Neutrino masses in lepton number violating mSUGRA

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Outline

- Review neutrino masses in Lepton number violating (LNV) SUSY.
- Additional issues in high scale models (mSUGRA).
- Fitting neutrino oscillation data in LNV mSUGRA.
  JHEP 0804:081,2008 (arXiv:0712.0852) B. Allanach, CHK
- Comments and Summary.

LNV in SUSY

- Assume Standard Model (SM) (super-) field content:

$$\begin{array}{lllllllll} Q & : & (3,2,\frac{1}{6}), & \bar{U} & : & (\bar{3},1,-\frac{2}{3}), & \bar{D} & : & (\bar{3},1,\frac{1}{3}), \\ L & : & (1,2,-\frac{1}{2}), & H_d & : & (1,2,-\frac{1}{2}), & & & \\ \bar{E} & : & (1,1,1), & H_u & : & (1,2,\frac{1}{2}). & & & \end{array}$$

- Most general superpotential leads to fast proton decay:

$$\begin{aligned} \mathcal{W}_{RPC} &= (Y_E)_{ij} L_i H_d \bar{E}_j + (Y_D)_{ij} Q_i H_d \bar{D}_j + (Y_U)_{ij} Q_i H_u \bar{U}_j - \mu H_d H_u, \\ \mathcal{W}_{RPV} &= \frac{1}{2}\lambda_{ijk} L_i L_j \bar{E}_k + \lambda'_{ijk} L_i Q_j \bar{D}_k - \mu_i L_i H_u + \frac{1}{2}\lambda''_{ijk} \bar{U}_i \bar{U}_j \bar{D}_k, \end{aligned}$$

  - Baryon parity forbids (including dimension 5) baryon number violating terms.
  - Lepton number violated, with Majorana neutrino masses.

Neutrino masses in LNV SUSY: tree level

- Neutrinos mix with neutralinos:
  $\chi = (-i\tilde{B}, \quad -i\tilde{W}^3, \quad \tilde{h}_u^0, \quad \tilde{h}_d^0, \quad \nu_j)^T$
- Involves bilinear μ_i and sneutrino VEVs v_i.

$$
\begin{aligned}
\mathcal{L}_m &= -\frac{1}{2}\chi^T \mathcal{M}_N \chi, \\
\mathcal{M}_N &= \begin{pmatrix} (\mathcal{M}_{\chi^0})_{4\times 4} & m^T \\ m & m_\nu \end{pmatrix} \\
&= \begin{pmatrix} \begin{pmatrix} M_1 & 0 & \frac{1}{2}gv_u & -\frac{1}{2}gv_d \\ 0 & M_2 & -\frac{1}{2}g_2v_u & \frac{1}{2}g_2v_d \\ \frac{1}{2}gv_u & -\frac{1}{2}g_2v_u & 0 & -\mu \\ -\frac{1}{2}gv_d & \frac{1}{2}g_2v_d & -\mu & 0 \end{pmatrix} & \begin{pmatrix} -\frac{1}{2}gv_j \\ \frac{1}{2}g_2v_j \\ -\mu_j \\ 0_j \end{pmatrix} \\ \begin{pmatrix} -\frac{1}{2}gv_i & \frac{1}{2}g_2v_i & -\mu_i & 0_i \end{pmatrix} & 0_{ij} \end{pmatrix}.
\end{aligned}
$$

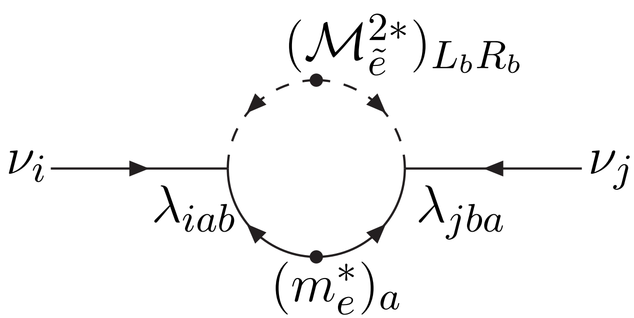
Seesaw mechanism: effective m_ν

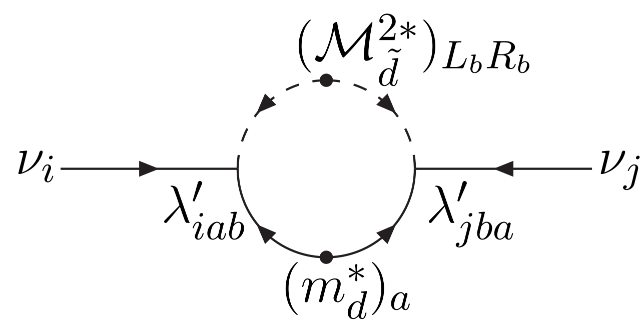
- ‘Electroweak seesaw’: neutralinos as the heavy fields
- At tree level ONE massive neutrino only : radiative corrections important

