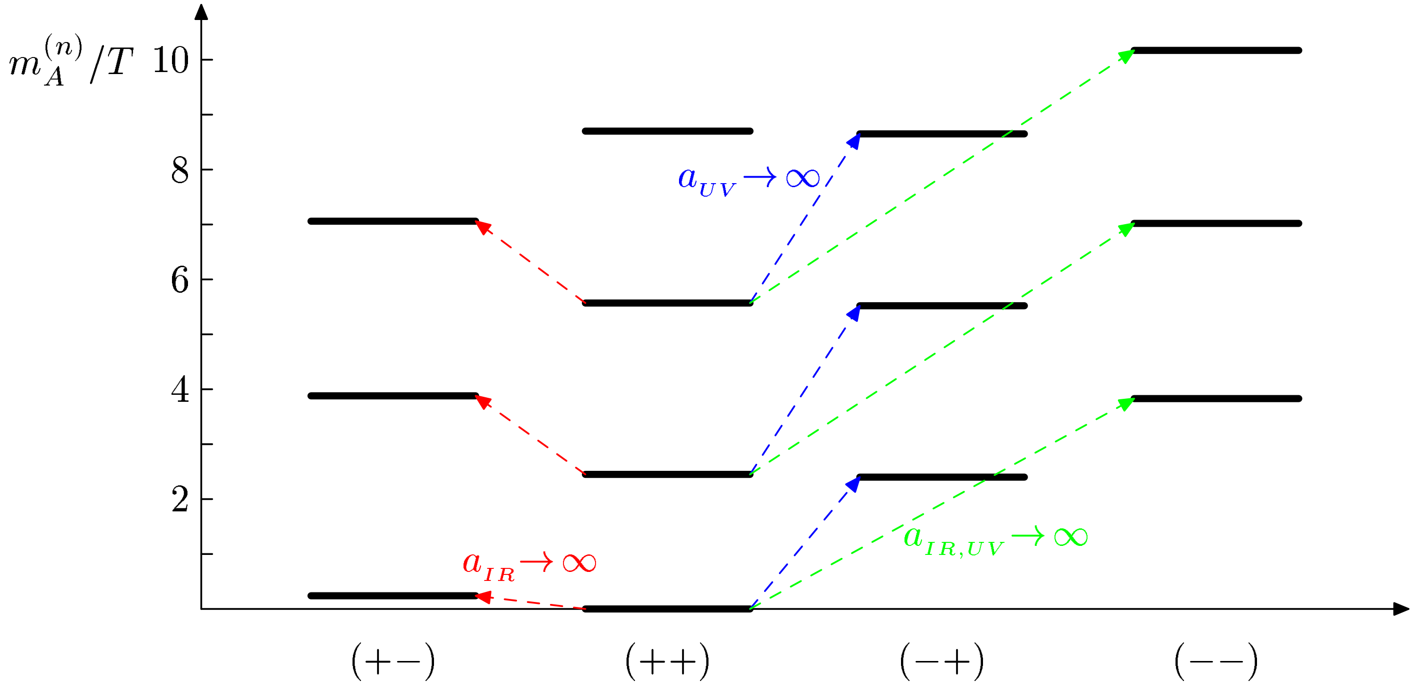
	UV(P)	**IR**(P')
$(\overrightarrow{W}_R)_\mu^{1,2}$	$-$	$+$
other gauge fields	$+$	$+$

- **TeV brane is $\mathbf{SU(2)}_R$ symmetric!**
  $\Longrightarrow$ Custodial symmetry on the TeV brane.

- $U(1)_R \times U(1)_{B-L} \rightarrow U(1)_Y$ **on UV brane**

*Agashe et.al. JHEP **0308**, 050 (2003)

Parity determined the KK masses of gauge bosons



- **No localized mass terms.**

For bulk fermion KK masses, Parity $+$ bulk Dirac mass parameter

- **5D action for a bulk fermion $\Psi(x^\mu, y) \equiv e^{2k|y|}\hat{\Psi}(x, z)$:**

$$\begin{aligned} S_{\text{fermion}} &= \int d^4x dy \sqrt{G}\left[i\overline{\Psi}\,\Gamma^{\underline{A}} e_{\underline{A}}{}^{A} \partial_A \Psi + m_D \overline{\Psi}\Psi \right] \\ &\supset \int d^4x dy \left[-\overline{\hat{\Psi}}_L \partial_y \hat{\Psi}_R + \overline{\hat{\Psi}}_R \partial_y \hat{\Psi}_L + m_D (\overline{\hat{\Psi}}_L \hat{\Psi}_R + \overline{\hat{\Psi}}_R \hat{\Psi}_L) \right] \end{aligned}$$

- **The $\mathbb{Z}_2 \times \mathbb{Z}_2'$ parity of Ψ_L is opposite to that of Ψ_R.**
- **The bulk Dirac mass m_D is $\mathbb{Z}_2 \times \mathbb{Z}_2'$-odd:**

$$m_D = c\,k\,\text{sign}(k).$$

- **Bulk dirac mass parameter c is not known.**

Extended Fermion sector

- **SM fermion : Zero mode of $(++)$ parity mode.**
- $\Psi_L^{(++)} \Longrightarrow \Psi_R^{(--)}$.
- **SM right-handed fermion ($SU(2)_L$ singlet) $\Longrightarrow$ another 5D fermion.**
  Doublet of $SU(2)_R$.
- $(-+)$ **parity of** $W_R^{\pm} \Longrightarrow$

$$Q_i = \begin{pmatrix} u_{iL}^{(++)} \\ d_{iL}^{(++)} \end{pmatrix}, \quad U_i = \begin{pmatrix} u_{iR}^{(++)} \\ D_{iR}^{(-+)} \end{pmatrix}, \quad D_i = \begin{pmatrix} U_{iR}^{(-+)} \\ d_{iR}^{(++)} \end{pmatrix},$$

$$L_i = \begin{pmatrix} \nu_{iL}^{(++)} \\ e_{iL}^{(++)} \end{pmatrix}, \quad N_i = \begin{pmatrix} \nu_{iR}^{(++)} \\ E_{iR}^{(-+)} \end{pmatrix}, \quad E_i = \begin{pmatrix} N_{iR}^{(-+)} \\ e_{iR}^{(++)} \end{pmatrix},$$

where $i = 1, 2, 3$ is the generation index.

