$T$-Parity in Little Higgs Models $^a$

David Krohn

Outline

- Review of little Higgs models and $T$-parity
- Anomalies and WZW terms
- Anomaly-free models and their obstacles
- Moose models preserving $T$-parity and their phenomenology
The motivation behind LH theories

• Little Higgs theories are models of EWSB designed to solve the little hierarchy problem \(^a\).

• They identify the Higgs with a pseudo Goldstone boson.

• The distinguishing feature of these theories is that by making use of particular gauge and global symmetries they keep the symmetry breaking scale associated with the Goldstones far above the electroweak scale (at around \(\mathcal{O}(10) \text{ TeV}\)).

• With the symmetry breaking scale around $10 \text{ TeV}$ it might seem that details of the UV physics of these theories are inaccessible at the LHC.

• However, effects from the UV physics can play an important role in LHC-scale phenomenology. Parities in particular are effected.
**$T$-parity**

- An important parity in LH model building is $T$-parity \(^a\) \(^b\), analogous to the $R$-parity of SUSY.

- This parity is useful because it
  - Helps models evade electroweak constraints
  - *Ensures a stable dark matter candidate*

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T-parity and anomalies

- Last year it was shown that quantum anomalies are important in discussing LH model building \(^a\)^ \(^b\).
- Quantum anomalies come from fermions in the UV description of a theory. These manifest themselves at low energies in Wess-Zumino-Witten (WZW) interaction terms.

- **These terms can violate the naive parities of a theory.**


Anomalies and parity

- The $\pi \to \gamma \gamma$ interaction from QCD is an example of a parity violated by anomalies. The chiral Lagrangian has a parity $\pi \to -\pi$ which is broken by interactions like $\pi \to \gamma \gamma$ coming from WZW terms.
• In the same way that $\pi \to \gamma\gamma$ violates $\pi \to -\pi$ parity in QCD, WZW terms in LH models can violate $T$-parity.

• Although this does not hurt electroweak constraints, it renders any dark matter candidate unstable.
We want to know what the requirement of a preserved $T$-parity can tell us about the phenomenology of LH models. We will consider:

- Models manifestly free of WZW terms
- Moose models with $\text{SU}(N)$ groups
Removing the WZW terms

• If WZW terms could be removed altogether there would be no problem with $T$-parity. This can be done most simply by using theories
  – With no UV condensing fermions
  – Based on anomaly-free groups
Linear UV completions

• LH theories can be given a linear UV completion with fundamental scalars. Such models are necessarily free of WZW terms because there are no UV fermions. However, stabilizing the electroweak scale requires that these scalars carry additional SUSY structure. This approach is taken in Csaki et al. \(^a\)

\(^a\)C. Csaki, J. Heinonen, M. Perelstein, and C. Spethmann, *A Weakly Coupled Ultraviolet Completion of the Littlest Higgs with T-Parity*, 0804.0622
Models with anomaly-free groups

- Another way to remove WZW terms is to build a model with anomaly free global symmetry groups.

- LH models based on anomaly free groups (SO and Sp) have been constructed \(^{a\ b\ c}\).

- Is it possible to UV complete these theories with condensing fermions?

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• Let’s see what happens with a $SO(N) \times SO(N) / SO(N)$ model. Our results can be generalized to other coset spaces.

• Begin by noting that a generic UV model with fermions will have $SU(N) \times SU(N)$ symmetry, which must be explicitly broken to $SO(N) \times SO(N)$. 
• One way to get the right global symmetry is to introduce Majorana masses. These will yield the desired global symmetry.

• Unfortunately, Majorana masses will prevent the vacuum from realizing the desired symmetry breaking pattern.

$$\text{SO}(N) \psi_R \psi_L \text{SO}(N) \not\Rightarrow \pi \text{SO}(N) \times \text{SO}(N)$$

Instead, we will only see $$\text{SU}(N) \times \text{SU}(N) \rightarrow \text{SO}(N) \times \text{SO}(N)$$
• This is because of the *persistent mass conjecture*: if the symmetry breaking pattern were $\text{SO}(N) \times \text{SO}(N) \rightarrow \text{SO}(N)$ we would expect massless Goldstones made out of very heavy fermions. This is a contradiction, and it tells us that the vacuum will not break symmetry in this way.
• Since Majorana masses don’t seem to work, we can look to higher dimensional operators to help achieve the desired symmetry breaking pattern.

