Fermions and Gauge Bosons in $SU(4)_L \times U(1)_X$ models with little Higgs

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Introduction

- As an alternative solution to the naturalness problem (and the fine-tuning issue), little Higgs models (LHM) has been recently introduced. (Arkani-Hamed, Cohen, and Georgi 2001)

- LHMs adopts the early idea that Higgs can be considered as a Nambu Goldstone boson from global symmetry breaking at some higher scale $\Lambda \sim 4\pi f$. (Dimopoulos, Preskill 1982; Georgi, Kaplan 1984; Banks 1984)

- Higgs acquires a mass radiatively through symmetry breaking at the EW scale $v$, by collective breaking.

- There are several LHMs such as “minimal moose”, “product group”, and “simple group” models.

- In this talk, we discuss the simple group model with $SU(4)_L \times U(1)_X$ electroweak gauge group (suggested by Kaplan and Schmaltz, 2003) by embedding the anomaly-free fermion spectra.
Little Higgs from $SU(4)$ Group

- Start from a non-linear sigma model $[SU(4)/SU(3)]^4$ with four complex quadruplets scalar fields $\Phi_{ij}$ ($i, j = 1, 2$)
  $\implies$ Diagonal $SU(4)$ is gauged.

- Gauge symmetry breaking: $SU(4)_L \times U(1)_X \rightarrow SU(2)_L \times U(1)_Y$
  $\implies$ 12 new gauge bosons with masses of order the scale $f$.

- Global symmetry breaking: $[SU(4)]^4 \rightarrow [SU(3)]^4$
  $\implies$ 12 of the 28 degrees of freedom in the $\Phi_{ij}$ are eaten by the Higgs mechanism when $SU(4)_W$ is broken.
  $\implies$ Remaining 16 consist of two complex doublets $h_u$ and $h_d$, three complex $SU(2)$ singlets $\sigma_1$, $\sigma_2$ and $\sigma_3$, and two real scalars $\eta_u$ and $\eta_d$.

(Kaplan and Schmaltz 2003)
Little Higgs from $SU(4)$ Group

One possible parametrization of the non-linear sigma model fields $\Phi_{ij}$ with general $f_{ij}$ ($SU(4)$ breaking is not aligned):

\[
\Phi_{11} = e^{+i\mathcal{H}_u f_{12}^1} \begin{pmatrix} 0 \\ 0 \\ f_{11} \\ f_{12} \\ 0 \end{pmatrix} \quad \Phi_{12} = e^{-i\mathcal{H}_u f_{11}^1} \begin{pmatrix} 0 \\ 0 \\ f_{12} \\ f_{11} \\ 0 \end{pmatrix} \\
\Phi_{21} = e^{+i\mathcal{H}_d f_{22}^2} \begin{pmatrix} 0 \\ 0 \\ 0 \\ f_{21} \\ h_1 \end{pmatrix} \quad \Phi_{22} = e^{-i\mathcal{H}_d f_{21}^2} \begin{pmatrix} 0 \\ 0 \\ 0 \\ f_{22} \\ 0 \end{pmatrix}
\]

where

\[
\mathcal{H}_u = \begin{pmatrix} 0 & 0 & h_1 & 0 \\ 0 & 0 & 0 & h_2 \\ h_1^\dagger & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} / 2f_1 \quad \mathcal{H}_d = \begin{pmatrix} 0 & 0 & 0 & h_2 \\ 0 & 0 & 0 & 0 \\ h_2^\dagger & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} / 2f_2
\]

\[
f_i^2 = \frac{1}{2} \sum_{j=1,2} f_{ij}^2, \quad \langle h_1 \rangle = \begin{pmatrix} v_u \\ 0 \end{pmatrix}, \quad \langle h_2 \rangle = \begin{pmatrix} 0 \\ v_d \end{pmatrix}
\]
Model Construction

- Electric charge generator: \( Q = a T_{3L} + \frac{1}{\sqrt{3}} b T_{8L} + \frac{1}{\sqrt{6}} c T_{15L} + X l_4 \)

- Correct embedding of the SM isospin \( SU(2)_L \) doublets gives \( a = 1 \) \( (Q = T_{3L} + Y) \).