3 SM particle mass $\Longleftarrow$ Localized Higgs field

- **The SM fermion mass $\Longleftarrow$ Yukawa interaction with the localized Higgs field:**

$$S_Y = -\int \mathrm{d}^4x\,\mathrm{d}y \frac{\delta(y-L)}{T}\left[\lambda^u_{5ij}\overline{\hat{u}}_{iR}\tilde{H}^\dagger\hat{Q}_{jL} + \lambda^d_{5ij}\overline{\hat{d}}_{iR}H^\dagger\hat{Q}_{jL} + h.c.\right],$$

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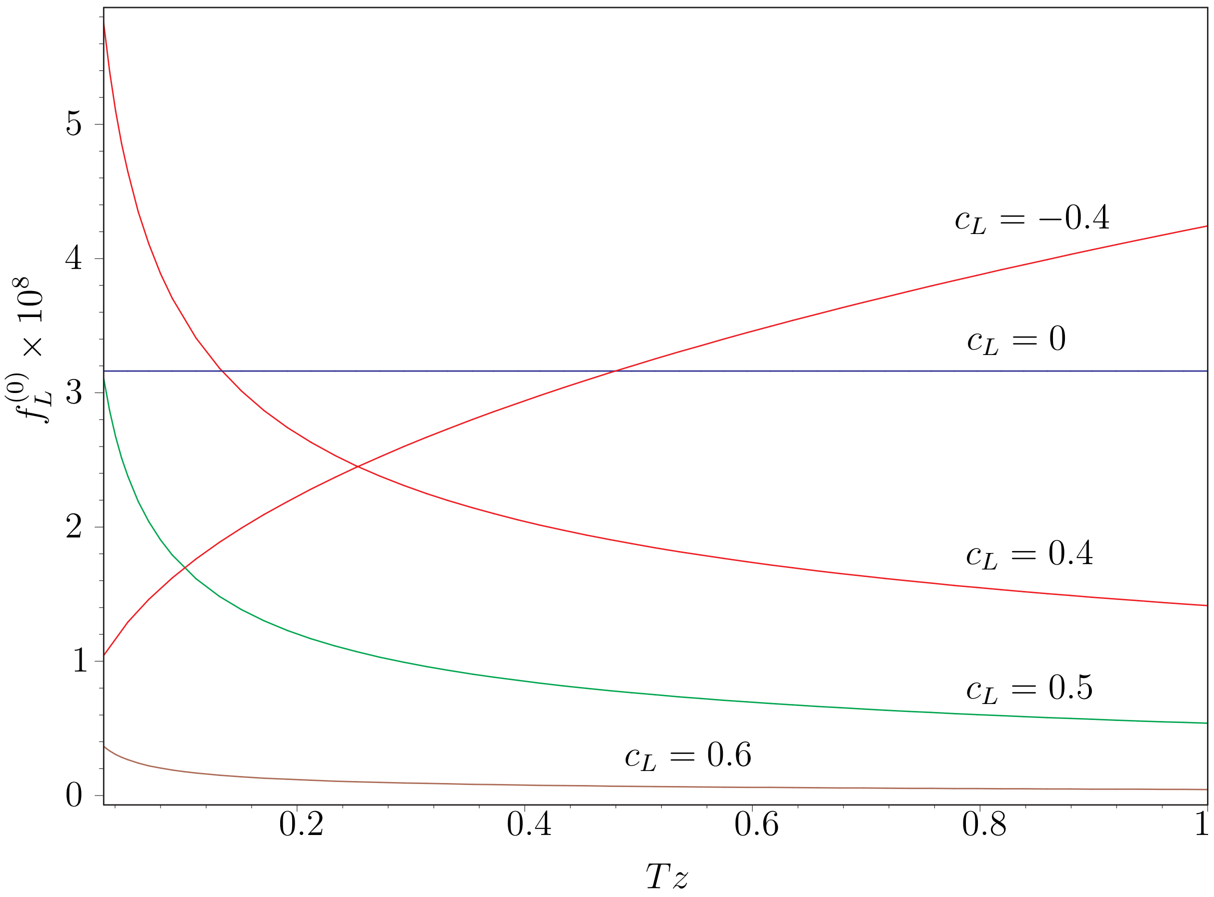
- **Higgs VEV of $\langle H \rangle = v \simeq 174\,\mathrm{GeV}$ generates the SM fermion mass matrix (non-diagonal):**

$$\left(M_f\right)_{ij} = v\lambda^f_{5ij} \, \frac{k}{T} \, f^{(0)}_R f^{(0)}_L \Big|_{\mathrm{TeV}} \equiv v\lambda^f_{5ij} F_R(c_{R_i}) F_L(c_{L_j}), \quad (f = u, d, \nu, e).$$

- $F_{L,R}$: **normalized zero-mode functions.**

Behavior of $F_L(c) = F_R(-c)$ as a function of c



UV brane **IR brane**

Light fermion **Heavy fermion**

Mass eigenstate $\Longrightarrow$ 2 independent mixing matrices

- **Mass eigenstates $\Longleftarrow$ bi-unitary transformation:**

$$U_{qR} M_q U_{qL}^\dagger = M_q^{(\mathrm{diag})},$$
$$\chi_{fL} = U_{fL}^\dagger \psi_{fL}^{(0)}, \quad \chi_{fR} = U_{fR}^\dagger \psi_{fR}^{(0)}.$$

- **NOTE!**

TWO RH mixing matrices, U_{uR} and U_{dR}.

