The muon g-2 and the bounds on the Higgs mass

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SUSY08
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The present experimental values:

$$a_e = 1159652180.73\ (28) \times 10^{-12}$$

0.24 parts per billion !! Hanneke et al., PRL100 (2008) 120801

$$a_\mu = 116592080\ (63) \times 10^{-11}$$

0.5 parts per million !! E821 – Final Report: PRD73 (2006) 072003

$$a_\tau = -0.018\ (17)$$

DELPHI - EPJC35 (2004) 159 [$a_\tau^{SM} = 117721(5) \times 10^{-8}$, Eidelman & MP '07]

The anomalous magnetic moment: the basics

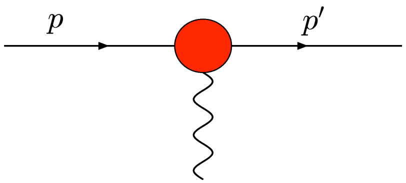
- The Dirac theory predicts for a lepton l=e,μ,τ:

$$\vec{\mu}_l = g_l \left(\frac{e}{2m_l c}\right) \vec{s} \qquad g_l = 2$$

- QFT predicts deviations from the Dirac value:

$$g_l = 2\,(1 + a_l)$$

- Study the photon-lepton vertex:



$$\bar{u}(p')\Gamma_\mu u(p) = \bar{u}(p')\left[\gamma_\mu F_1(q^2) + \frac{i\sigma_{\mu\nu}q^\nu}{2m} F_2(q^2) + \ldots\right] u(p)$$

$$F_1(0) = 1 \qquad F_2(0) = a_l$$

The QED contribution to a_μ

$a_\mu^{QED} = (1/2)(\alpha/\pi)$ Schwinger 1948

$+ \ 0.765857410\ (27)\ (\alpha/\pi)^2$

Sommerfield; Petermann; Suura & Wichmann '57; Elend '66; MP '04

$+ \ 24.05050964\ (43)\ (\alpha/\pi)^3$

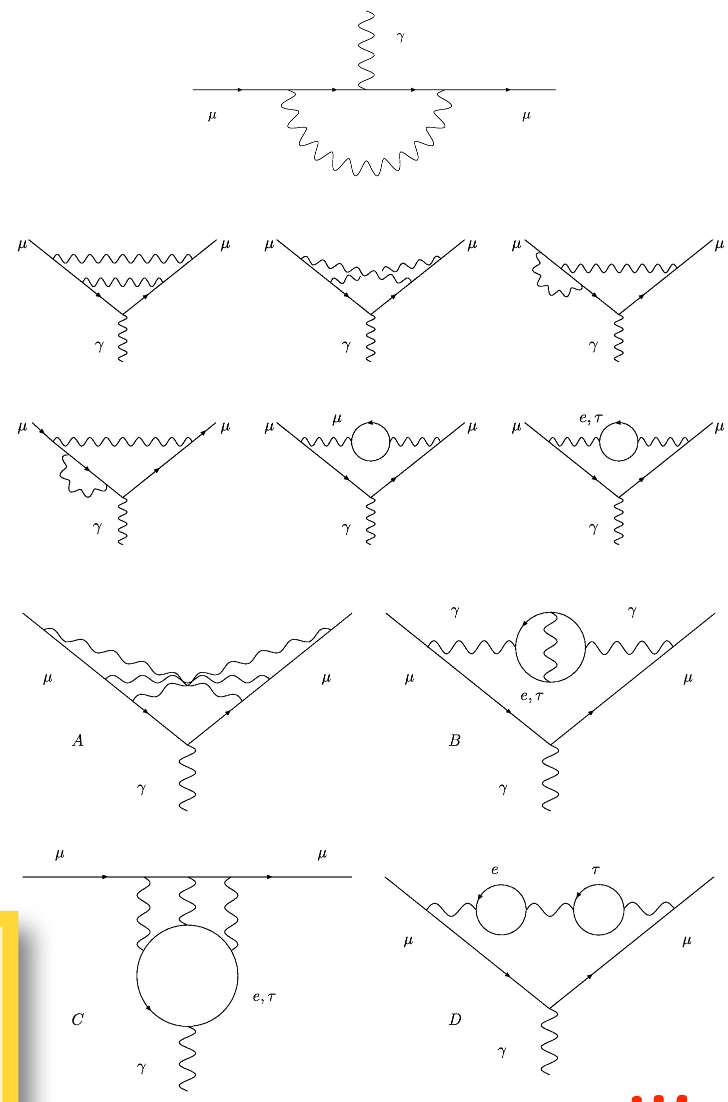
Remiddi, Laporta, Barbieri … ; Czarnecki, Skrzypek; MP '04; Friot, Greynat, de Rafael '05

$+ \ 130.805\ (8)\ (/\pi)^4$ Revised!

