Running of bottom quark mass in the MSSM

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in collaboration with

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Bottom quark mass

- Bottom quark mass in SM
  - $m_b^{\overline{\text{MS}}}(m_b)$ known with 4-loop accuracy [J. H. Kühn, M. Steinhauser, C. Sturm '07]
  - $m_b^{\overline{\text{DR}}}(M_{\text{SUSY}})$ input parameter for MSSM analyses
- Bottom quark in MSSM (models with large $\tan\beta$)
  - SUSY mass spectrum sensitive to bottom Yukawa coupling
  - $Y_b(\mu) \leftrightarrow m^{\overline{\text{DR}}}(\mu)$ affected by large SUSY radiative corrections
- (SUSY)GUT models $\Rightarrow$ predictions for $m_{\text{t}}, m_{\text{b}}/m_\tau$ [talks by S. Raby, G. Ross, S. Autush]

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$$\Rightarrow \quad {m_b}^{\overline{\mathrm{DR}}}(M_{\mathrm{GUT}}) \leftrightarrow {m_b}^{\overline{\mathrm{DR}}}(M_{\mathrm{SUSY}}) \leftrightarrow {m_b}^{\overline{\mathrm{MS}}}(m_b)$$

with high accuracy

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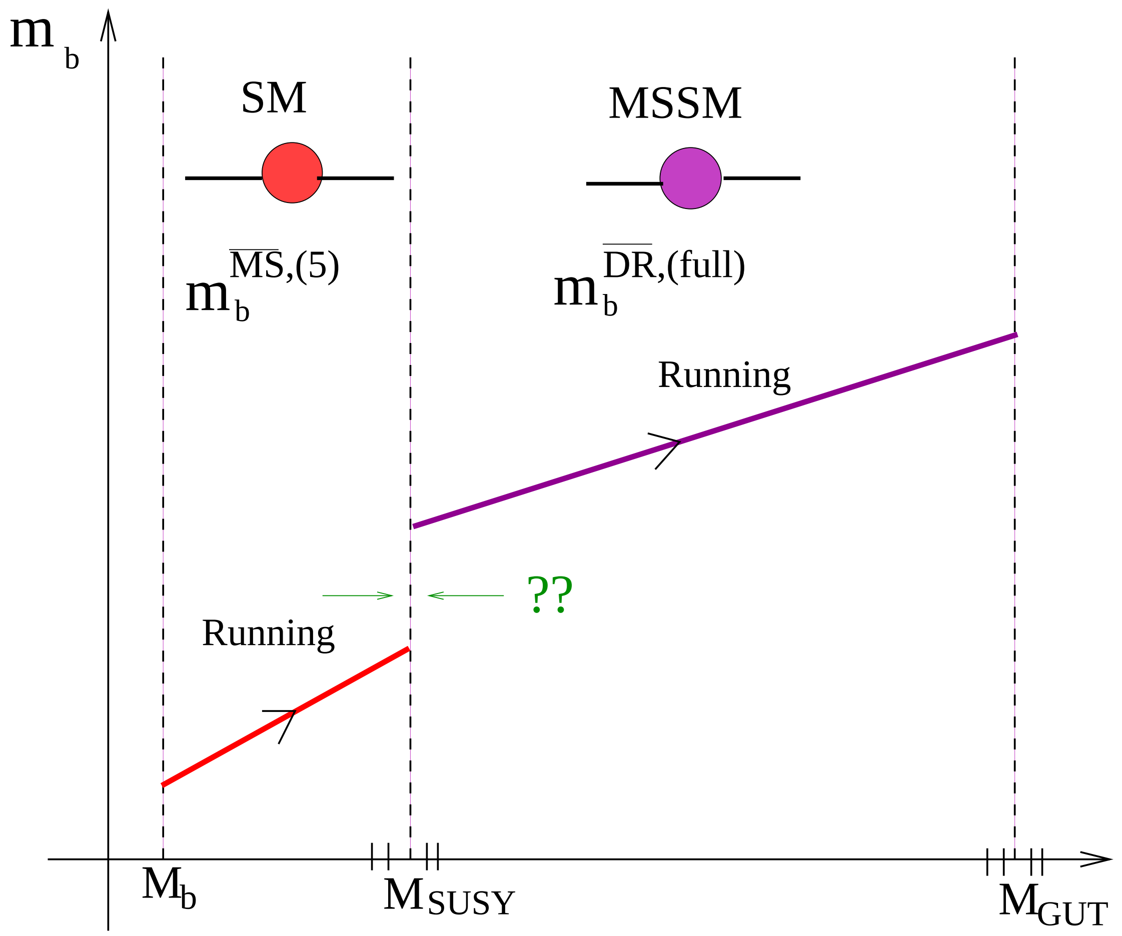
  - State of the art: 2-loop RG running $\oplus$ 1-loop threshold corrections at $\mu = M_Z$
  - Our aim: improve theoretical accuracy on ${m_b}^{\overline{\mathrm{DR}}}(M_{\mathrm{GUT}})$

Evolution of parameters

- Input parameters: $\alpha_s^{\overline{\text{MS}},(5)}(\text{M}_\text{Z})$, $m_b{}^{\overline{\text{MS}},(5)}(m_b)$ $\Rightarrow$
  Output parameters: $\alpha_s^{\overline{\text{DR}},(\text{full})}(\text{M}_\text{GUT})$, $m_b{}^{\overline{\text{DR}},(\text{full})}(\text{M}_\text{GUT})$



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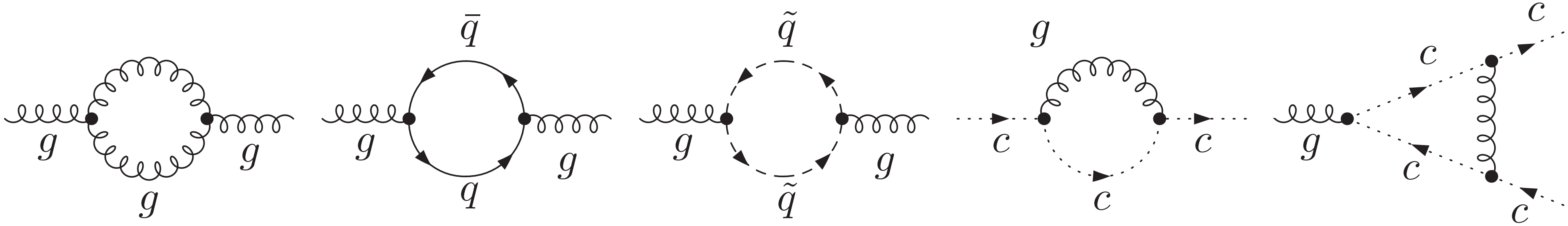

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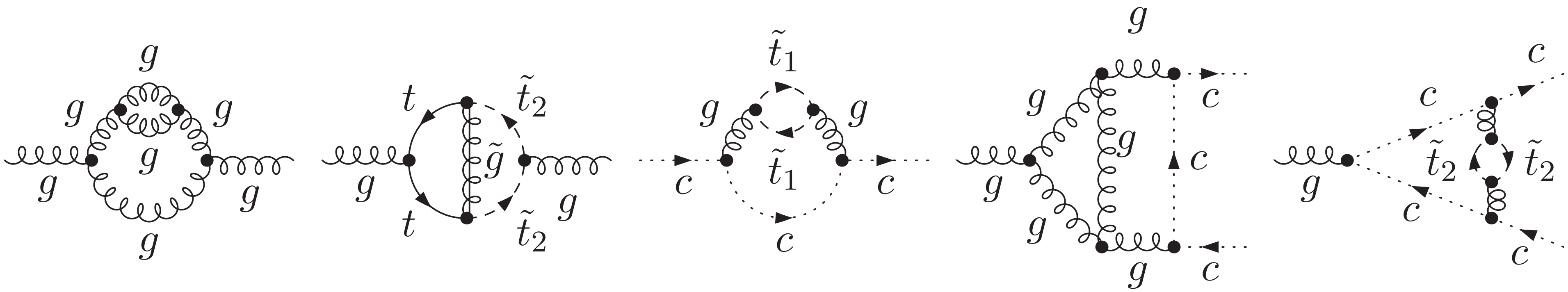

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3-loops:

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- 3-loop β_s and γ_m in the MSSM
  - $\simeq$ 13.000 diagrams
  - Computer programs: QGRAF, FORM, MINCER, MATAD, EXP, ...