- **Observed mixing matrices**

$$V^{\mathrm{CKM}} = U_{uL}^\dagger U_{dL}, \quad U_{\mathrm{PMNS}} = U_{eL}^\dagger U_{\nu L}$$

Too many free parameters

- **18 Dirac mass parameters**

$$c_{Q_i}, \quad c_{U_i}, \quad c_{D_i}, \quad c_{L_i}, \quad c_{E_i}, \quad c_{N_i}, \quad i = 1, 2, 3.$$

- **5D Yukawa couplings**

$$\lambda^u_{5ij}, \quad \lambda^d_{5ij}, \quad \lambda^e_{5ij}, \quad \lambda^\nu_{5ij},$$

- **No proper prediction.**

Natural assumptions can lead to a specified model.

- **Assumption 1 : Universal 5D Yukawa structure.**

$$\lambda_{5ij}^{f} = \lambda_5 \sim \mathcal{O}(1).$$

- **Assumption 2: No fine-tuned cancellation.**
  - **No cancellation in explaining the observed V_{CKM} and $U_{\rm PMNS}$.**
  - **No order changing for $V^{\rm CKM} = U_{uL}^{\dagger} U_{dL}, \quad U_{\rm PMNS} = U_{eL}^{\dagger} U_{\nu L}$.**

$$0.1\Big|_{\rm CKM} = 1.0 - 0.9 : \textbf{ NOT ALLOWED!}$$

Quark sector can be fixed only by these two assumptions.*

- **hierarchical quark masses & small mixing angles.**
- **Mixing matrices:**

$$(U_{qL})_{ij(i\le j)} \approx \frac{F_L(c_{Q_i})}{F_L(c_{Q_j})}, \quad (U_{qR})_{ij(i\le j)} \approx \frac{F_R(c_{A_i})}{F_R(c_{A_j})}.$$

- **bulk Dirac mass parameters**

$$\begin{aligned} c_{Q_1} &\simeq 0.61, & c_{Q_2} &\simeq 0.56, & c_{Q_3} &\simeq 0.3\,^{+0.02}_{-0.04}, \\ c_{D_1} &\simeq -0.66, & c_{D_2} &\simeq -0.61, & c_{D_3} &\simeq -0.56\,, \\ c_{U_1} &\simeq -0.71, & c_{U_2} &\simeq -0.53, & 0 \lesssim c_{U_3} &\lesssim 0.2. \end{aligned}$$

*Chang, Kim, Yamaguchi, PRD73

Lepton sector: Not totally fixed!

- **Large mixing angles.**

$$U_{\rm PMNS} \simeq \begin{pmatrix} 0.8 & 0.5 & U_{e3} \\ 0.4 & 0.6 & 0.7 \\ 0.4 & 0.6 & 0.7 \end{pmatrix},$$

- **Two assumptions are not enough to fix all the parameters.**

4 Misalignment b/w the mass and gauge eigenstates.

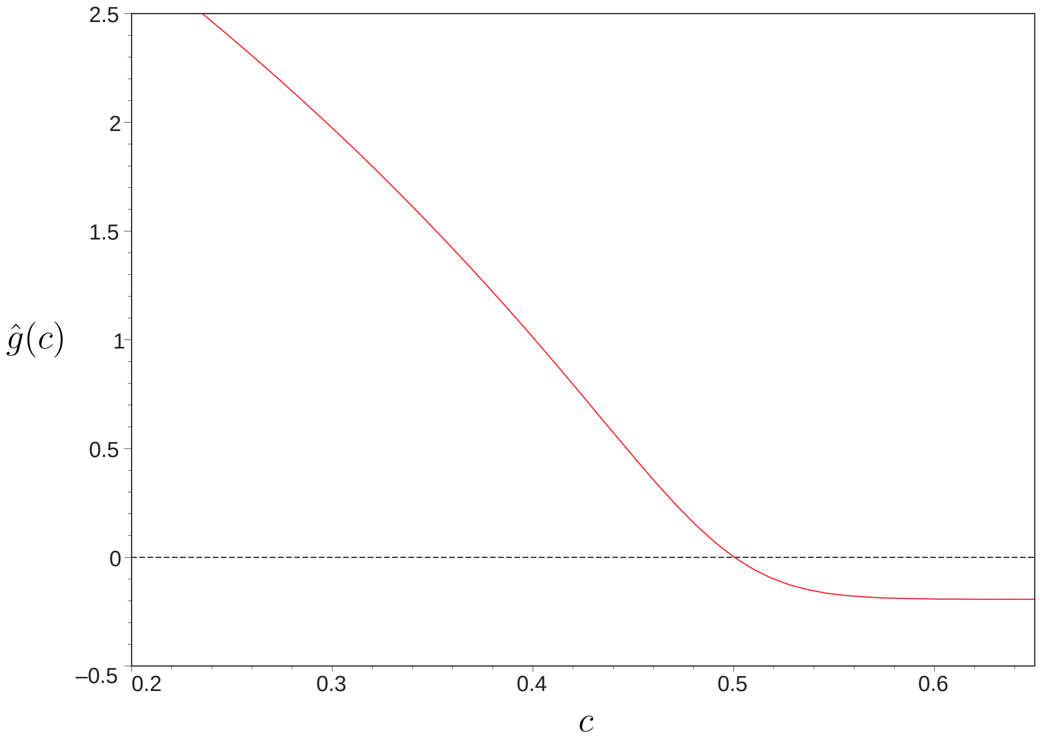
- **The 4D effective gauge interaction for a fermion:**

