Fermions and Gauge Bosons in $SU(4)_L \times U(1)_X$ models with little Higgs

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June 19, 2008

< 2008 SUSY Conference in Seoul >

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Introduction

- As an alternative solution to the naturalness problem (and the fine-tuning issue), little Higgs models (LHM) has been recently introduced.
  (Arkani-Hamed, Cohen, and Georgi 2001)
- LHMs adopts the early idea that Higgs can be considered as a Nambu Goldstone boson from global symmetry breaking at some higher scale $\Lambda \sim 4\pi f$.
  (Dimopoulos, Preskill 1982; Georgi, Kaplan 1984; Banks 1984)
- Higgs acquires a mass radiatively through symmetry breaking at the EW scale v, by collective breaking.
- There are several LHMs such as “minimal moose”, “product group”, and “simple group” models.
- In this talk, we discuss the simple group model with $SU(4)_L \times U(1)_X$ electroweak gauge group (suggested by Kaplan and Schmaltz, 2003) by embedding the anomaly-free fermion spectra.

Little Higgs from $SU(4)$ Group

- Start from a non-linear sigma model $[SU(4)/SU(3)]^4$ with four complex quadruplets scalar fields Φ_{ij} $(i,j=1,2)$

  $\Longrightarrow$ Diagonal $SU(4)$ is gauged.

- Gauge symmetry breaking: $SU(4)_L \times U(1)_X \rightarrow SU(2)_L \times U(1)_Y$

  $\Longrightarrow$ 12 new gauge bosons with masses of order the scale f.

- Global symmetry breaking: $[SU(4)]^4 \rightarrow [SU(3)]^4$

  $\Longrightarrow$ 12 of the 28 degrees of freedom in the Φ_{ij} are eaten by the Higgs mechanism when $SU(4)_w$ is broken.

  $\Longrightarrow$ Remaining 16 consist of two complex doublets h_u and h_d, three complex $SU(2)$ singlets σ_1, σ_2 and σ_3, and two real scalars η_u and η_d.

  (Kaplan and Schmaltz 2003)

Little Higgs from $SU(4)$ Group

One possible parametrization of the non-linear sigma model fields Φ_{ij} with general f_{ij} ($SU(4)$ breaking is not aligned):

$$\Phi_{11} = e^{+i\mathcal{H}_u \frac{f_{12}}{f_{11}}} \begin{pmatrix} 0 \\ 0 \\ f_{11} \\ 0 \end{pmatrix} \qquad \Phi_{12} = e^{-i\mathcal{H}_u \frac{f_{11}}{f_{12}}} \begin{pmatrix} 0 \\ 0 \\ f_{12} \\ 0 \end{pmatrix}$$

$$\Phi_{21} = e^{+i\mathcal{H}_d \frac{f_{22}}{f_{21}}} \begin{pmatrix} 0 \\ 0 \\ 0 \\ f_{21} \end{pmatrix} \qquad \Phi_{22} = e^{-i\mathcal{H}_d \frac{f_{21}}{f_{22}}} \begin{pmatrix} 0 \\ 0 \\ 0 \\ f_{22} \end{pmatrix}$$

where

$$\mathcal{H}_u = \begin{pmatrix} 0 & 0 & h_1 & 0 \\ 0 & 0 & & 0 \\ h_1^\dagger & & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} /2f_1 \qquad \mathcal{H}_d = \begin{pmatrix} 0 & 0 & 0 & h_2 \\ 0 & 0 & 0 & \\ 0 & 0 & 0 & 0 \\ h_2^\dagger & & 0 & 0 \end{pmatrix} /2f_2$$

$$f_i^2 = \frac{1}{2}\sum_{j=1,2} f_{ij}^2, \qquad \langle h_1 \rangle = \begin{pmatrix} v_u \\ 0 \end{pmatrix}, \qquad \langle h_2 \rangle = \begin{pmatrix} 0 \\ v_d \end{pmatrix}$$