    [Noguiera; Vermaseren; Larin, Tkachov; Steinhauser; Seidensticker, Harlander; ...]

Relation $m^{\overline{\mathrm{DR}}} \leftrightarrow m^{\overline{\mathrm{MS}}}$

- Extract $m_{\mathrm{b}}^{\overline{\mathrm{DR}}}(M_{\mathrm{SUSY}})$ from accurately determined $m_{\mathrm{b}}^{\overline{\mathrm{MS}}}(m_b)$

$$m_b^{\overline{\mathrm{DR}}}(\mu) = m_b^{\overline{\mathrm{MS}}}(\mu)\left[1 + \delta_m^{(1l)}(\alpha_e) + \delta_m^{(2l)}(\alpha_s^{\overline{\mathrm{MS}}}, \alpha_e) + \delta_m^{(3l)}(\alpha_s^{\overline{\mathrm{MS}}}, \alpha_e, \eta_i)\right]\Big|_{\mu=\mu_S},$$

$\{\alpha_s^{\overline{\mathrm{DR}}}, \alpha_e, \eta_i\}\Big|_{\mu=\mu_S}$ have to be known.

  - Log contributions absent (mass-independent schemes)

- 2-step approach for computing $\boldsymbol{m}_{\mathrm{b}}^{\overline{\mathrm{DR}}}(M_Z)$ [H. Baer et al '02]
  - Running of $m_b(\mu)$ and conversion between $\overline{\mathrm{MS}} \leftrightarrow \overline{\mathrm{DR}}$
- 3-loop accuracy needed to accomodate with the new data

Evaluation of $m_b^{\overline{\mathrm{DR}},(5)}(M_{\mathrm{Susy}})$

Input parameters:

$\alpha_s^{\overline{\mathrm{MS}}}(M_Z) = 0.1189 \pm 0.001$ [Bethke '06]

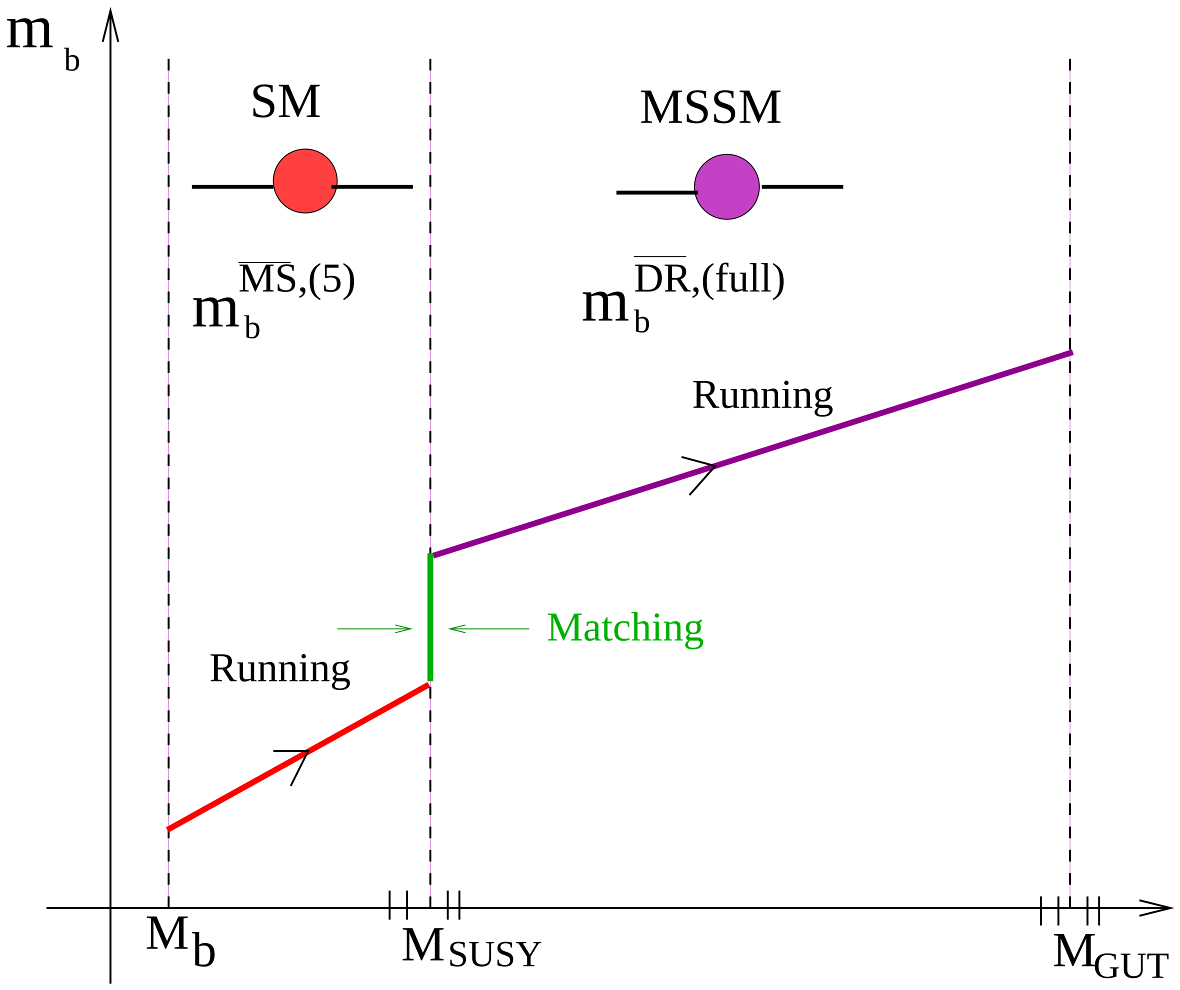
$m_b^{\overline{\mathrm{MS}}}(m_b) = 4.164 \pm 0.025$ GeV [Kühn, Steinhauser, Sturm '07]

3-loop order predictions for $m_b^{\overline{\mathrm{DR}}}(\mu)$

μ (GeV)	$m_b^{\overline{\mathrm{DR}}}(\mu)$
91.1876	$2.804(16)(20)$
350	$2.528(17)(18)$
500	$2.467(17)(18)$
800	$2.394(17)(17)$
1000	$2.361(17)(17)$
2000	$2.268(17)(16)$

Matching

- Effective Field Theory:



Matching

- Effective Field Theory:

$$\mathcal{L}_{\mathrm{MSSM}}(\alpha_s^{(\mathrm{full})}, \ldots) \quad \rightarrow \quad \mathcal{L}(\alpha_s^{(5)}, \ldots) \quad \text{at energy } \mu$$

- "Matching" : low energy physics must be unchanged !!

$$\begin{aligned} \alpha_s^{(5)} &= \zeta_s \, \alpha_s^{(\mathrm{full})} \\ m_b^{(5)} &= \zeta_m \, m_b^{(\mathrm{full})} \\ &\vdots \\ \zeta_s &= \zeta_s(\alpha_s, M_{\mathrm{SUSY}}, m_t, \mu) \\ \zeta_m &= \zeta_m(\alpha_s, M_{\mathrm{SUSY}}, m_t, \mu) \\ \zeta_{s,m} &= \text{matching coefficient} \end{aligned}$$

μ not predicted by theory

Matching coefficients

Relate Green functions computed in the full and effective theory.