$$\mathcal{L}_{4D} = \mathbf{g_{SM}} \bar{\psi}^{(0)}_{\mathbf{jL}} \mathbf{i}\gamma^\mu \psi^{(0)}_{\mathbf{jL}} \mathbf{A}^{(0)}_\mu + \mathbf{g_{SM}} \sum_{\mathbf{n=1}}^{\infty} \hat{\mathbf{g}}^{(\mathbf{n})}_{\mathbf{j}}(\mathbf{c_{Q_i}})\, \bar{\psi}^{(0)}_{\mathbf{jL}} \mathbf{i}\gamma^\mu \psi^{(0)}_{\mathbf{jL}} \mathbf{A}^{(\mathbf{n})}_\mu,$$

- **Effective coupling $\hat{g}^{(n)}_j$, normalized by the SM coupling, depends on c.**

$$\hat{\mathbf{g}}^{(\mathbf{n})}_{\mathbf{i}}(\mathbf{c_{Q_i}}) = \sqrt{\mathbf{kL}} \int \mathbf{d(kz)} \left[\mathbf{f}^{(0)}_{\mathbf{L}}(\mathbf{z}, \mathbf{c_{Q_i}})\right]^{\mathbf{2}} \mathbf{f}^{(\mathbf{n})}_{\mathbf{A}}(\mathbf{z}) \,.$$

Behavior of $\hat{g}(c)$.



heavier fermion **lighter fermion**

- $\hat{g}(c = 0.5) = 0$.
- **Heavier ($c < 0.5$) SM fermion has larger $\hat{g}$.**

Mixing

- **In terms of mass eigenstates, we have NC mixing mediated by KK gauge bosons**

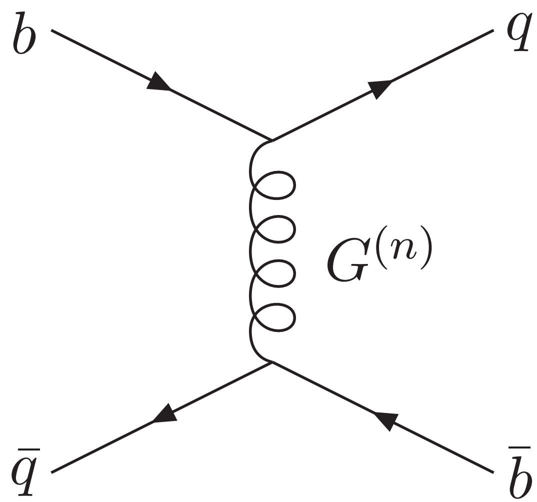
$$\mathcal{L}_{4D} \supset -\frac{1}{2}\sum_{i,j,n} \Big[\; g\left(K^{(n)}_{Qij}\,\bar{d}_{iL}\gamma^\mu d_{jL} + K^{(n)}_{Lij}\,\bar{e}_{iL}\gamma^\mu e_{jL}\right) W^{(n)}_{3L\mu} - g_X\left(K^{(n)}_{Qij}\,\bar{d}_{iL}\gamma^\mu d_{jL} - K^{(n)}_{Lij}\,\bar{e}_{iL}\gamma^\mu e_{jL}\right) B^{(n)}_{X\mu}\Big],$$

- **Mixing matrix is**

$$\begin{aligned} K^{(n)}_{Qij} &= \sum_{k=1}^{3} \left(U^\dagger_{dL}\right)_{ik} \hat{g}^{(n)}(c_{Q_k}) \left(U_{dL}\right)_{kj} \;, \\ K^{(n)}_{Lij} &= \sum_{k=1}^{3} \left(U^\dagger_{eL}\right)_{ik} \hat{g}^{(n)}(c_{L_k}) \left(U_{eL}\right)_{kj} \;, \end{aligned} \tag{1}$$

- **NOTE: if all c are the same so that all $\hat{g}$'s are common, NO mixing.**

Tree level mixing for FCNC through KK gauge bosons.



- **Tree level contributions to** $B_q^0 - \bar{B}_q^0$**,** $b \to s l^+ l^-$**,** $\mu \to eee \ldots$ **etc.**
- **Suppressed by the TeV mass of KK gauge bosons.**

5 Constraints from lepton violating processes

- **Considering $\tau \to 3\mu$, $\mu \to 3e$.**
- **Fix the lepton sector:**

$$U_{eL} \sim \begin{pmatrix} 1 & 0.001 & 0.1 \\ 0.1 & 1 & 1 \\ 0.1 & 1 & 1 \end{pmatrix}, \quad (U_{eR})_{ij} \approx \frac{F_R(c_{E_i})}{F_R(c_{E_j})},$$

$$c_{L_1} \simeq 0.59, \quad c_{L_2} \simeq 0.5, \quad c_{L_3} \simeq 0.5,$$

$$c_{E_1} \simeq -0.74, \quad c_{E_2} \simeq -0.65, \quad c_{E_3} \simeq -0.55.$$

- $\hat{g}(0.5) = 0$.