$$\mathcal{M}^{\nu}_{\text{eff}} = \frac{(M_1 g_2^2 + M_2 g^2)}{2\mu[v_u v_d (M_1 g_2^2 + M_2 g^2) - \mu M_1 M_2]} \begin{pmatrix} \Lambda_e\Lambda_e & \Lambda_e\Lambda_\mu & \Lambda_e\Lambda_\tau \\ \Lambda_\mu\Lambda_e & \Lambda_\mu\Lambda_\mu & \Lambda_\mu\Lambda_\tau \\ \Lambda_\tau\Lambda_e & \Lambda_\tau\Lambda_\mu & \Lambda_\tau\Lambda_\tau \end{pmatrix},$$

$$\Lambda_i \equiv \mu v_i - v_d \mu_i, \qquad i = \{e, \mu, \tau\}.$$

- E.g. of radiative corrections: mass corrections





Neutrino masses in LNV mSUGRA

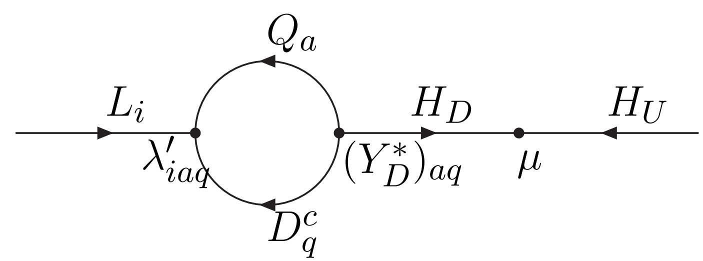
- At $M_X \sim 10^{16}$GeV
  - R-parity conserving (RPC) parameters: $m_0, M_{1/2}, A_0,$ sgnμ.
  - 0 bilinear LNV pars $\mu_i, \tilde{D}_i, m^2_{L_i H_d}$: rotated away.
  - LNV pars and lepton mixing angles:
    - 2 non-zero trilinear LNV pars: $\lambda_{ijk}, \lambda'_{ijk}$.
    - other LNV pars are assumed to be subdominant in neutrino mass generation, but not necessarily absent.
    - 3 charged lepton mixing angles define Z_l.

At M_Z: tan$\beta = v_u/v_d$.

Neutrino masses in LNV mSUGRA

- Renormalization effect important, e.g.

$$16\pi^2 \frac{d}{dt}\mu_i \simeq -\mu(\lambda_{ijk}(Y_E^*)_{jk} + 3\lambda'_{ijk}(Y_D^*)_{jk})$$



- Tree level m_ν dominates in general: suppression by interplay among LNV parameters.

Oscillation data fit in LNV mSUGRA

- Specify RPC parameters (SPS1a benchmark)

  $m_0 = 100$GeV, $M_{1/2} = 250$GeV, $A_0 = -100$GeV,

  sgn$\mu = +$, tan$\beta = 10$

- Parameters to be fitted using MINUIT:

$$\theta^l_{12}, \theta^l_{13}, \theta^l_{23}, \Lambda_1, \Lambda_2, \in \{\lambda_{ijk}, \lambda'_{ijk}\} \quad @ \quad M_X.$$

- Rotate to diagonal charged lepton basis using θ^l @ M_X.
- Renormalization using a RPV version of SOFTSUSY.
- Fit b.c.'s @ M_Z, EWSB @ M_{SUSY}.
- Calculate m_ν @ M_{SUSY}, and minimize χ^2 of:

  $\Delta m^2_{21} = 7.9^{+0.27}_{-0.28} \times 10^{-5}$eV^2, $|\Delta m^2_{31}| = 2.6 \pm 0.2 \times 10^{-3}$eV^2,

  sin$^2\theta_{12} = 0.31 \pm 0.02$, sin$^2\theta_{23} = 0.47^{+0.08}_{-0.07}$, sin$^2\theta_{13} = 0^{+0.008}_{-0.0}$

Technical aside

- Strong cancellation between CPE and CPO sneutrino-higgs contributions:
  - cancellation among CPE and CPO sneutrinos exact in the RPC limit.
  - numerical fluctuations could spoil minimization.
  - expand analytically the $\tilde{\nu}$-higgs terms about the RPC limit.
- Scale dependence of charged lepton mixing matrix:
  - Z_l scale independent in the RPC limit (c.f. quark sector).
  - Z_l scale dependence $\propto \Lambda^2$: negligible.