- If we assume that all gauge bosons have electric charges 0 and \( \pm 1 \) only, there are not more than four different possibilities (Ponce, Gutiérrez, and Sánchez 2004):

\[
\begin{align*}
  b = c &= 1 & b = c &= -1 & b = 1 & c = -2 & b = -1 & c = 2 \\
  (u, d, D, D')^T &; (d, u, U, U')^T &; (u, d, D, U)^T &; (u, d, U, D)^T
\end{align*}
\]

- Our choice is to have the duplicated extra heavy fermions which can remove the quadratic divergences due to their SM fermion partners.

- For \( b = -1 & c = 2 \), \( X(\Phi_1) = -\frac{1}{2}, X(\Phi_2) = \frac{1}{2}; X(\psi_L) = \frac{1}{6}, X(\psi_R) = Q \)

  ※ Our quantum number assignments differ from those of the original paper by Kaplan & Schmaltz.
Gauge Boson Sector

- Covariant derivative for 4-plets: \( D_\mu = \partial_\mu + igT^\alpha A^\alpha_\mu + ig_X X A^X_\mu \)

- There are 15 gauge bosons associated with \( SU(4)_L \):

\[
T^\alpha A^\alpha_\mu = \frac{1}{\sqrt{2}} \begin{pmatrix}
Z_1^0_\mu & W^+_\mu & Y^0_\mu & X'^+_\mu \\
W^-_\mu & Z_2^0_\mu & X^-_1_\mu & Y'0_\mu \\
\bar{Y}^0_\mu & X^+_1_\mu & Z_3^0_\mu & W'_+^0_\mu \\
X'^-_\mu & \bar{Y}'0_\mu & W'_-^0_\mu & Z_4^0_\mu
\end{pmatrix}
\]

where

\[
Z_{1\mu}^0 = A_3^\mu / \sqrt{2} + A_8^\mu / \sqrt{6} + A_{15}^\mu / \sqrt{12} \\
Z_{2\mu}^0 = -A_3^\mu / \sqrt{2} + A_8^\mu / \sqrt{6} + A_{15}^\mu / \sqrt{12} \\
Z_{3\mu}^0 = -2A_8^\mu / \sqrt{6} + A_{15}^\mu / \sqrt{12} \\
Z_{4\mu}^0 = -3A_{15}^\mu / \sqrt{12}
\]
Gauge Boson Sector

- **Physical Gauge Boson Masses:**

  ▶ \( M^2_W = \frac{1}{4} g^2 v^2 \), \( M^2_{W'} = \frac{1}{4} g^2 \left( 4f^2 - v^2 \right) \)

  ▶ \( M^2_X = \frac{1}{4} g^2 \left( 4f^1 - v^2_1 + v^2_2 \right) \), \( M^2_{X'} = \frac{1}{4} g^2 \left( 4f^2_2 + v^2_1 - v^2_2 \right) \)

  ▶ \( M^2_Y = g^2 f^2_1 \), \( M^2_{Y'} = g^2 f^2_2 \)

  ▶ \( M^2_Z = \frac{g^2 v^2}{4c^2_W} \left( 1 - \frac{t^4_W}{4} \frac{v^2}{f^2} \right) \), \( M^2_{Z'} = g^2 \left( 1 + t^2 \right)f^2 - M^2_Z \)