- Computations:

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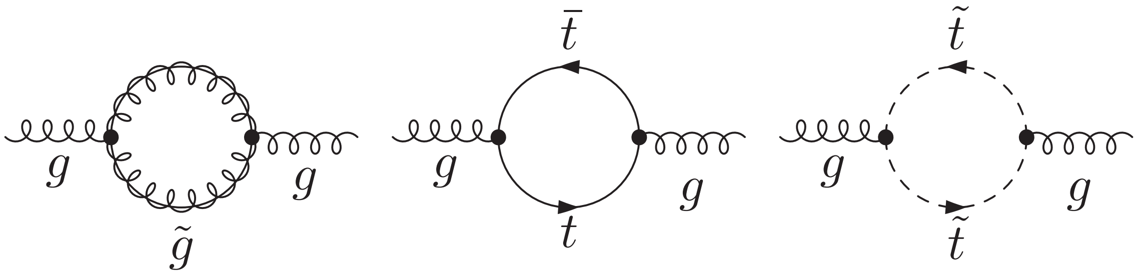
  - 1-loop ζ_s and ζ_m in MSSM [Pierce et al '95]



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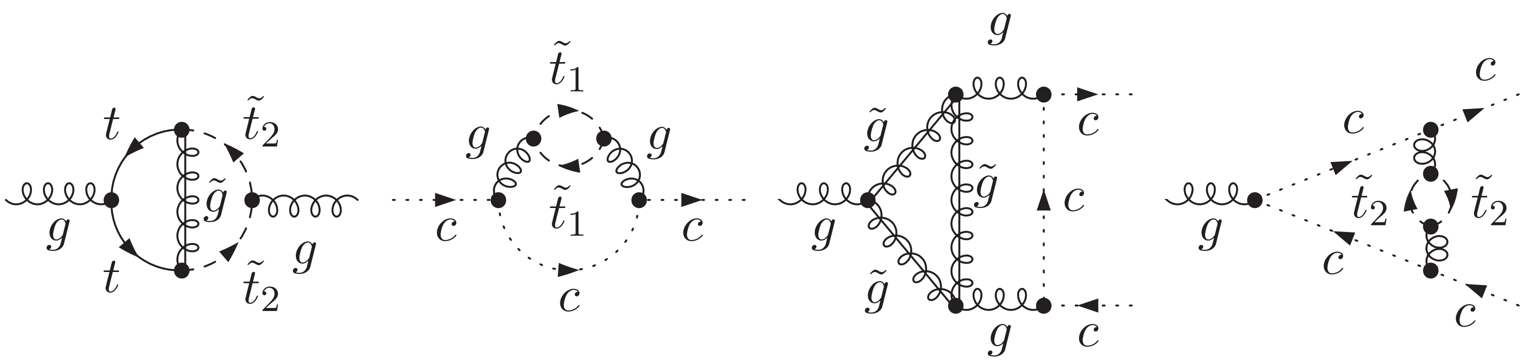

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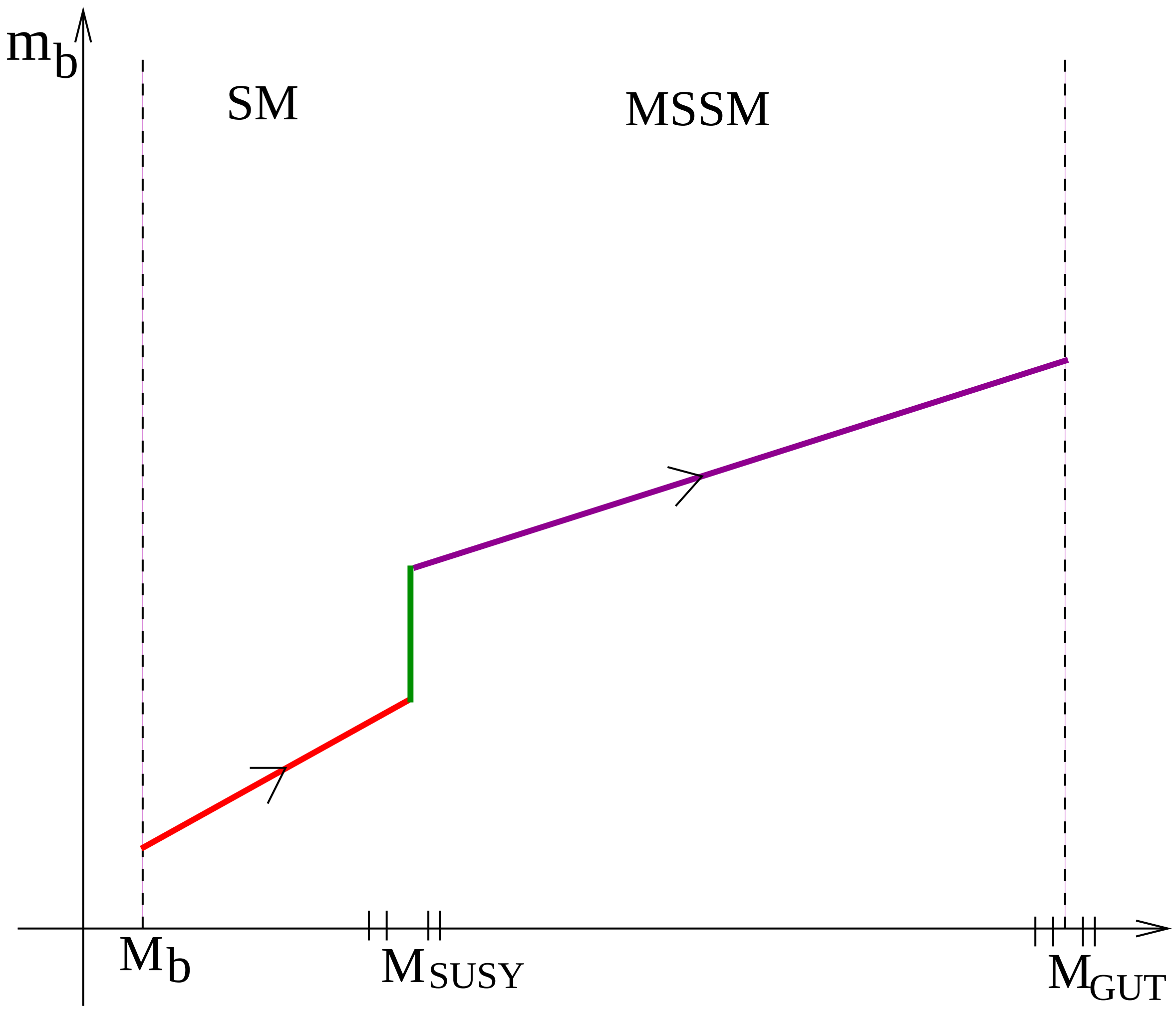