• Dimension-six operators can reduce the global symmetry to $\text{SO}(N) \times \text{SO}(N)$:

$$\mathcal{L} \supset \frac{y^2}{M^2} \psi^T_L \psi_L \overline{\psi}^T_L \overline{\psi}_L + L \rightarrow R$$

However, they must have unnaturally large coefficients.

• Any trick that could naturally generate these operators without inducing a large fermion mass term would be an important tool for constructing little Higgs theories. We know of no realistic implementation to achieve this.
• We saw that the simplest solution to the question of $T$-parity, making a model free of WZW terms, can be difficult to realize.

• Now let’s look at moose models. We will find that these models have WZW terms, yet they can be arranged in a way that preserves $T$-parity.
Multi-link moose models

- Some LH theories are built from moose models \(^a\). These take a symmetry breaking pattern and copy it multiple times, gauging some linear combination of global symmetries.
- The multi-link model can be made free of gauge anomalies. However, this is not true for global anomalies.

- Multi-link moose models will always have WZW terms because these are the result of the theory’s global symmetry.

- Let’s see how $T$-parity acts on such a model. Consider the case of a $SU(3) \times SU(3)$ moose model with two sites. Label the link fields $\pi_1$ and $\pi_2$, and gauge the combination of the upper left $SU(2)$ and the diagonal $U(1)$. The Higgs lives in the Goldstone fields:

$$\pi \rightarrow \begin{pmatrix}
\begin{array}{c|c}
 d & h \\
\hline
h^\dagger & d
\end{array}
\end{pmatrix}$$

and we identify the combination $\pi_1 - \pi_2$ as that containing the SM Higgs.
- \( T \)-Parity is normally defined to take

\[
U_{1/2} \rightarrow \Omega U_{1/2}^\dagger \Omega, \quad A_{L/R} \rightarrow A_{R/L}
\]

where

\[
\Omega = \begin{pmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & -1
\end{pmatrix}
\]

- Under this transformation the kinetic terms in the Lagrangian go into themselves. The WZW terms, however, go

\[
\mathcal{L}_{WZW}(\pi, A_L, A_R) \xrightarrow{T\text{-Parity}} -\mathcal{L}_{WZW}(\pi, A_L, A_R)
\]
• It looks like $T$-parity is broken. However, we can change the definition of $T$-parity to involve an interchange of the $\pi$ fields. The new definition of $T$-parity takes

$$U_{1/2} \rightarrow \Omega U_{2/1} \Omega, \ A_{L/R} \rightarrow A_{R/L}$$

• This is an exact symmetry of both the kinetic and WZW terms of the theory. Therefore, the particles odd under this symmetry are stable against decay. They could be dark matter candidates.
• For $T$-parity to be exact in a moose model, it must be reflected in a UV relabeling symmetry of the theory.

• If we see missing energy signals at the LHC and we suspect that we see a LH theory, the constraints from quantum anomalies already tells us something about the UV model.
• Models with exact $T$-parity must be symmetric in their gauge groups, UV fermions, and SM fermions.
• Keeping a theory free of gauge anomalies may require heavy spectator fermions. These must also be symmetric in a theory.
• The next two slides contain example models.
Figure 1: A more realistic moose with exact $T$-parity, which is manifested by a $180^\circ$ rotation. The middle site are necessary for the model to have SM fermions.
Figure 2: Another model with exact $T$-parity, which here comes from a reflection about the $y$-axis. Here we have added spectator fermions necessary to make the model free of gauge anomalies.
Conclusions

- $T$-parity can be (*but is not necessarily*) violated by WZW terms.
- Completely removing all WZW terms is difficult, and may not be achievable without invoking SUSY.
- Models with WZW terms yet preserving $T$-parity can be constructed with moose models.
- The requirement of $T$-parity constrains what form these models can take.