  \( M^2_{Z''} = \frac{1}{2} g^2 f^2 \)

where

\[ t \equiv g_x / g, \quad c_W \equiv \cos \theta_W = \sqrt{(1 + t^2) / (1 + 2t^2)} \]

\[ v^2_1 \equiv v^2_u - \frac{v^2_u}{12f^2_1} \left( \frac{f^2_{12}}{f^2_{11}} + \frac{f^2_{11}}{f^2_{12}} - 1 \right), \quad v^2_2 \equiv v^2_d - \frac{v^2_d}{12f^2_2} \left( \frac{f^2_{22}}{f^2_{21}} + \frac{f^2_{21}}{f^2_{22}} - 1 \right) \]

\[ f^2 = f^2_1 + f^2_2 \gg \Delta f^2 = f^2_1 - f^2_2, \quad v^2 = v^2_1 + v^2_2 \gg \Delta v^2 = v^2_1 - v^2_2, \]
Fermion Sector

- There are several possible ways to construct anomaly-free fermion spectra for $b = -1$ & $c = 2$.
- We choose the following set of fermion spectra which would be feasible enough to complete this little Higgs model (Kong 2003):

<table>
<thead>
<tr>
<th></th>
<th>$U(1)_Y$-states</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\frac{1}{6}[Q]$</td>
</tr>
<tr>
<td>2</td>
<td>$\frac{1}{6}[2Q]$</td>
</tr>
<tr>
<td>3</td>
<td>$\frac{-1}{2}[3L]$</td>
</tr>
<tr>
<td>6</td>
<td>$\frac{-2}{3}(\bar{u},\bar{c},\bar{t},\bar{U},\bar{C},\bar{T})$</td>
</tr>
<tr>
<td>6</td>
<td>$\frac{1}{3}(e^+,\mu^+,\tau^+)$</td>
</tr>
</tbody>
</table>

⇒ Anomaly cancellation is achieved when $N_f = N_c = 3 \Rightarrow$ one of the best features of this model.
Interaction Lagrangian

- **Charged Current**

\[
L_{CC} = - \frac{g}{\sqrt{2}} [\bar{u} \gamma^\mu (1 - \gamma_5) d + \bar{\nu} \gamma_\mu (1 - \gamma_5) e] W^+ + \text{(terms with } X^{(i)}, Y^{(i)}, \text{ and } W') + h.c.
\]

- **Neutral Current**

\[
L_{NC} = - e Q (\bar{\psi} \gamma^\mu \psi) A_\mu + \frac{g}{4\sqrt{2}} (\bar{\psi} \gamma^\mu (1 - \gamma_5) \psi) Z_\mu''
\]

\[
- \frac{g}{2c_W} \left[(\bar{\psi} \gamma^\mu (g_V - g_A \gamma_5) \psi) Z_\mu + (\bar{\psi} \gamma^\mu (g'_V - g'_A \gamma_5) \psi) Z'_\mu\right]
\]

<table>
<thead>
<tr>
<th>\psi</th>
<th>( g_V )</th>
<th>( g_A )</th>
<th>( g'_V )</th>
<th>( g'_A )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( u )</td>
<td>( \frac{1}{2} - \frac{4}{3} s_W^2 + \frac{5}{6} t s_W s_\theta )</td>
<td>( \frac{1}{2} - \frac{t}{2} s_W s_\theta )</td>
<td>( \frac{1}{2} - \frac{4}{3} s_W^2 ) s_\theta - \frac{5}{6} s_W</td>
<td>( \frac{1}{2} s_\theta + \frac{t}{2} s_W )</td>
</tr>
<tr>
<td>( d )</td>
<td>( -\frac{1}{2} + \frac{4}{3} s_W^2 - \frac{t}{6} s_W s_\theta )</td>
<td>( -\frac{1}{2} + \frac{t}{2} s_W s_\theta )</td>
<td>( -\frac{1}{2} + \frac{2}{3} s_W^2 ) s_\theta + \frac{t}{6} s_W</td>
<td>( -\frac{1}{2} s_\theta - \frac{t}{2} s_W )</td>
</tr>
<tr>
<td>( \nu )</td>
<td>( \frac{1}{2} - \frac{1}{2} s_W s_\theta )</td>
<td>( \frac{1}{2} - \frac{t}{2} s_W s_\theta )</td>
<td>( \frac{1}{2} s_\theta + \frac{t}{2} s_W )</td>
<td>( \frac{1}{2} s_\theta + \frac{t}{2} s_W )</td>
</tr>
<tr>
<td>( e )</td>
<td>( -\frac{1}{2} + 2 s_W^2 - \frac{3t}{2} s_W s_\theta )</td>
<td>( -\frac{1}{2} + \frac{t}{2} s_W s_\theta )</td>
<td>( -\frac{1}{2} + 2 s_W^2 ) s_\theta + \frac{3}{2} s_W</td>
<td>( -\frac{1}{2} s_\theta - \frac{t}{2} s_W )</td>
</tr>
</tbody>
</table>