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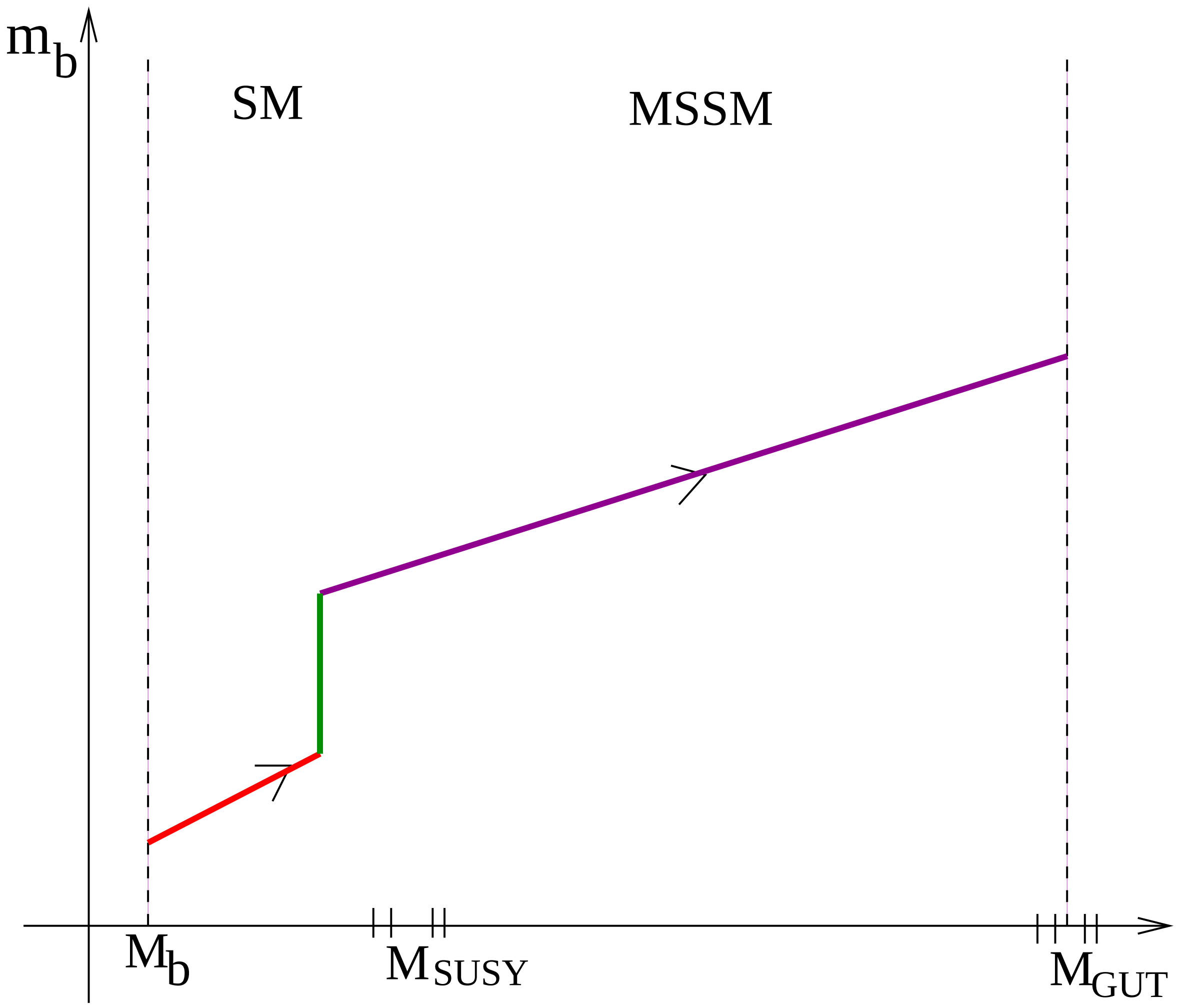

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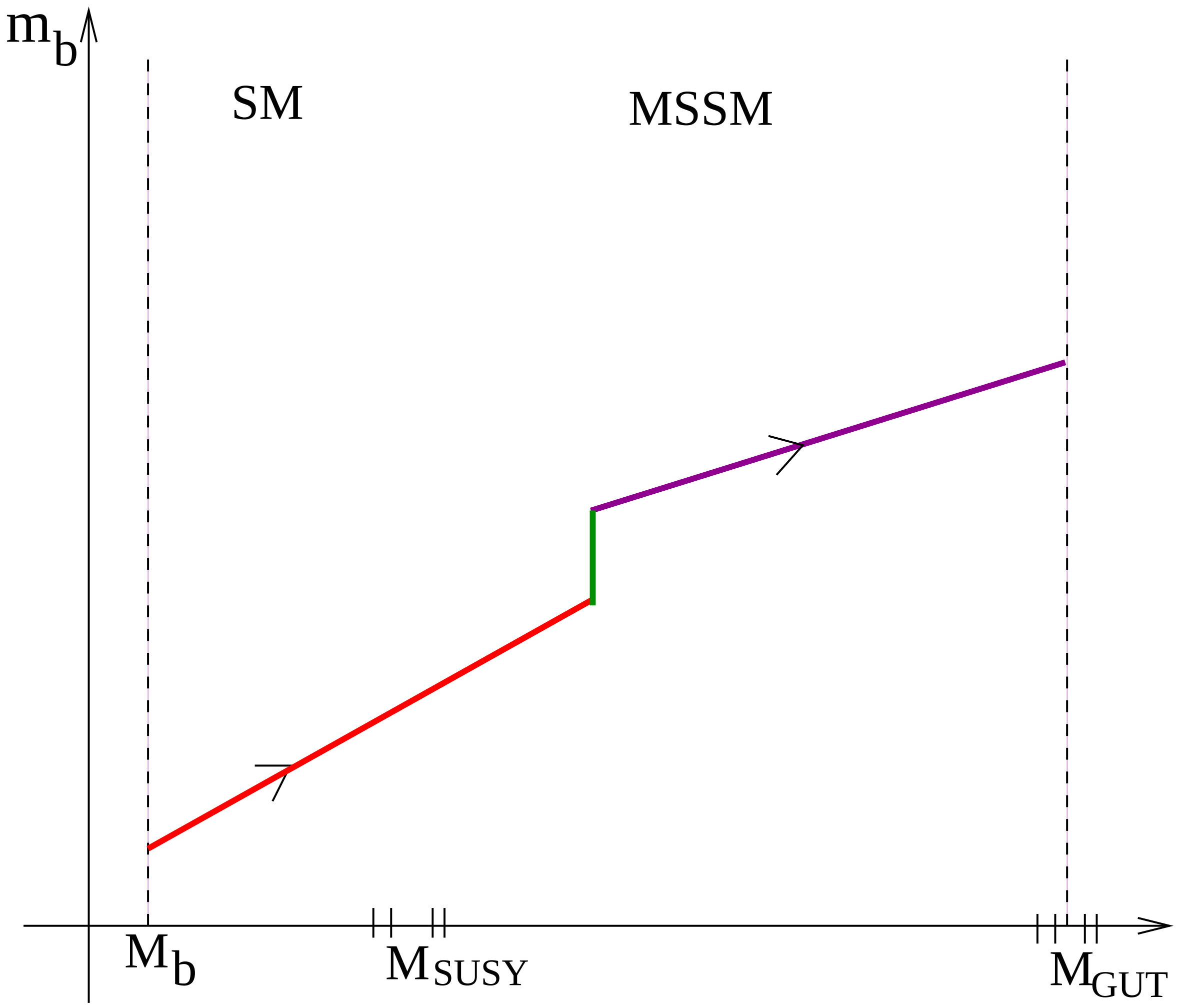

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Input parameters:

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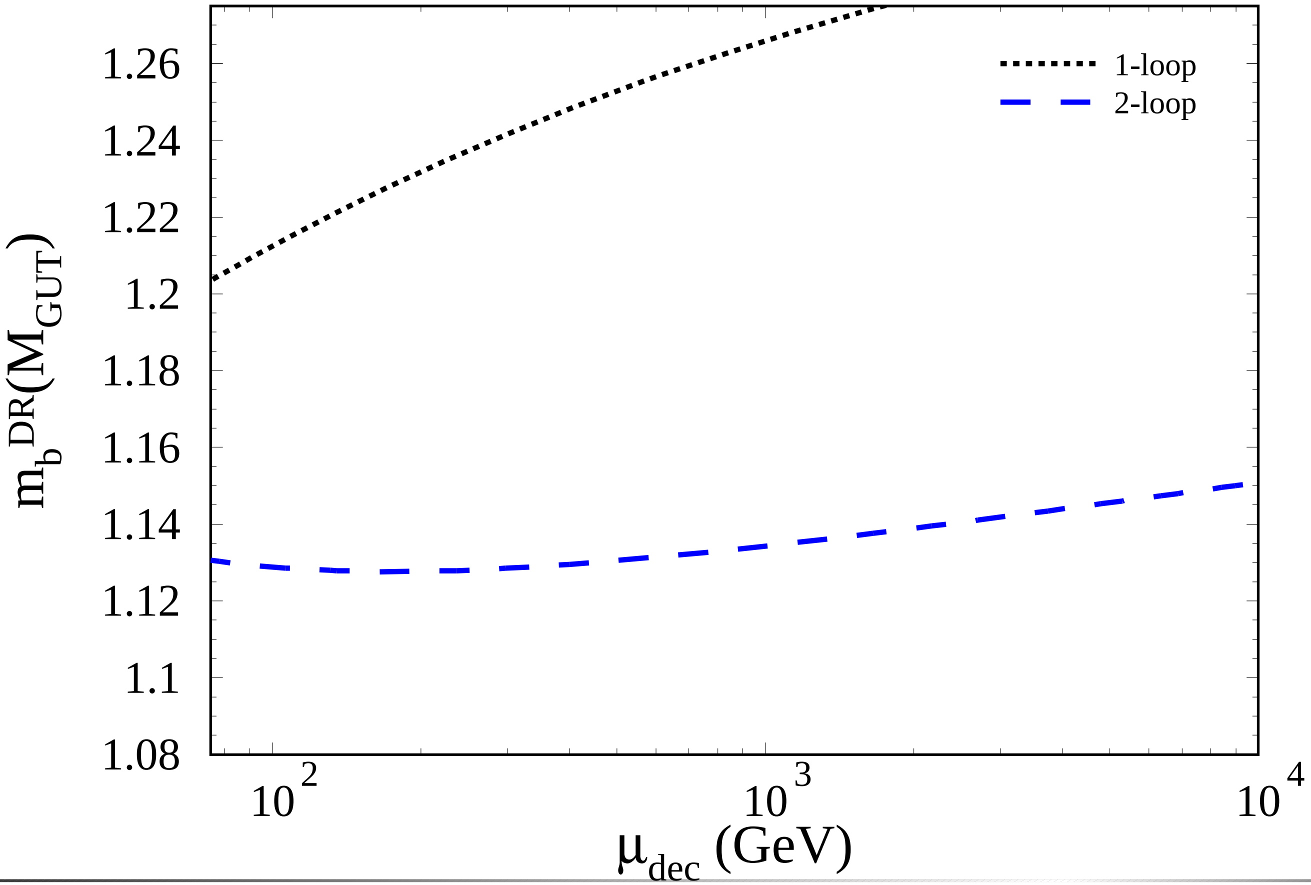
Susy masses: SPS1a' scenario



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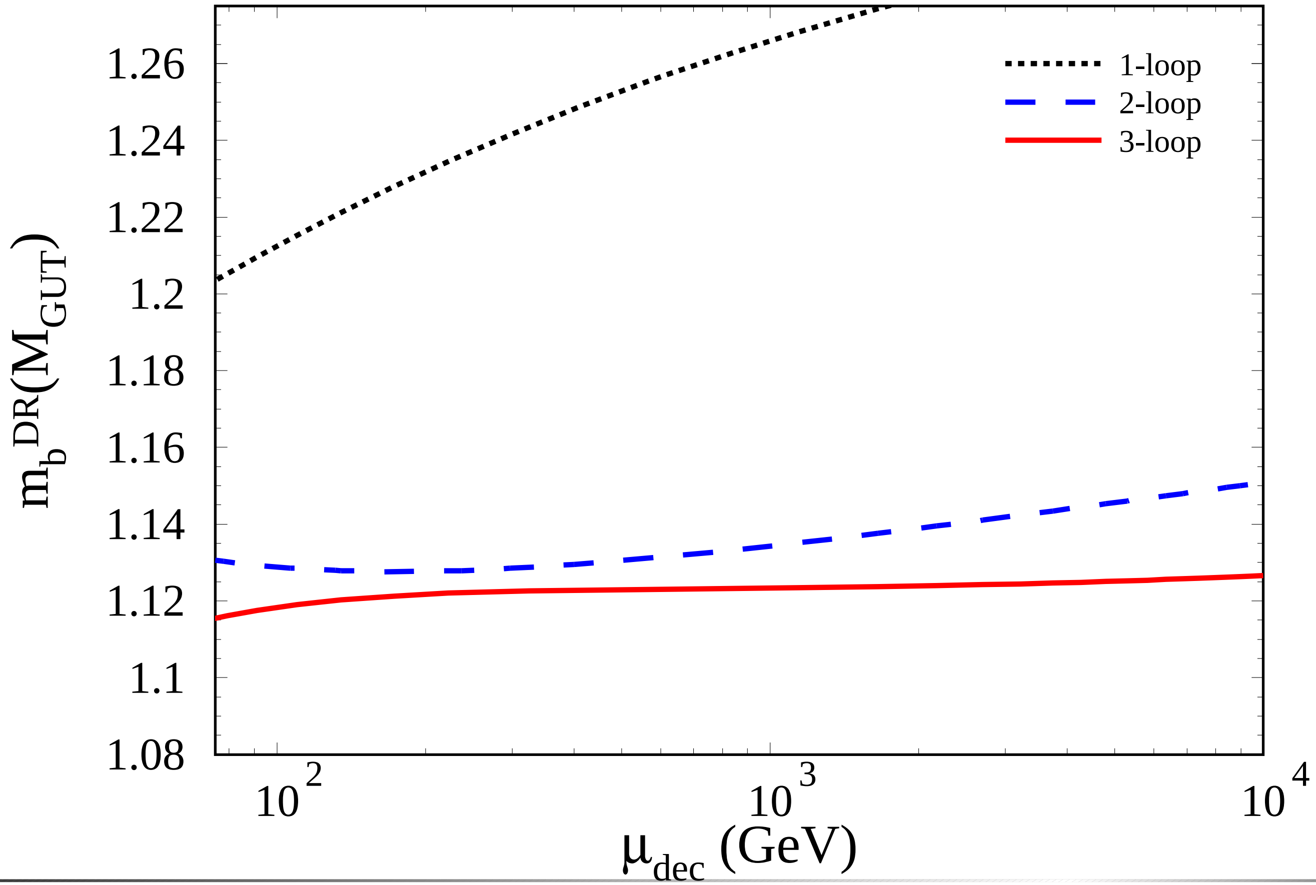