Model Construction

- Electric charge generator: $Q = aT_{3L} + \frac{1}{\sqrt{3}} bT_{8L} + \frac{1}{\sqrt{6}} cT_{15L} + XI_4$
- Correct embedding of the SM isospin $SU(2)_L$ doublets gives $a = 1$ ($Q = T_{3L} + Y$).
- If we assume that all gauge bosons have electric charges 0 and ± 1 only, there are not more than four different possibilities (Ponce, Gutiérrez, and Sánchez 2004):

$$\begin{matrix} b = c = 1 \\ (u, d, D, D')^T \end{matrix} ; \quad \begin{matrix} b = c = -1 \\ (d, u, U, U')^T \end{matrix} ; \quad \begin{matrix} b = 1 \;\&\; c = -2 \\ (u, d, D, U)^T \end{matrix} ; \quad \begin{matrix} b = -1 \;\&\; c = 2 \\ (u, d, U, D)^T \end{matrix}$$

- Our choice is to have the duplicated extra heavy fermions which can remove the quadratic divergences due to their SM fermion partners.
- For $b = -1 \& c = 2$, $X(\Phi_{1j}) = -\frac{1}{2}, X(\Phi_{2j}) = \frac{1}{2};\ X(\psi_L) = \frac{1}{6}, X(\psi_R) = Q$

  ⋇ Our quantum number assignments differ from those of the original paper by Kaplan & Schmaltz.

Gauge Boson Sector

- Covariant derivative for 4-plets: $D_\mu = \partial_\mu + igT^\alpha A_\mu^\alpha + ig_X X A_\mu^X$
- There are 15 gauge bosons associated with $SU(4)_L$:

$$T^\alpha A_\mu^\alpha = \frac{1}{\sqrt{2}} \begin{pmatrix} Z_{1\mu}^0 & W_\mu^+ & Y_\mu^0 & X_\mu^{\prime +} \\ W_\mu^- & Z_{2\mu}^0 & X_{1\mu}^- & Y_\mu^{\prime 0} \\ \bar{Y}_\mu^0 & X_{1\mu}^+ & Z_{3\mu}^0 & W_\mu^{\prime +} \\ X_\mu^{\prime -} & \bar{Y}_\mu^{\prime 0} & W_\mu^{\prime -} & Z_{4\mu}^0 \end{pmatrix}$$

where

$$Z_1^{0\mu} = A_3^\mu/\sqrt{2} + A_8^\mu/\sqrt{6} + A_{15}^\mu/\sqrt{12} \qquad Z_2^{0\mu} = -A_3^\mu/\sqrt{2} + A_8^\mu/\sqrt{6} + A_{15}^\mu/\sqrt{12}$$

$$Z_3^{0\mu} = -2A_8^\mu/\sqrt{6} + A_{15}^\mu/\sqrt{12} \qquad Z_4^{0\mu} = -3A_{15}^\mu/\sqrt{12}$$

Gauge Boson Sector

- Physical Gauge Boson Masses:
  - $M_W^2 = \frac{1}{4} g^2 v^2, \qquad M_{W'}^2 = \frac{1}{4} g^2 \left(4f^2 - v^2\right)$
  - $M_X^2 = \frac{1}{4} g^2 \left(4f_1^2 - v_1^2 + v_2^2\right), \qquad M_{X'}^2 = \frac{1}{4} g^2 \left(4f_2^2 + v_1^2 - v_2^2\right)$
  - $M_Y^2 = g^2 f_1^2, \qquad M_{Y'}^2 = g^2 f_2^2$
  - $M_Z^2 = \frac{g^2 v^2}{4c_W^2} \left(1 - \frac{t_W^4}{4} \frac{v^2}{f^2}\right), \qquad M_{Z'}^2 = g^2(1+t^2)f^2 - M_Z^2 \qquad M_{Z''}^2 = \frac{1}{2} g^2 f^2$