Numerical results @ SPS1a

Normal hierarchy						
Λ_1	Λ_2	θ^l_{12}	θ^l_{13}	θ^l_{23}	Δ_{FT}	χ^2
$\lambda'_{233} = -2.49978 \times 10^{-6}$	$\lambda_{233} = 4.06508 \times 10^{-5}$	0.459520	0.388989	0.304863	8.09	-
$\lambda'_{233} = -2.50019 \times 10^{-6}$	$\lambda_{211} = 4.06533 \times 10^{-5}$	1.98935	1.08162	0.632130	8.10	-
$\lambda'_{233} = -3.41336 \times 10^{-6}$	$\lambda_{321} = 9.86746 \times 10^{-5}$	0.448321	0.400030	2.89062	12.4	-
$\lambda'_{122} = -1.14066 \times 10^{-4}$	$\lambda_{122} = 4.06346 \times 10^{-5}$	1.19298	0.190538	1.17391	11.8	-
$\lambda'_{122} = -8.97777 \times 10^{-5}$	$\lambda_{123} = 1.02771 \times 10^{-4}$	2.10672	0.174800	1.18124	9.44	-
$\lambda'_{122} = -8.59626 \times 10^{-5}$	$\lambda_{133} = 4.09647 \times 10^{-5}$	0.997963	0.281922	0.417935	8.00	-
Inverted hierarchy						
Λ_1	Λ_2	θ^l_{12}	θ^l_{13}	θ^l_{23}	Δ_{FT}	χ^2
$\lambda'_{233} = -5.69116 \times 10^{-6}$	$\lambda_{233} = 1.36023 \times 10^{-4}$	1.55779	0.815115	0.146126	755	0.01
$\lambda'_{233} = -5.69126 \times 10^{-6}$	$\lambda_{211} = 1.32365 \times 10^{-4}$	1.38843	0.760045	0.140903	758	0.06
$\lambda'_{233} = -5.67940 \times 10^{-6}$	$\lambda_{123} = 1.42938 \times 10^{-4}$	1.81386	-0.757538	0.141975	726	0.05
$\lambda'_{122} = -1.96283 \times 10^{-4}$	$\lambda_{122} = 1.24824 \times 10^{-4}$	0.134765	0.101938	0.798282	988	0.43
$\lambda'_{122} = -1.93673 \times 10^{-4}$	$\lambda_{132} = 1.47364 \times 10^{-4}$	3.03234	0.0866645	0.931616	743	2.85
$\lambda'_{122} = -1.96175 \times 10^{-4}$	$\lambda_{123} = 1.45708 \times 10^{-4}$	0.144386	0.0943266	0.689708	736	0.52

Comment

- Mild tuning in normal hierarchy. Stronger fine tuning in inverted hierarchy.
- Near tri-bi maximal mixing accidental.
- Small LNV pars: no significant effect on SUSY production/ decay except LSP.
- Complicated LSP decay channels due to large θ^l's.
- For $\tilde{l}$ LSP, expect 2-bd decay, promptly at the interaction point.
- For $\tilde{\chi}$ LSP, can decay with a displaced vertex of $\mathcal{O}(0.1)$mm.
  - displacement not immediately obvious, but could be searched for.

Summary

- Lepton number violating SUSY models provides alternative See-Saw mechanism to neutrino masses.
- Possible to promote to unified models, e.g. minimal supergravity (mSUGRA), with additional issues.
- Performed quantitative study of mSUGRA with 2 GUT-scale trilinear LNV couplings and three charged lepton mixing angles.

Thank you.