Kinoshita & Lindquist '81, … , Kinoshita & Nio '04, '05; Aoyama, Hayakawa, Kinoshita & Nio, June & Dec 2007

$+ \ 663\ (20)\ (\alpha/\pi)^5$ In progress

Kinoshita et al. '90, Yelkhovsky, Milstein, Starshenko, Laporta, Karshenboim,…, Kataev, Kinoshita & Nio March '06.



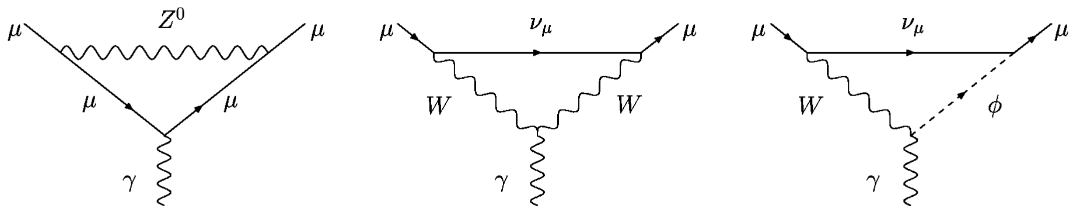
Adding up, I get:

$a_\mu^{QED} = 116584718.09\ (14)(04) \times 10^{-11}$

mainly from 5-loop unc ↲ ↳ from new $\delta\alpha$('08)

with α=1/137.035999084(51) [0.37 ppb]

- **One-loop term:**



$$a_\mu^{\mathrm{EW}}(\text{1-loop}) = \frac{5 G_\mu m_\mu^2}{24\sqrt{2}\pi^2}\left[1 + \frac{1}{5}\left(1 - 4\sin^2\theta_W\right)^2 + O\left(\frac{m_\mu^2}{M_{Z,W,H}^2}\right)\right] \approx 195\times10^{-11}$$

1972: Jackiv, Weinberg; Bars, Yoshimura; Altarelli, Cabibbo, Maiani; Bardeen, Gastmans, Lautrup; Fujikawa, Lee, Sanda.

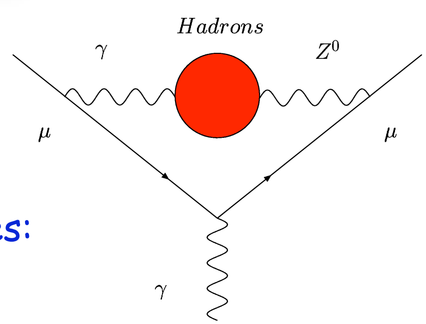
- **One-loop plus higher-order terms:**

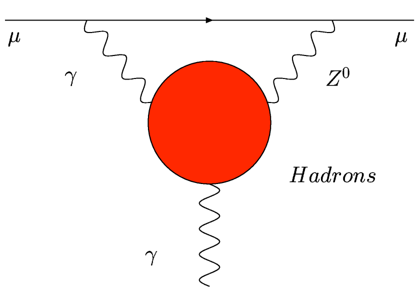
$a_\mu^{\mathrm{EW}} = 154\ (2)\ (1) \times 10^{-11}$

Kukhto et al. '92; Czarnecki, Krause, Marciano '95; Knecht, Peris, Perrottet, de Rafael '02; Czarnecki, Marciano, Vainshtein '02; Degrassi, Giudice '98; Heinemeyer, Stockinger, Weiglein '04; Gribouk, Czarnecki '05; Vainshtein '03.