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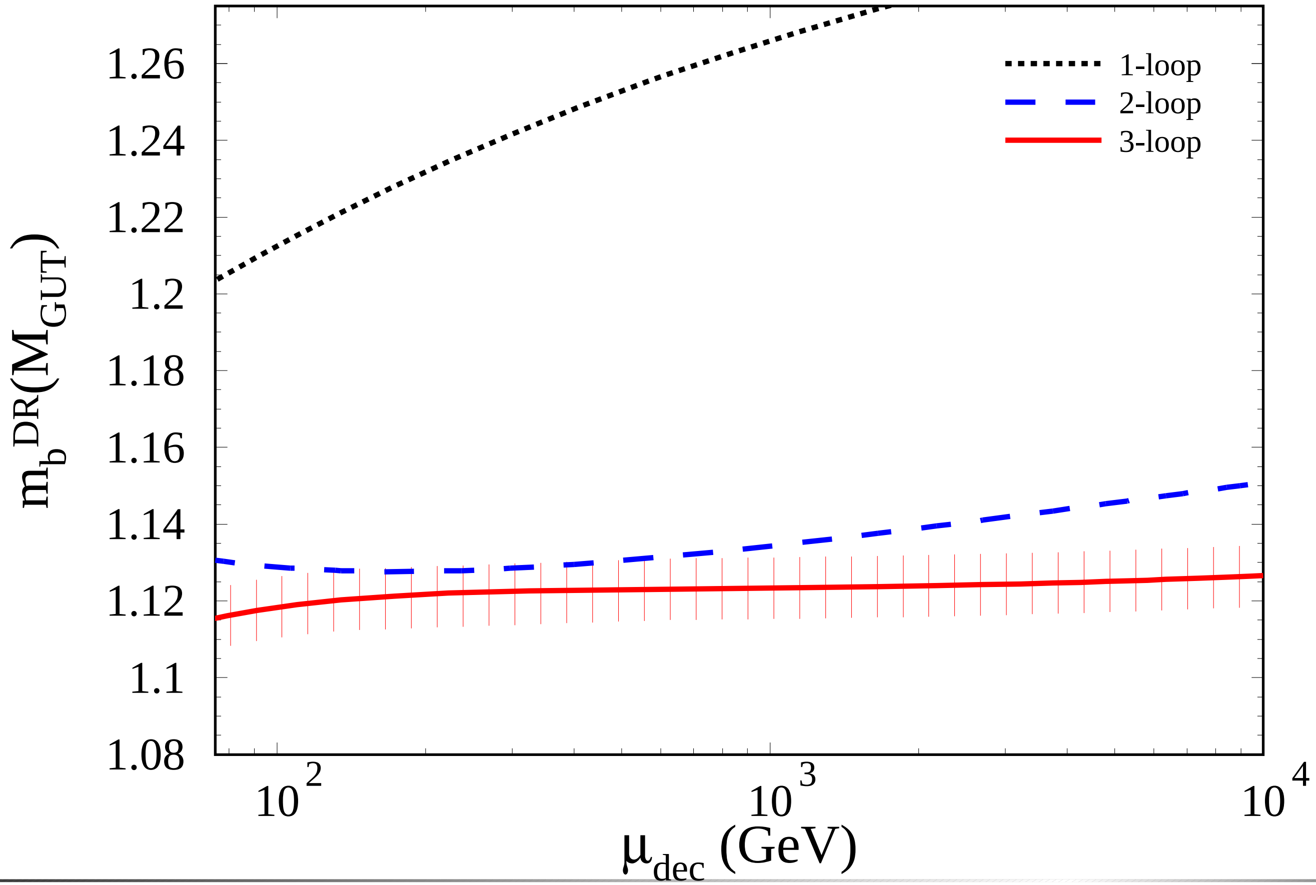

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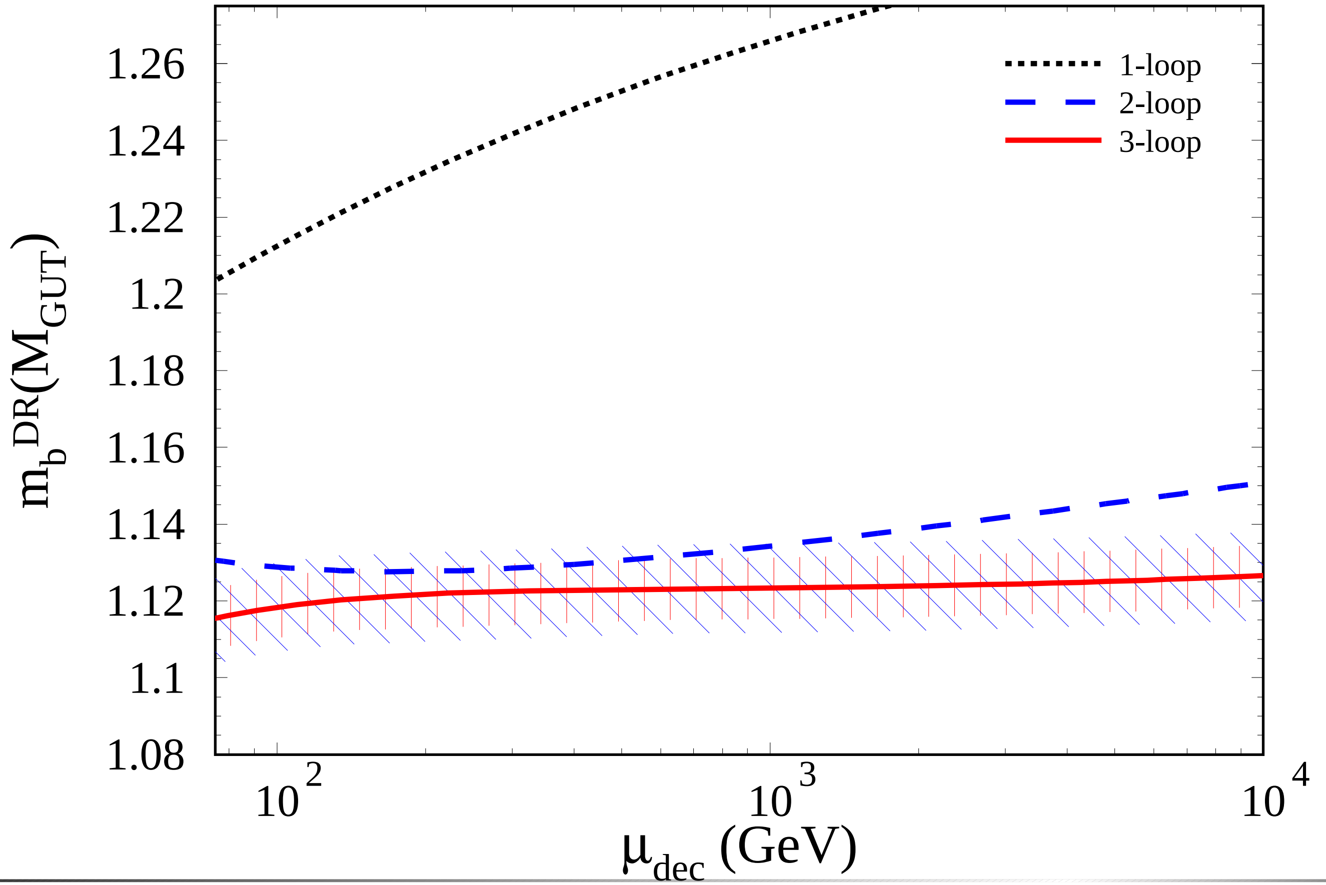