  where

$$t \equiv g_X/g, \quad c_W \equiv \cos\theta_W = \sqrt{(1+t^2)/(1+2t^2)},$$

$$v_1^2 \equiv v_u^2 - \frac{v_u^2}{12 f_1^2} \left(\frac{f_{12}^2}{f_{11}^2} + \frac{f_{11}^2}{f_{12}^2} - 1 \right), \quad v_2^2 \equiv v_d^2 - \frac{v_d^2}{12 f_2^2} \left(\frac{f_{22}^2}{f_{21}^2} + \frac{f_{21}^2}{f_{22}^2} - 1 \right)$$

$$f^2 = f_1^2 + f_2^2 \gg \triangle f^2 = f_1^2 - f_2^2, \; v^2 = v_1^2 + v_2^2 \gg \triangle v^2 = v_1^2 - v_2^2,$$

Fermion Sector

- There are several possible ways to construct anomaly-free fermion spectra for $b = -1$ & $c = 2$.
- We choose the following set of fermion spectra which would be feasible enough to complete this little Higgs model (Kong 2003):

	$U(1)_Y$-states		
$(3_C, 4_L, \frac{1}{6})$	$\frac{1}{6}[Q]$	$\frac{2}{3}(T)$	$\frac{-1}{3}(B)$
$2\ (3_C, \bar{4}_L, \frac{1}{6})$	$2\ \frac{1}{6}[2\ Q]$	$2\ \frac{-1}{3}(D, S)$	$2\ \frac{2}{3}(U, C)$
$3\ (l_C, 4_L, \frac{-1}{2})$	$3\ \frac{-1}{2}[3\ L]$	$3\ \ 0(3\ N)$	$3\ \ \text{-1}(3\ E^-)$
$6\ (\bar{3}_C, 1_L, \frac{-2}{3})$	$6\ \frac{-2}{3}\ (\bar{u}, \bar{c}, \bar{t}, \bar{U}, \bar{C}, \bar{T})$		
$6\ (\bar{3}_C, 1_L, \frac{1}{3})$	$6\ \frac{1}{3}\ (\bar{d}, \bar{s}, \bar{b}, \bar{D}, \bar{S}, \bar{B})$		
$6\ (1_C, 1_L, 1)$	$3\ 1\ (e^+, \mu^+, \tau^+)$		$3\ \ 1(3\ E^+)$

$\Longrightarrow$ Anomaly cancellation is achieved when $N_f = N_c = 3 \Rightarrow$ one of the best features of this model.

Interaction Lagrangian

- Charged Current

$$\mathcal{L}_{CC} = -\frac{g}{\sqrt{2}}\left[\bar{u}\gamma^{\mu}(1-\gamma_5)d + \bar{\nu}\gamma_{\mu}(1-\gamma_5)e\right]W^{+} + (\text{terms with } X^{(\prime)}, Y^{(\prime)}, \text{and } W') + h.c.$$

- Neutral Current

$$\begin{aligned}\mathcal{L}_{NC} &= -eQ\left(\bar{\psi}\gamma^{\mu}\psi\right)A_{\mu} + \frac{g}{4\sqrt{2}}\left(\bar{\psi}\gamma^{\mu}(1-\gamma_5)\psi\right)Z''_{\mu} \\ &\quad -\frac{g}{2c_W}\left[\left(\bar{\psi}\gamma^{\mu}(g_V - g_A\gamma_5)\psi\right)Z_{\mu} + \left(\bar{\psi}\gamma^{\mu}(g'_V - g'_A\gamma_5)\psi\right)Z'_{\mu}\right]\end{aligned}$$