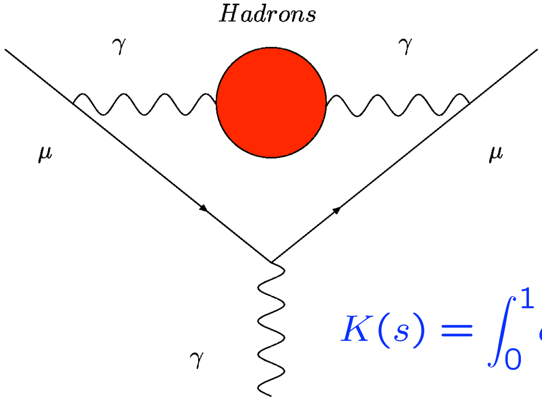
(2): Higgs mass, M_{top} error, 3-loop nonleading logs

(1): Hadronic loop uncertainties:





The hadronic leading-order (HLO) contribution

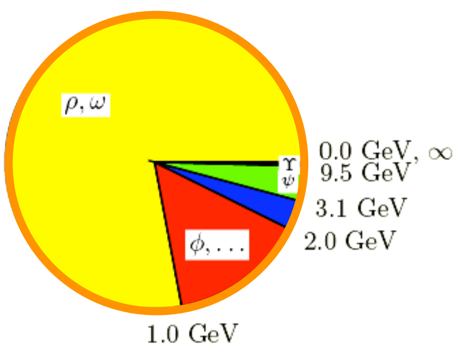


$$a_\mu^{\mathrm{HLO}} = \frac{1}{4\pi^3}\int_{4m_\pi^2}^{\infty} ds\, K(s)\, \sigma^{(0)}(s) = \frac{\alpha^2}{3\pi^2}\int_{4m_\pi^2}^{\infty} \frac{ds}{s} K(s)\, R(s)$$

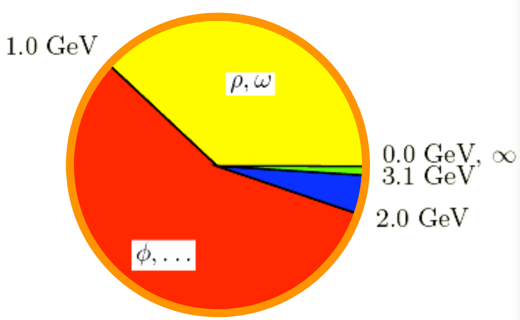
$$K(s) = \int_0^1 dx \frac{x^2(1-x)}{x^2+(1-x)s/m_\mu^2}$$

Bouchiat & Michel 1961; Gourdin & de Rafael 1969

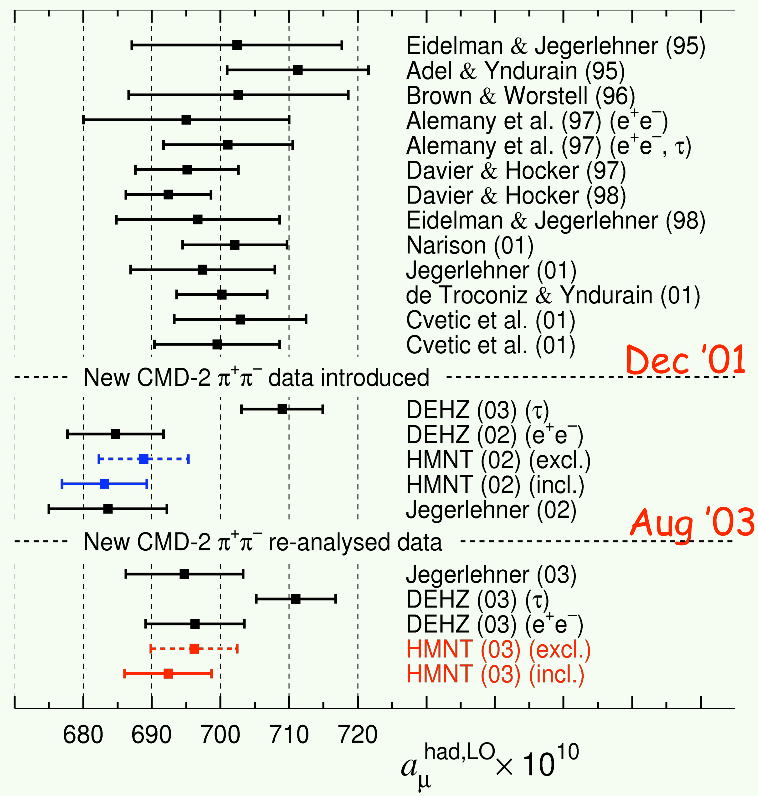
Central values



Errors^2



F. Jegerlehner, PhiPsi 08, Frascati, April 2008



Hagiwara et al., PRD 69 (2004) 093003

The HLO contribution: e^+e^- data

$a_\mu^{HLO} = 6909\,(39)_{exp}\,(19)_{rad}\,(7)_{qcd} \times 10^{-11}$ S. Eidelman, ICHEP06; M. Davier, TAU06