\[
s_\theta = t_W^2 \sqrt{1 - t_W^2 v^2 / (2c_W f^2)}
\]
Electroweak Constraints

- Custodial $SU(2)$ symmetry violating shift in the $Z$ mass:

$$\delta \rho = \alpha T \simeq \frac{g_X^4}{4(g^2 + g_X^2)^2} \frac{\nu^2}{f^2} \quad (f^2 = f_1^2 + f_2^2, \; \nu^2 = \nu_1^2 + \nu_2^2)$$

- For $M_H = 117$ GeV, $T \leq 0.06$ (95% C.L.)
  $\implies f_{SU(4)} \geq 1.7$ TeV \quad ($M_{Z'} \geq 1.3$ TeV, \quad $M_{Z''} \geq 790$ GeV)
  $\implies$ comparable to $f_{SU(3)} \geq 2.3$ TeV

- For $M_H = 300$ GeV, $T \leq 0.14$ (95% C.L.)
  $\implies f_{SU(4)} \geq 1.1$ TeV \quad ($M_{Z'} \geq 870$ GeV, \quad $M_{Z''} \geq 520$ GeV)
  $\implies$ comparable to $f_{SU(3)} \geq 1.5$ TeV
Electroweak Constraints

- Effective weak charge in atomic parity violation:

\[ Q_W = -2 [(2Z + N)C_{1u} + (Z + 2N)C_{1d}] \quad (C_{1q} = 2g_A^q g_Y^q) \]

- For \( \Delta Q_W(Cs) = Q_W^{exp} - Q_W^{SM} = 0.55 \pm 0.49 \),
  \[ 0.82 \text{ TeV} \leq f_{SU(4)} \leq 3.4 \text{ TeV} \]
  \[ \Rightarrow 0.63 \text{ TeV} \leq M_{Z'} \leq 2.7 \text{ TeV}, \quad 0.38 \text{ TeV} \leq M_{Z''} \leq 1.6 \text{ TeV} \]

- Direct experimental constraint on four-fermion operator from left-left currents:

\[ \eta_{LL}^{eq} (\bar{e}_L \gamma_{\mu} e_L)(\bar{q}_L \gamma_{\mu} q_L), \quad \eta_{LL}^{eq} \sim 1/f^2 \]

- For \( \eta_{LL}^{eq} = 0.01 \pm 0.20 \) (Cheung 2001),
  \[ M_{Z''} \geq 500 \text{ GeV} \quad (f_{SU(4)} \geq 1.1 \text{ TeV}) \]
As an alternative to SUSY, a little Higgs model is proposed to describe a TeV scale effective theory.

It is natural to complete the LHM with all nice features of the SM such as the gauge anomaly cancellation conditions which dictate the SM fermionic spectrum.

Sensible phenomenological analysis of such models has to start with feasible complete global quantum number assignment to all multiplets.

Custodial $SU(2)$ symmetry violating shift in the $Z$ mass gives $\Lambda_{SU(4)} \sim 4\pi f \geq 14$ TeV for $M_H \leq 300$ GeV.

Further phenomenological study is in progress.