6 Difficult to probe at LHC: Proton phobic

- **NO coupling of $G^{(1)} - g - g$ due to orthonormality of profiles of these particles.**
- **Suppressed couplings with light quarks:**

$$\frac{g^{qqG^{(1)}}}{g_{SM}} \sim 0.1.$$

7 Effects on $B_q^0 - \bar{B}_q^0$ mixing

- **Recently CDF measured $B_s^0 - \bar{B}_s^{0*}$ and Belle and BaBar measured $B_d^0 - \bar{B}_d^0$.**

$$\begin{aligned} \Delta M_d^{\rm exp} &= (0.507 \pm 0.004)\,{\rm ps}^{-1}\,, \\ \Delta M_s^{\rm exp} &= \left[17.33^{+0.42}_{-0.21}({\rm stat}) \pm 0.07({\rm syst})\right]\,{\rm ps}^{-1}\,. \end{aligned}$$

- **From the transition amplitude $\langle B_q^0|\mathcal{H}_{\rm eff}^{\Delta B=2}|\bar{B}_q^0\rangle = 2M_{B_q}M_{b\bar{b}}^q$, we have two observables:**

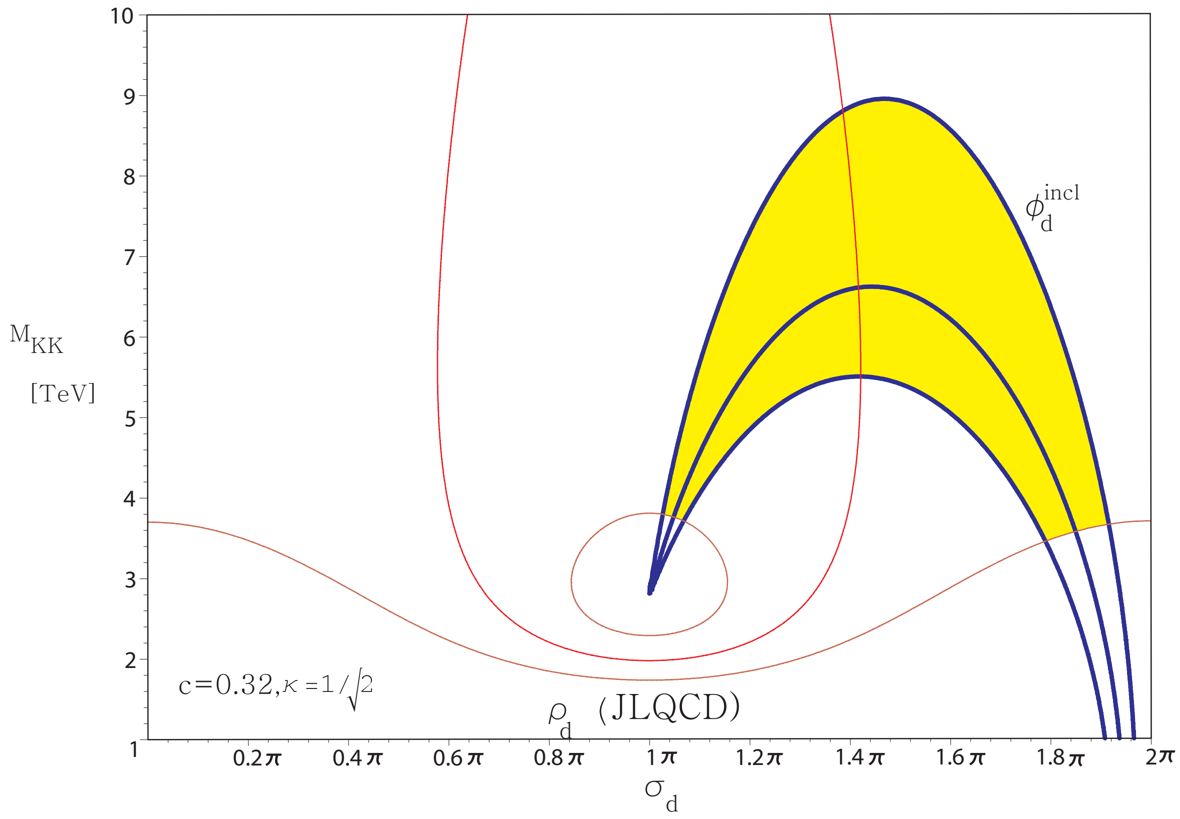
$$\begin{aligned} \rho_q \equiv \frac{\Delta M_q}{\Delta M_q^{\rm SM}} &= \left|1 + \frac{M_{b\bar{b}}^{q,{\rm RS}}}{M_{b\bar{b}}^{q,{\rm SM}}}\right|, \\ \phi_q &\equiv \arg(M_{b\bar{b}}^q) = \phi_q^{\rm SM} + \phi_q^{\rm NP}\,. \end{aligned}$$

- **CP-violating phase for d quark in NP**

$$\phi_d^{\rm NP}\Big|_{\rm incl} = -(10.1 \pm 4.6)^\circ, \quad \phi_d^{\rm NP}\Big|_{\rm excl} = -(2.5 \pm 8.0)^\circ.$$

*CDF coll.`http://fpcp2006.triumf.ca`

Allowed parameter space from $B_d^0 - \bar{B}_d^0$



- **Allowed parameter space of $(\sigma_d,\ M_{KK})$ by ρ_d and $\phi_d^{NP}|_{\rm incl}$.**
- **$\phi_d^{\rm NP}$ constraint is quite strong.**

8 Effects on $B \to K^* l^+ l^-$

- **Clean signatures.**
- **Larger BR than the decay into K boson.**
- **a vector boson $K^*(\to \pi K)$: angular analysis is possible, i.e., $A_{\rm FB}$.**

Parametrization of NP

$$\mathcal{M}_{\rm new} = \frac{G_F \alpha}{\sqrt{2}\pi} V_{tb} V_{ts}^* \Big[\ C_{LL}^{ij} (\bar{s}_L \gamma^\mu b_L)(\bar{l}_L^i \gamma^\mu l_L^j) + C_{LR}^{ij} (\bar{s}_L \gamma^\mu b_L)(\bar{l}_R^i \gamma^\mu l_R^j)$$

$$+ C_{RL}^{ij} (\bar{s}_R \gamma^\mu b_R)(\bar{l}_L^i \gamma^\mu l_L^j) + C_{RR}^{ij} (\bar{s}_R \gamma^\mu b_R)(\bar{l}_R^i \gamma^\mu l_R^j) \Big] \, .$$