Decay widths

λ_{ijk}	Channel	BR	Channel	BR
λ_{122}	$e^- \ \nu_\mu \ \mu^+$	0.006	$\mu^- \ \nu_e \ \mu^+$	0.006
λ_{132}	$e^- \ \nu_\tau \ \mu^+$	0.007	$\tau^- \ \nu_e \ \mu^+$	0.007
λ_{123}	$e^- \ \nu_\mu \ \tau^+$	0.029	$\mu^- \ \nu_e \ \tau^+$	0.029
λ_{133}	$e^- \ \nu_\tau \ \tau^+$	0.034	$\tau^- \ \nu_e \ \tau^+$	0.034
λ_{231}	$\mu^- \ \nu_\tau \ e^+$	0.005	$\tau^- \ \nu_\mu \ e^+$	0.005
λ_{232}	$\mu^- \ \nu_\tau \ \mu^+$	0.027	$\tau^- \ \nu_\mu \ \mu^+$	0.028
λ_{233}	$\mu^- \ \nu_\tau \ \tau^+$	0.138	$\tau^- \ \nu_\mu \ \tau^+$	0.140

(a) Normal hierarchy. Best fit: $\lambda_{233} = 4.07e^{-5}, \lambda'_{233} = -2.50e^{-6}$, $\theta^l_{12} = 0.460, \theta^l_{13} = 0.389, \theta^l_{23} = 0.305$ @ SPS1a.

λ_{ijk}	Channel	BR	Channel	BR
λ_{122}	$e^- \ \nu_\mu \ \mu^+$	0.063	$\mu^- \ \nu_e \ \mu^+$	0.063
λ_{132}	$e^- \ \nu_\tau \ \mu^+$	0.056	$\tau^- \ \nu_e \ \mu^+$	0.057
λ_{123}	$e^- \ \nu_\mu \ \tau^+$	0.067	$\mu^- \ \nu_e \ \tau^+$	0.067
λ_{133}	$e^- \ \nu_\tau \ \tau^+$	0.059	$\tau^- \ \nu_e \ \tau^+$	0.060

(b) Inverted hierarchy. Best fit: $\lambda_{233} = 1.36023e^{-4}, \lambda'_{233} = -5.69116e^{-6}$, $\theta^l_{12} = 1.55779, \theta^l_{13} = 0.815155, \theta^l_{23} = 0.146126$ @ SPS1a.

Alignment

Tree level effective mass matrix

$$(\mathcal{M}^{\nu}_{\text{eff}})_{in} \quad \propto \quad \Big[\lambda_{ijk}(Y_E^*)_{jk}\lambda_{nlm}(Y_E^*)_{lm}\Big].$$

One loop mass corrections

$$(m_\nu)_{in} \quad \simeq \quad \sum_{j,k,l,m}\Big\{\frac{1}{(4\pi)^2}\lambda_{ijk}\lambda_{nlm}\frac{(m_e^*)_{jm}(\mathcal{M}^{2*}_{\tilde{e}_{LR}})_{lk}}{\overline{\mathcal{M}}^2_{\tilde{e}_{LL}}-\overline{\mathcal{M}}^2_{\tilde{e}_{RR}}}\ln\Big(\frac{(\overline{\mathcal{M}}^2_{\tilde{e}_{LL}})}{(\overline{\mathcal{M}}^2_{\tilde{e}_{RR}})}\Big)\Big[1+\mathcal{O}\Big(\frac{\delta\mathcal{M}^2}{\overline{\mathcal{M}}^2}\Big)\Big]$$
$$+(i\leftrightarrow n)\Big\}.$$

One LNV coupling: i=n, j=l, k=m:

$$\mathcal{M}^{\nu}_{\text{eff}} \quad \propto \quad m_\nu + m_\nu\mathcal{O}\Big(\frac{\delta\mathcal{M}^2}{\overline{\mathcal{M}}^2}\Big).$$

Tree- loop mass ratio

Tree level mass scale

$$m_\nu^{tree} \quad \simeq \quad -\frac{8\pi\alpha_{GUT}}{5M_{1/2}}[\frac{v_d}{16\pi^2}]^2[\ln\frac{M_X}{M_Z}]^2[\lambda_{ijq}(Y_E^*)_{jq}]^2 f^2(\frac{\mu^2}{m_0^2};\frac{A_0^2}{m_0^2};\frac{\tilde{B}}{m_0^2};\tan\beta),$$

Loop level mass scale

$$m_\nu^{loop} \quad \equiv \quad \sum_{i=1}^{3}\left(\Sigma_{Dii}\right) \simeq 2\sum_{i,j,k}\lambda_{iL_jR_k}\lambda_{iL_kR_j}\frac{m_{f_j}^*}{(4\pi)^2}\frac{(\mathcal{M}_{\tilde{f}}^{2*})_{L_kR_k}}{(\mathcal{M}_{\tilde{f}}^2)_{L_kL_k}-(\mathcal{M}_{\tilde{f}}^2)_{R_kR_k}}\ln\frac{(\mathcal{M}_{\tilde{f}}^2)_{L_kL_k}}{(\mathcal{M}_{\tilde{f}}^2)_{R_kR_k}}$$