$= 6894\,(42)_{exp}\,(18)_{rad} \times 10^{-11}$ Hagiwara, Martin, Nomura, Teubner, PLB649(2007)173

$= 6923\,(60)_{tot} \times 10^{-11}$ F. Jegerlehner, PhiPsi 08, Frascati, April 2008

$= 6944\,(48)_{exp}\,(10)_{rad} \times 10^{-11}$ de Troconiz & Yndurain, PRD71 (2005) 73008

- Radiative Corrections (Luminosity, ISR, Vacuum Polarization, **FSR**) are a very delicate issue! Are they all under control?
- CMD2's 1998 $\pi^+\pi^-$ data in the ρ energy range, published in 2007, agree well with their earlier (1995) ones.
- SND's $\pi^+\pi^-$ 2006 data reanalysis appears to be in good agreement with CMD2.

- The RADIATIVE RETURN (ISR) Method: KLOE & BABAR. Collider operates at fixed energy but s_π can vary continuously. Important independent method made possible by beautiful interplay between theory and experiment.

- Discrepancies between KLOE's (2001) and CMD2's results even if their contributions to a_μ^{HLO} are similar (see table).

- Comparison in the range $s_\pi \in [0.37, 0.93]$ GeV^2:

$a_\mu^{\pi\pi} = (3786 \pm 27_{stat} \pm 23_{sys+th}) \times 10^{-11}$	CMD2 (95)	PLB578 (2004) 285
$a_\mu^{\pi\pi} = (3771 \pm 19_{stat} \pm 27_{sys+th}) \times 10^{-11}$	CMD2 (95+98)	S.Eidelman, ICHEP '06
$a_\mu^{\pi\pi} = (3756 \pm 8_{stat} \pm 48_{sys+th}) \times 10^{-11}$	KLOE	G.Venanzoni, ICHEP '04
$a_\mu^{\pi\pi} = (3768 \pm 13_{stat} \pm 47_{sys+th}) \times 10^{-11}$	SND (revised)	S.Eidelman, ICHEP '06

- PhiPsi08: KLOE presented an update of its 2001 data analysis (some differences in $a_\mu^{\pi\pi}$ w.r.t. published value) & the new 2002 data analysis. Final results coming soon...

The HLO contribution: Tau-decay data

- TAU DATA: Several data sets (Aleph, Cleo, Opal, Belle).
- The tau data of ALEPH and CLEO are significantly higher than CMD2 e^+e^- ones above ~ 0.85 GeV. KLOE confirms this discrepancy with the tau data.
- The recent $a_\mu^{\pi\pi}$ tau result of BELLE (arXiv:0805.3773) is in agreement with the previous Aleph-Cleo-Opal one, even if deviations from ALEPH's spectral functions are observed.
- Latest value, still (Davier, Eidelman, Hoecker, Zhang, EPJC31 (2003) 503):

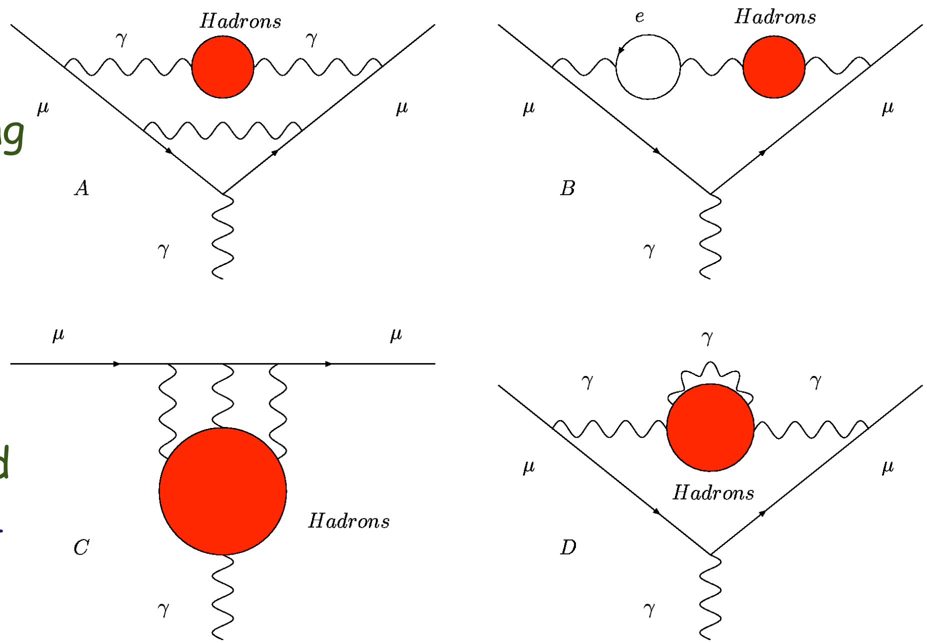
$$a_\mu^{HLO} = 7110\ (58) \times 10^{-11}$$