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In this model, we have

$$C_{LL}^{ij} \simeq \left(\frac{3.5\,\text{TeV}}{M_A} \right)^2 (g^2 - g_X^2)\, \kappa_Q^2\, \hat{g}(c_{Q_3}) K_{Lij},$$

$$C_{LR}^{ij} \simeq \left(\frac{3.5\,\text{TeV}}{M_A} \right)^2 g_X^2 \kappa_Q^2 \hat{g}(c_{Q_3}) K_{Eij},$$

$$C_{RL}^{ij} \simeq 2 \left(\frac{3.5\,\text{TeV}}{M_A} \right)^2 g_X^2 \kappa_D^2 \hat{g}(c_{D_3}) K_{Lij},$$

$$C_{RR}^{ij} \simeq 2 \left(\frac{3.5\,\text{TeV}}{M_A} \right)^2 (\tilde{g}^2 - g_X^2) \kappa_D^2 \hat{g}(c_{D_3}) K_{Eij} ,$$

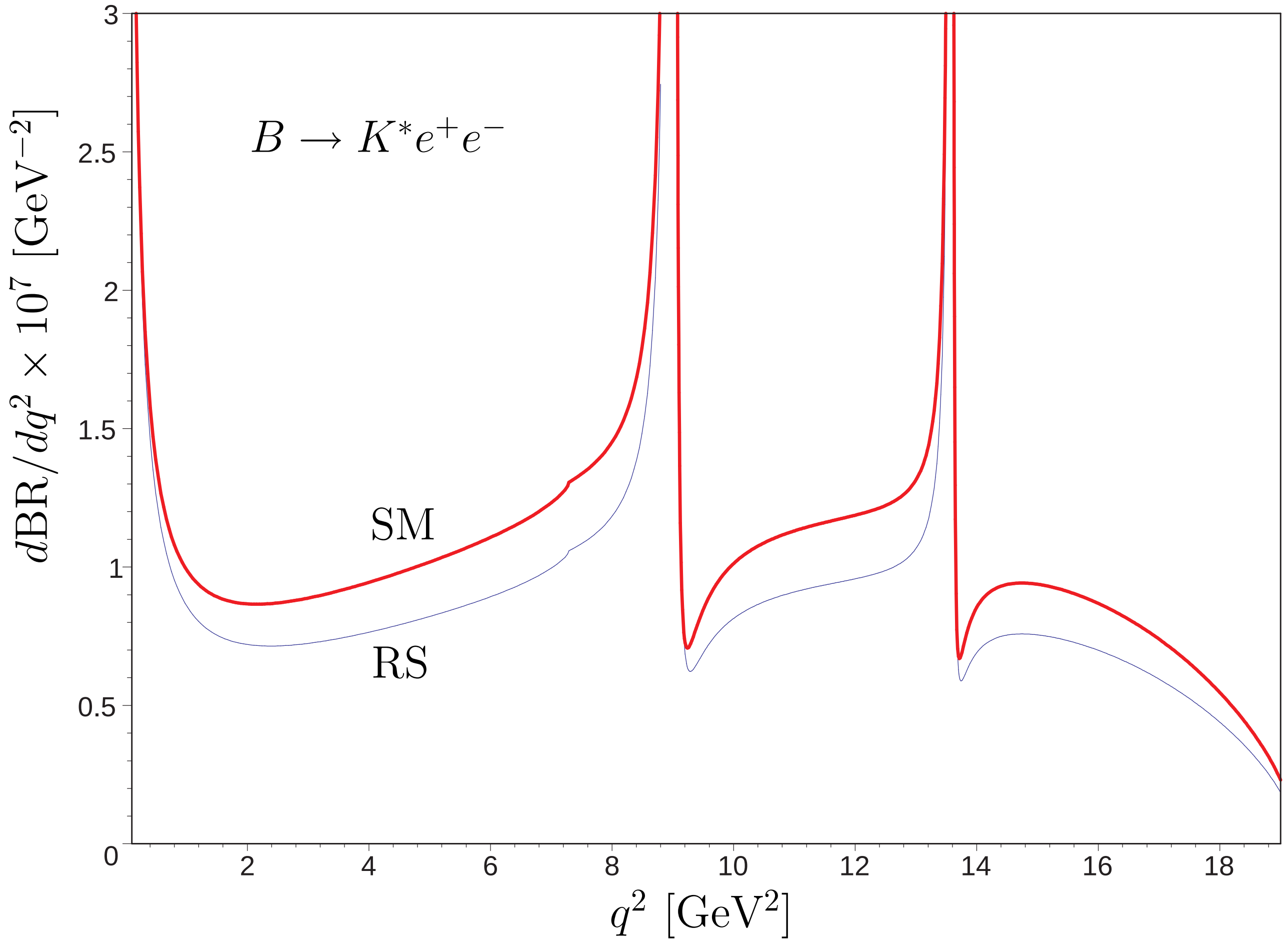
Numerical values for $M_A^{(1)} = 2\,\mathrm{TeV}$

	e^+e^-	$e^+\mu^-$	$e^+\tau^-$	$\mu^+\mu^-$	$\mu^+\tau^-$	$\tau^+\tau^-$
C_{LL}	-0.3	$\pm 7 \times 10^{-3}$	± 0.05	-7×10^{-3}	$\pm 6 \times 10^{-3}$	-4×10^{-5}
C_{RL}	0.02	$\pm 5 \times 10^{-4}$	$\pm 4 \times 10^{-3}$	6×10^{-4}	$\pm 5 \times 10^{-4}$	3×10^{-6}
C_{LR}	-0.1	± 0.01	$\pm 10^{-3}$	-0.1	± 0.01	-0.1
C_{RR}	0.03	$\pm 3 \times 10^{-3}$	$\pm 2 \times 10^{-4}$	0.03	$\pm 3 \times 10^{-3}$	0.02