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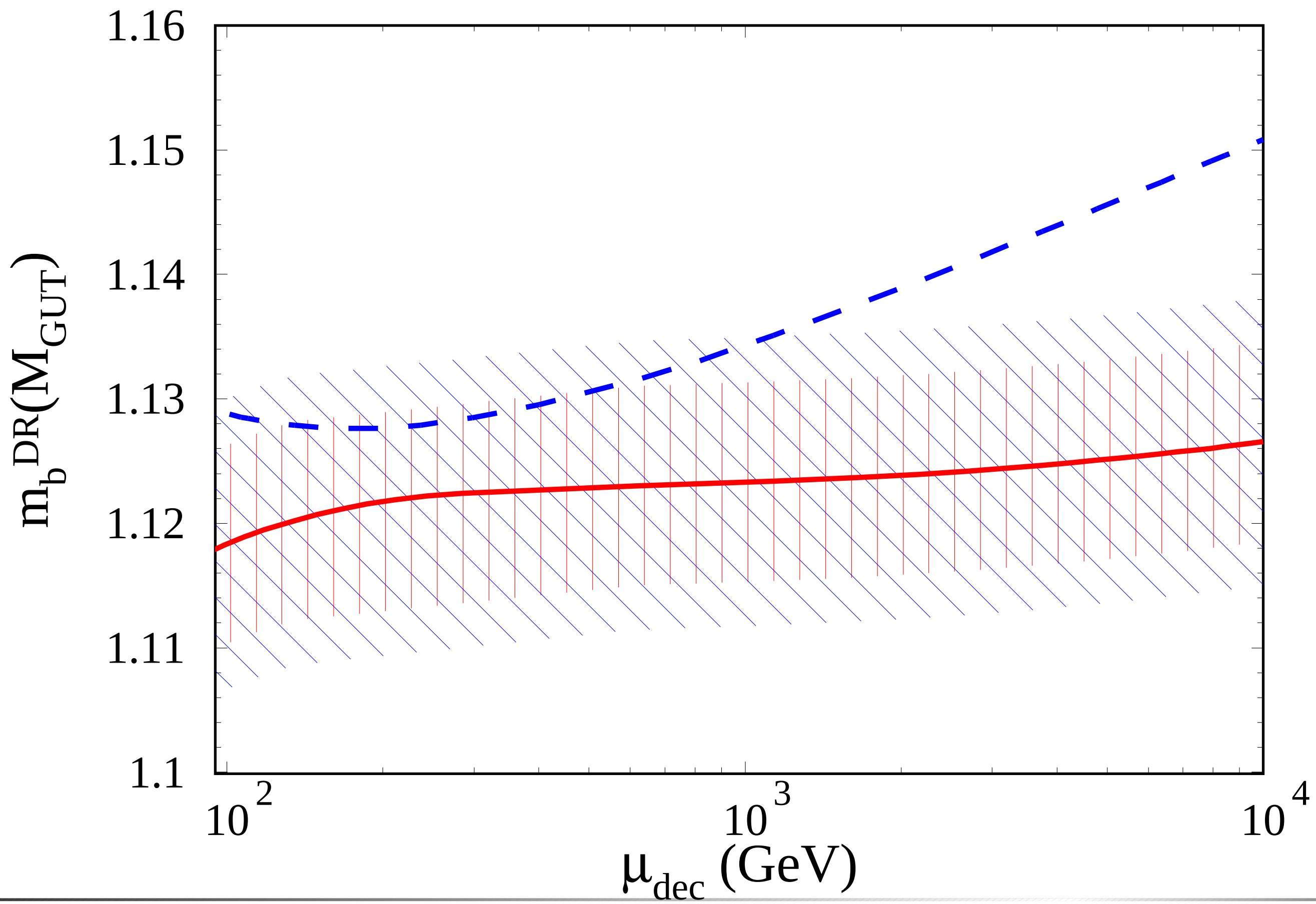

$\alpha_s(M_{\mathrm{GUT}})$

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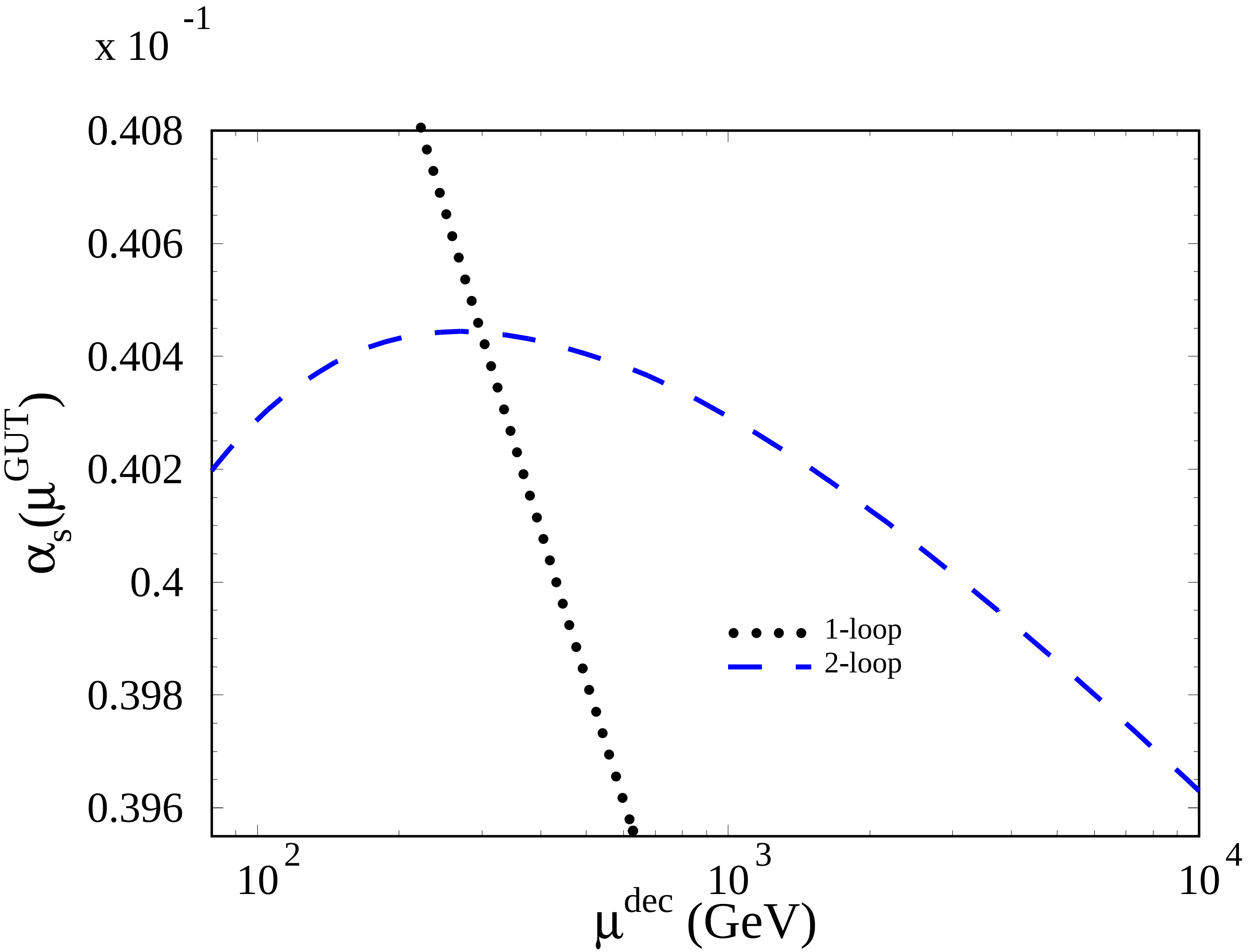