Tree-loop mass ratio $m_\nu^{tree}/m_\nu^{loop}$

$$\simeq \quad -n_c\frac{\alpha_{GUT}\ln^2(M_X/M_Z)}{10\pi M_{1/2}(A_0-\mu\tan\beta)}\frac{(\mathcal{M}_{\tilde{f}}^2)_{L_kL_k}-(\mathcal{M}_{\tilde{f}}^2)_{R_kR_k}}{\ln((\mathcal{M}_{\tilde{f}}^2)_{L_kL_k}/(\mathcal{M}_{\tilde{f}}^2)_{R_kR_k})}f^2(\frac{\mu^2}{m_0^2};\frac{A_0^2}{m_0^2};\frac{\tilde{B}}{m_0^2};\tan\beta)$$

Experimental contraints

- Bounds on FCNC, rare decays, etc Allanach, Dedes, Dreiner PRD 60 .

$$
\begin{array}{rcll}
 & BR_{exp}(\mu \to e\gamma) & < 1.2 \times 10^{-11} & 90\%\text{CL} \quad \text{MEGA PRD 65},\\
 & BR_{exp}(\tau \to e\gamma) & < 1.1 \times 10^{-7} & 90\%\text{CL} \quad \text{BABAR PRL 96},\\
 & BR_{exp}(\tau \to \mu\gamma) & < 6.8 \times 10^{-8} & 90\%\text{CL} \quad \text{BABAR PRL 95},\\
 & BR_{exp}(B_s \to \mu^+\mu^-) & < 1.0 \times 10^{-7} & 95\%\text{CL} \quad \text{CDF 2006},\\
2.76 \times 10^{-4} < & BR(b \to s\gamma) & < 4.34 \times 10^{-4} & 2\sigma
\end{array}
$$

Allanach, Bernhardt, Dreiner, CHK, Richardson PRD 75 .

- Normal hierarchy $\frac{m_3}{m_2} = 5.74 \pm 0.32,\ @\ 1\sigma.$
- Inverted hierarchy $\frac{m_2}{m_1} = 1.0151 \pm 0.0769.\ @\ 1\sigma.$

Fine-tuning measure

- Ellis, Enqvist, Nanopoulos, Zwirner Mod PLA 1

$$\Delta_{FT} = \left|\frac{\partial \ln(\mathcal{O}(\lambda'))}{\partial \ln \lambda'}\right|,$$

- Our numerical results:

$$\begin{aligned}\Delta_{FT} &\simeq \left|\frac{\ln(\mathcal{O}(\lambda'_a)) - \ln(\mathcal{O}(\lambda'_b))}{\ln \lambda'_a - \ln \lambda'_b}\right|, \\ \lambda'_a &= \lambda'_b \times (1 - 2.0 \times 10^{-4}).\end{aligned}$$

LNV in MSSM

- Can suppress proton decay by R-parity Farrar, Fayet PLB 76
  - $$\begin{array}{rcl}(L,\bar{E},Q,\bar{U},\bar{D}) & \rightarrow & -\quad (L,\bar{E},Q,\bar{U},\bar{D}). \\ (H_u,H_d) & \rightarrow & +\quad (H_u,H_d).\end{array}$$
  - $\mathcal{W}_{LNV} = \mathcal{W}_{BNV} = 0$: R-parity conservation (RPC).

- Also possible to forbid $\mathcal{W}_{BNV}$ alone - Baryon parity Ibanez, Ross PLB 260 .
  - $$\begin{array}{rcl}(Q,\bar{U},\bar{D}) & \rightarrow & -\quad (Q,\bar{U},\bar{D}). \\ (L,\bar{E},H_u,H_d) & \rightarrow & +\quad (L,\bar{E},H_u,H_d).\end{array}$$
  - $\mathcal{W}_{LNV} \neq 0$: lepton number violation (LNV).
  - L_i and H_d have same transformation properties: $H_d \equiv L_0$.

Neutrino masses in LNV mSUGRA

How many *dominant* trilinear LNV parameters @ M_X ?

- One λ or λ' in a weak interaction basis:
  - Renormalization effect: tree level mass matrix dominates.
  - Alignment: tree mass and loop corrections tend to be proportional.
- One λ *and* λ' in a weak interaction basis:
  - Partial cancellation may suppress tree level mass.
  - Alignment also weakens.