- **Only e^+e^- mode has sizable correction since**

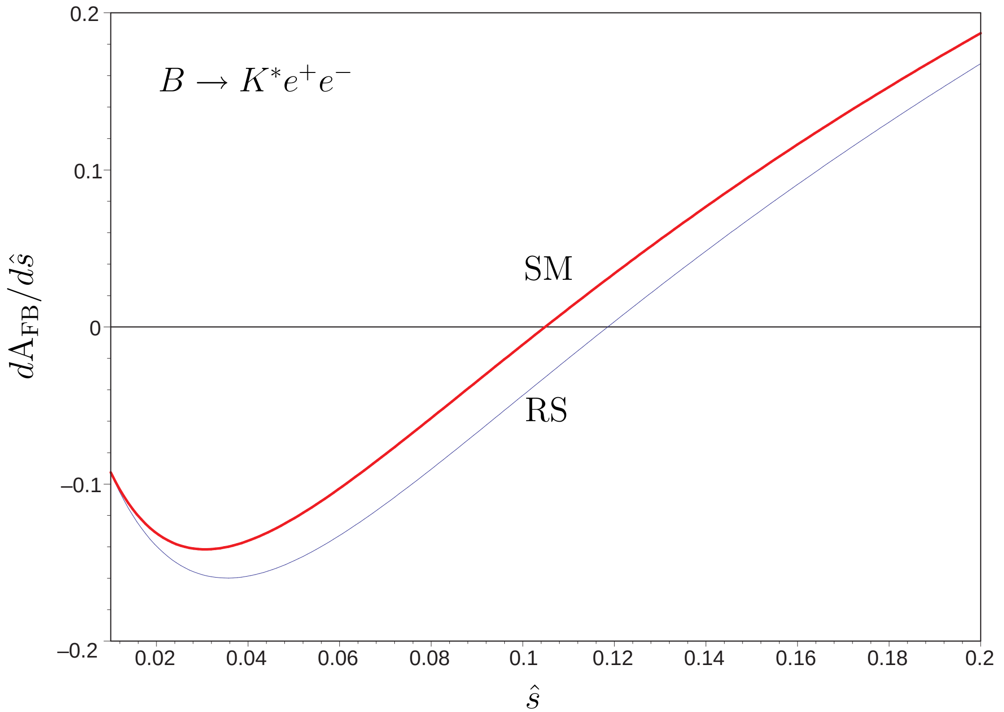
$$\hat{g}(c_{L_1}) \simeq -0.2, \quad \hat{g}(c_{L_2}) \simeq \hat{g}(c_{L_3}) \simeq 0.$$

$d\mathrm{BR}/dq^2$ for $B \to K^* e^+ e^-$



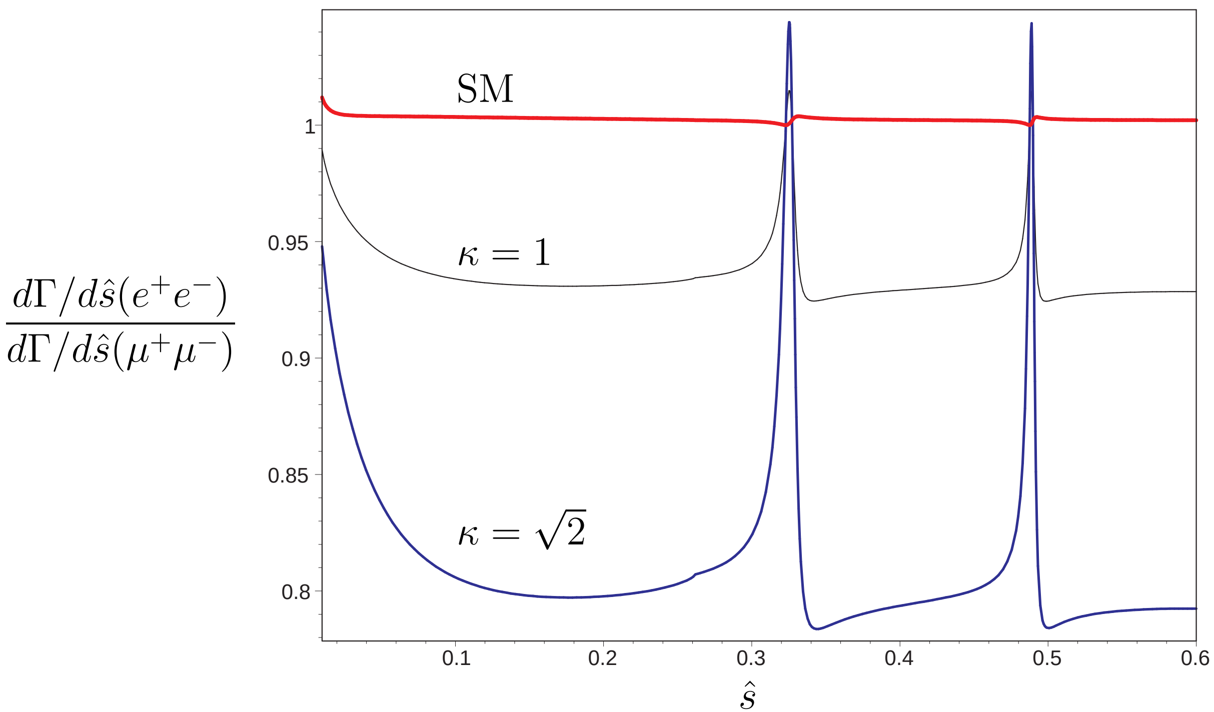
Too large hadronic uncertainty in the calculation of form factors!