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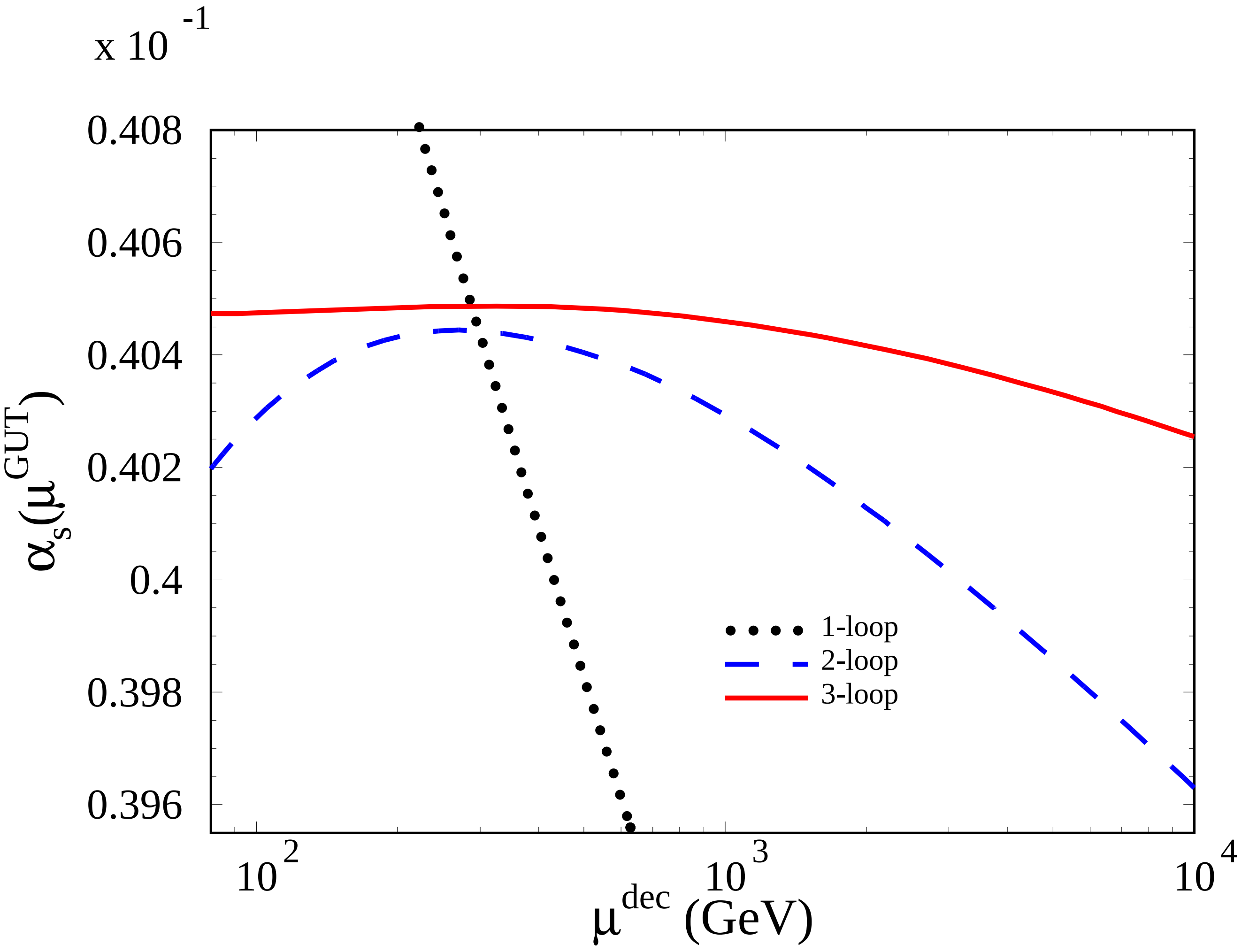

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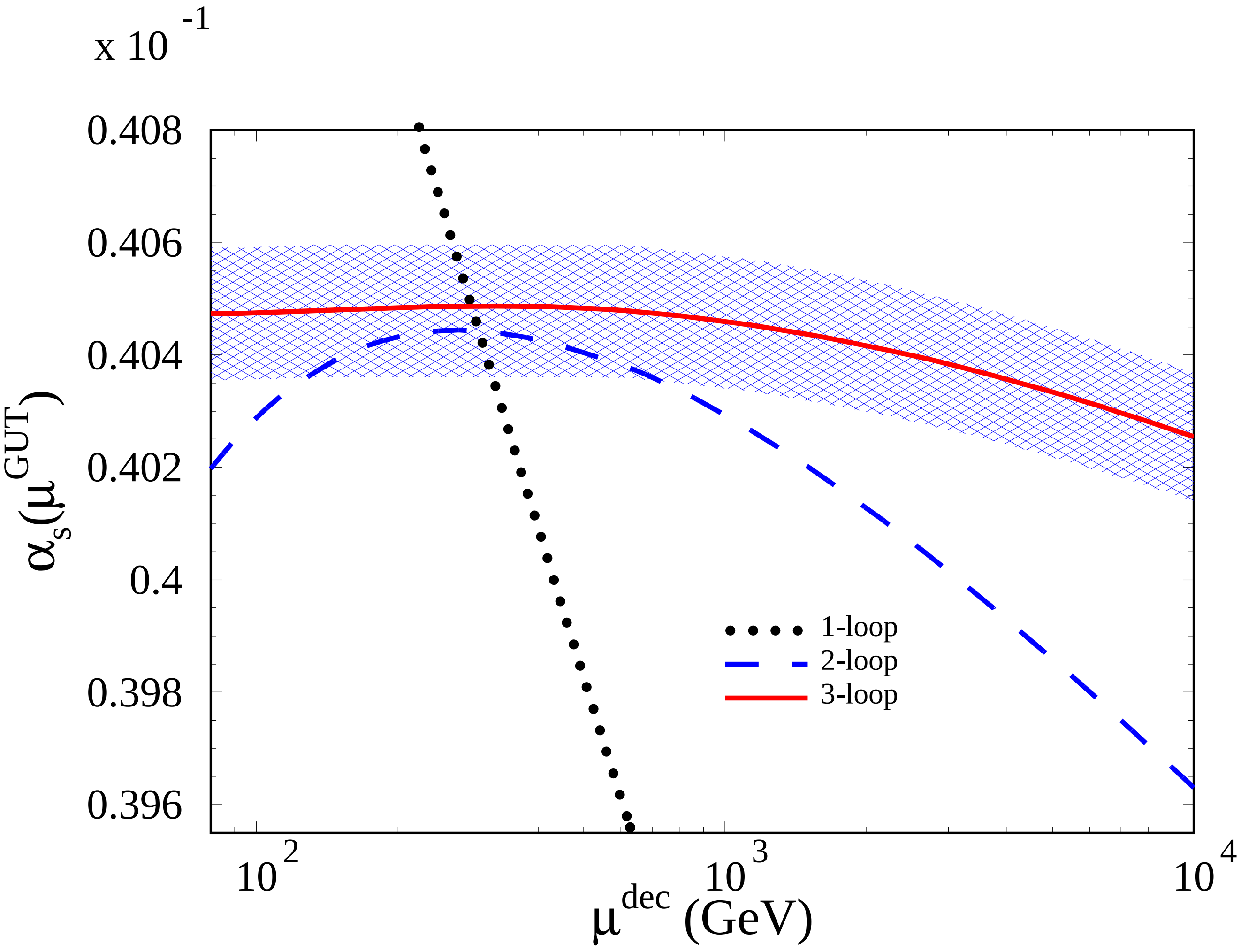

Conclusions

- $\alpha_s^{\overline{\rm DR}}(M_{\rm GUT})$ and $m_b^{\overline{\rm DR}}(M_{\rm GUT})$ to 3-loop accuracy (analytical results)

$$\alpha_s^{\overline{\rm DR}}(M_{\rm GUT}) = 0.0403 \pm 0.0001$$
$$m_b^{\overline{\rm DR}}(M_{\rm GUT}) = 1.12 \pm 0.08 \pm 0.12 \text{ GeV}$$

- 3-loop effects comparable with experimental accuracy on α_s and/or m_b