ψ	g_V	g_A	g'_V	g'_A
u	$\frac{1}{2}-\frac{4}{3}s_W^2+\frac{5t}{6}s_W s_\theta$	$\frac{1}{2}-\frac{t}{2}s_W s_\theta$	$\left(\frac{1}{2}-\frac{4}{3}s_W^2\right)s_\theta-\frac{5t}{6}s_W$	$\frac{1}{2}s_\theta+\frac{t}{2}s_W$
d	$-\frac{1}{2}+\frac{4}{3}s_W^2-\frac{t}{6}s_W s_\theta$	$-\frac{1}{2}+\frac{t}{2}s_W s_\theta$	$\left(-\frac{1}{2}+\frac{2}{3}s_W^2\right)s_\theta+\frac{t}{6}s_W$	$-\frac{1}{2}s_\theta-\frac{t}{2}s_W$
ν	$\frac{1}{2}-\frac{t}{2}s_W s_\theta$	$\frac{1}{2}-\frac{t}{2}s_W s_\theta$	$\frac{1}{2}s_\theta+\frac{t}{2}s_W$	$\frac{1}{2}s_\theta+\frac{t}{2}s_W$
e	$-\frac{1}{2}+2s_W^2-\frac{3t}{2}s_W s_\theta$	$-\frac{1}{2}+\frac{t}{2}s_W s_\theta$	$\left(-\frac{1}{2}+2s_W^2\right)s_\theta+\frac{3}{2}s_W$	$-\frac{1}{2}s_\theta-\frac{t}{2}s_W$

$$s_\theta = t_W^2\sqrt{1-t_W^2 v^2/(2c_W f^2)}$$

Electroweak Constraints

- Custodial $SU(2)$ symmetry violating shift in the Z mass:

$$\delta\rho = \alpha T \simeq \frac{g_X^4}{4(g^2+g_X^2)^2}\frac{v^2}{f^2} \qquad (f^2 = f_1^2 + f_2^2,\ v^2 = v_1^2 + v_2^2)$$

- For $M_H = 117$ GeV, $T \leq 0.06$ (95% C.L.)

  $\Longrightarrow f_{SU(4)} \geq 1.7$ TeV $\quad (M_{Z'} \geq 1.3$ TeV, $\quad M_{Z''} \geq 790$ GeV$)$

  $\Longrightarrow$ comparable to $f_{SU(3)} \geq 2.3$ TeV

- For $M_H = 300$ GeV, $T \leq 0.14$ (95% C.L.)

  $\Longrightarrow f_{SU(4)} \geq 1.1$ TeV $\quad (M_{Z'} \geq 870$ GeV, $\quad M_{Z''} \geq 520$ GeV$)$

  $\Longrightarrow$ comparable to $f_{SU(3)} \geq 1.5$ TeV

Electroweak Constraints

- Effective weak charge in atomic parity violation:

$$Q_W = -2\,[(2Z+N)C_{1u} + (Z+2N)C_{1d}] \quad (C_{1q} = 2g_A^e g_V^q)$$

- For $\Delta Q_W(Cs) = Q_W^{exp} - Q_W^{\rm SM} = 0.55 \pm 0.49$, $\quad 0.82\ \text{TeV} \leq f_{SU(4)} \leq 3.4\ \text{TeV}$

$\Longrightarrow 0.63\ \text{TeV} \leq M_{Z'} \leq 2.7\ \text{TeV}, \quad 0.38\ \text{TeV} \leq M_{Z''} \leq 1.6\ \text{TeV}$

- Direct experimental constraint on four-fermion operator from left-left currents:

$$\eta_{LL}^{eq}\,(\bar{e}_L \gamma_\mu e_L)\,(\bar{q}_L \gamma_\mu q_L)\,, \quad \eta_{LL}^{eq} \sim 1/f^2$$

- For $\eta_{LL}^{eq} = 0.01 \pm 0.20$ (Cheung 2001), $\quad M_{Z''} \geq 500\ \text{GeV} \quad (f_{SU(4)} \geq 1.1\ \text{TeV})$

Summary

- As an alternative to SUSY, a little Higgs model is proposed to describe a TeV scale effective theory.
- It is natural to complete the LHM with all nice features of the SM such as the gauge anomaly cancellation conditions which dictate the SM fermionic spectrum.
- Sensible phenomenological analysis of such models has to start with feasible complete global quantum number assignment to all multiplets.
- Custodial $SU(2)$ symmetry violating shift in the Z mass gives $\Lambda_{SU(4)} \sim 4\pi f \geq 14$ TeV for $M_H \leq 300$ GeV.
- Further phenomenological study is in progress.