$dA_{\rm FB}/d\hat{s}$ $(\hat{s} \equiv q^2/m_B^2)$ for $B \to K^* e^+ e^-$



- **Good probe is the position of $\hat{s}_0$ such that $A_{\rm FB}(\hat{s}_0) = 0$.**
- **$\hat{s}_0$: negligible hadronic uncertainty.**
  : moves to positive direction ($\sim 18\%$).

Best observable: Sizable effect for ee but negligible for $\mu\mu$.



- **Ratio $\Longrightarrow$ removing the hadronic uncertainty.**
- **Deviation can reach up to 20%.**

Dependence on $M_A^{(1)}$: partially integrated Γ over $\hat{s} \in [0.1,\ 4\, m_c^2/m_B^2]$

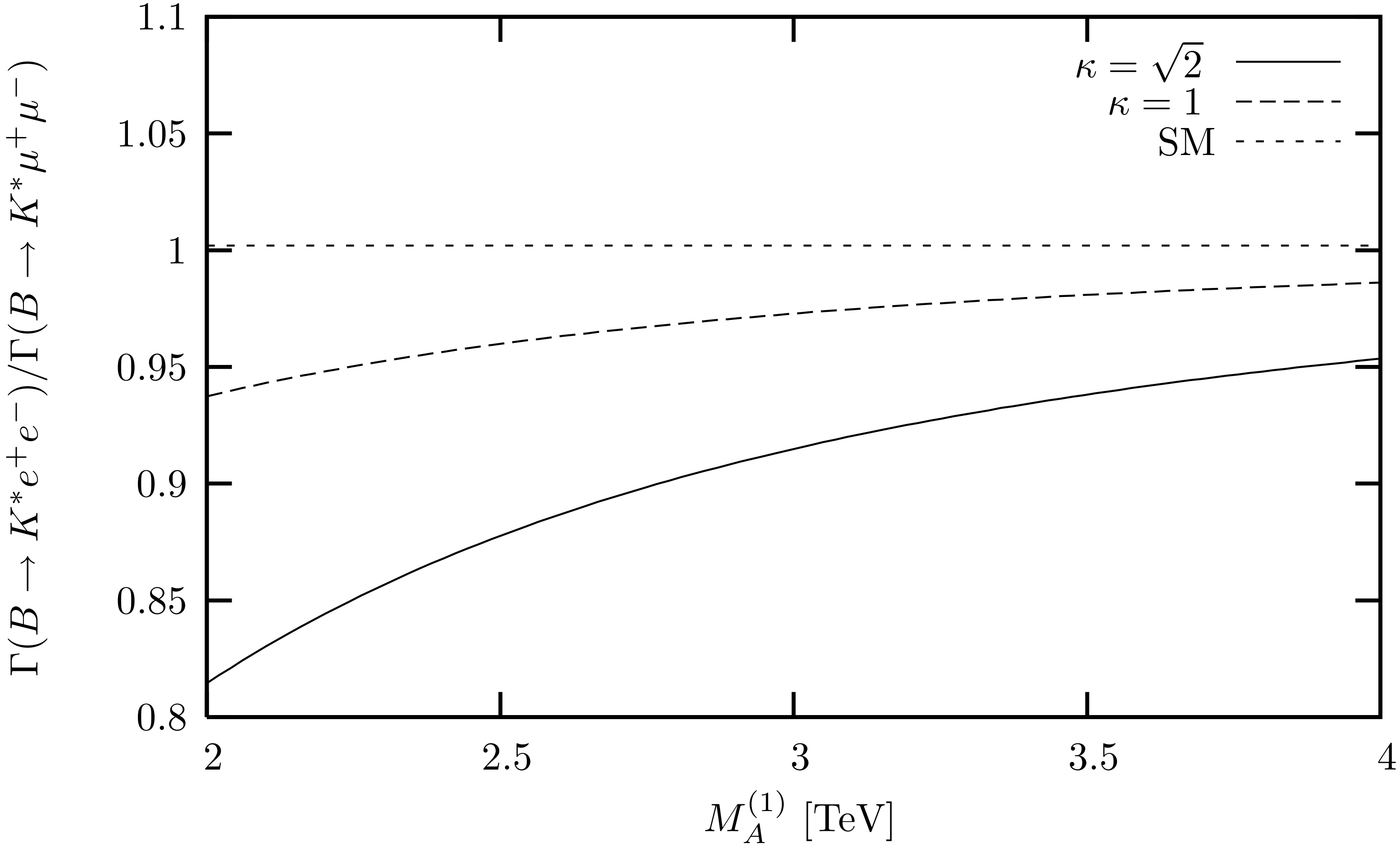


Figure 1: $d\mathrm{A}_{\mathrm{FB}}/d\hat{s}$ as a function of $\hat{s}$ for $B \to K^* e^+ e^-$.

9 Conclusions

- **A bulk RS model with custodial symmetry can solve the gauge hierarchy and SM fermion mass hierarchy problems.**
- **With two natural assumptions and the constraint from lepton-violating processes, we fix all the parameters for the fermion sector.**
- **Misalignment between the localized mass eigenstate and the bulk gauge eigenstate causes the tree level contribution to some FCNC, mediated by KK gauge bosons.**
- $B \to K^* l^+ l^-$ **can provide one of the most sensitive observables to probe the new FCNC.**