- Inconsistencies in the e^+e^- or tau data? Are all possible isospin-breaking (IB) effects taken into account? Recent additional IB corrections somewhat reduce the diff. with e^+e^- data. Also, recent claims that e^+e^- and tau data are consistent after IB effects & vector meson mixings are considered (Marciano&Sirlin '88; Cirigliano, Ecker, Neufeld '01-'02, Flores-Baez et al. '06 &' 07, Benayoun et al.'07, Davier@Glasgow g-2 workshop, Oct '07).

The hadronic higher-order (HHO) contributions

Vacuum Polarization

$O(\alpha^3)$ contributions of diagrams containing hadronic vacuum polarization insertions:



$$a_\mu^{HHO}(vp) = -98\ (1) \times 10^{-11}$$

Krause '96, Alemany et al. '98, Hagiwara et al. '03 & '06

Shifts by ~ -3×10^{-11} if tau data are used instead of the e^+e^- ones   Davier & Marciano '04

Light-by-Light

The contribution of the hadronic l-b-l diagrams had a troubled life. The latest values vary between:

$a_\mu^{HHO}(lbl) = +80\ (40) \times 10^{-11}$	Knecht & Nyffeler '02
$a_\mu^{HHO}(lbl) = +136\ (25) \times 10^{-11}$	Melnikov & Vainshtein '03
$a_\mu^{HHO}(lbl) = +110\ (40) \times 10^{-11}$	Bijnens & Prades '07

based also on Hayakawa, Kinoshita '98 & '02; Bijnens, Pallante, Prades '96 & '02;

This contribution will likely become the ultimate limitation of the SM prediction.

The muon g-2: Standard Model vs. Experiment

Adding up all the above contribution we get the following SM predictions for a_μ and comparisons with the measured value:

	$a_\mu^{\rm SM} \times 10^{11}$	$\Delta a_\mu \times 10^{11}$	σ
[1]	116 591 793 (60)	287 (87)	3.3
[2]	116 591 778 (61)	302 (88)	3.4
[3]	116 591 807 (72)	273 (96)	2.8
[4]	116 591 828 (63)	252 (89)	2.8
[5]	116 591 991 (70)	89 (95)	0.9

with $a_\mu^{\rm HHO}(\rm lbl) = 110\ (40) \times 10^{-11}$.

$\Delta a_\mu = a_\mu^{\rm EXP} - a_\mu^{\rm SM}$.

[1] Eidelman at ICHEP06 & Davier at TAU06 (update of ref. [5]).
[2] Hagiwara, Martin, Nomura, Teubner, PLB649 (2007) 173.
[3] F. Jegerlehner, PhiPsi 08, Frascati, April 2008.
[4] J.F. de Troconiz and F.J. Yndurain, PRD71 (2005) 073008.
[5] Davier, Eidelman, Hoecker and Zhang, EPJC31 (2003) 503 (τ data).

The th. error is now the same (or even smaller) as the exp. one!

The muon g-2 and the bounds on the Higgs mass

MP, W.J. Marciano & A. Sirlin

arXiv:0804.1142 (PRD, to appear)

How do we explain Δa_μ?

- Δa_μ can be explained in many ways: errors in HHO-LBL, QED, EW, HHO-VP, g-2 EXP, HLO; or NP (see Nomura's talk).
- Can Δa_μ be due to hypothetical changes in the hadronic $\sigma(s)$?
- An upward shift of $\sigma(s)$ also induces an increase of $\Delta\alpha_{had}^{(5)}(M_Z)$.
- Consider:

$$
\begin{aligned}
a &= \int_{4m_\pi^2}^{s_u} ds\, f(s)\, \sigma(s), \qquad f(s) = \frac{K(s)}{4\pi^3},\ s_u < M_Z^2, \\
b &= \int_{4m_\pi^2}^{s_u} ds\, g(s)\, \sigma(s), \qquad g(s) = \frac{M_Z^2}{(M_Z^2 - s)(4\alpha\pi^2)},
\end{aligned}
$$

and the increase

$$\Delta\sigma(s) = \epsilon\sigma(s)$$

($\varepsilon>0$), in the range:

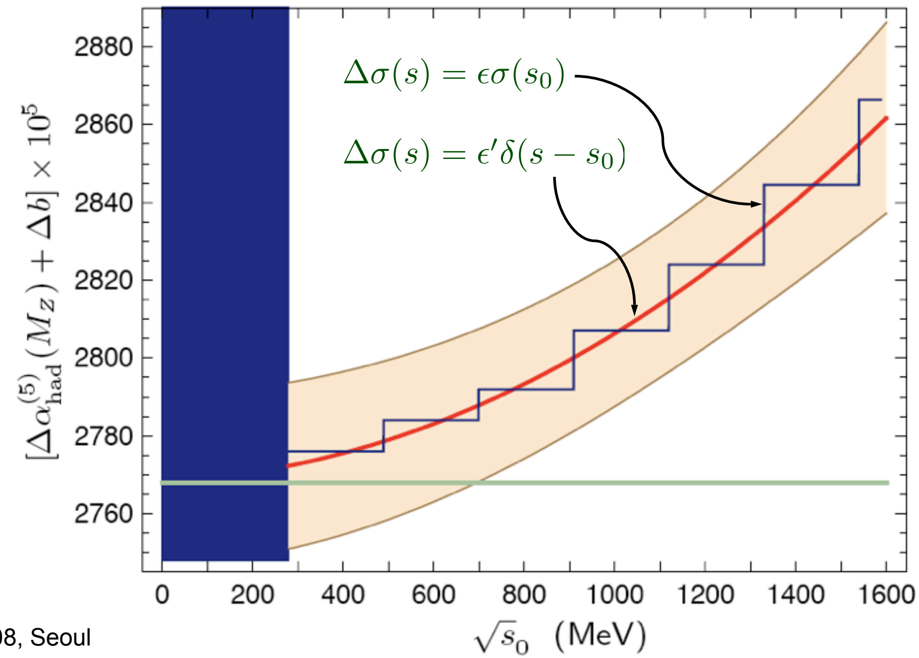
$$\sqrt{(s)} \in [\sqrt{s_0} - \delta/2, \sqrt{s_0} + \delta/2] \quad \longrightarrow$$

Shifts of a_μ^{HLO} and $\Delta\alpha_{had}^{(5)}(M_Z)$

- If this shift $\Delta\sigma(s)$ in $[\sqrt{s_0}-\delta/2, \sqrt{s_0}+\delta/2]$ is adjusted to bridge the g-2 discrepancy, the value of $\Delta\alpha_{had}^{(5)}(M_Z)$ increases by:

$$\Delta b(\sqrt{s_0},\delta) = \Delta a_\mu \frac{\int_{\sqrt{s_0}-\delta/2}^{\sqrt{s_0}+\delta/2} g(t^2)\,\sigma(t^2)\,t\,dt}{\int_{\sqrt{s_0}-\delta/2}^{\sqrt{s_0}+\delta/2} f(t^2)\,\sigma(t^2)\,t\,dt}$$

- Adding this shift to $\Delta\alpha_{had}^{(5)}(M_Z) = 0.02768(22)$ [HMNT07], with $\Delta a_\mu = 302(88) \times 10^{-11}$ [HMNT07], we obtain:



EW Bounds on the SM Higgs mass

- The dependence of SM predictions on the Higgs mass, via loops, provides a powerful tool to set bounds on its value.
- Comparing the theoretical predictions of M_W and $\sin^2\theta_{\rm eff}^{\rm lept}$
  [convenient formulae in terms of M_H, M_{top}, $\Delta\alpha_{had}^{(5)}(M_Z)$ and $\alpha_s(M_Z)$ by Degrassi, Gambino, MP, Sirlin '98; Degrassi, Gambino '00; Ferroglia, Ossola, MP, Sirlin '02; Awramik, Czakon, Freitas, Weiglein '04 & '06]

with

M_W	$= 80.398\ (25)$ GeV	[LEP+Tevatron]
$\sin^2\theta_{\rm eff}^{\rm lept}$	$= 0.23153\ (16)$	[LEP+SLC]

and

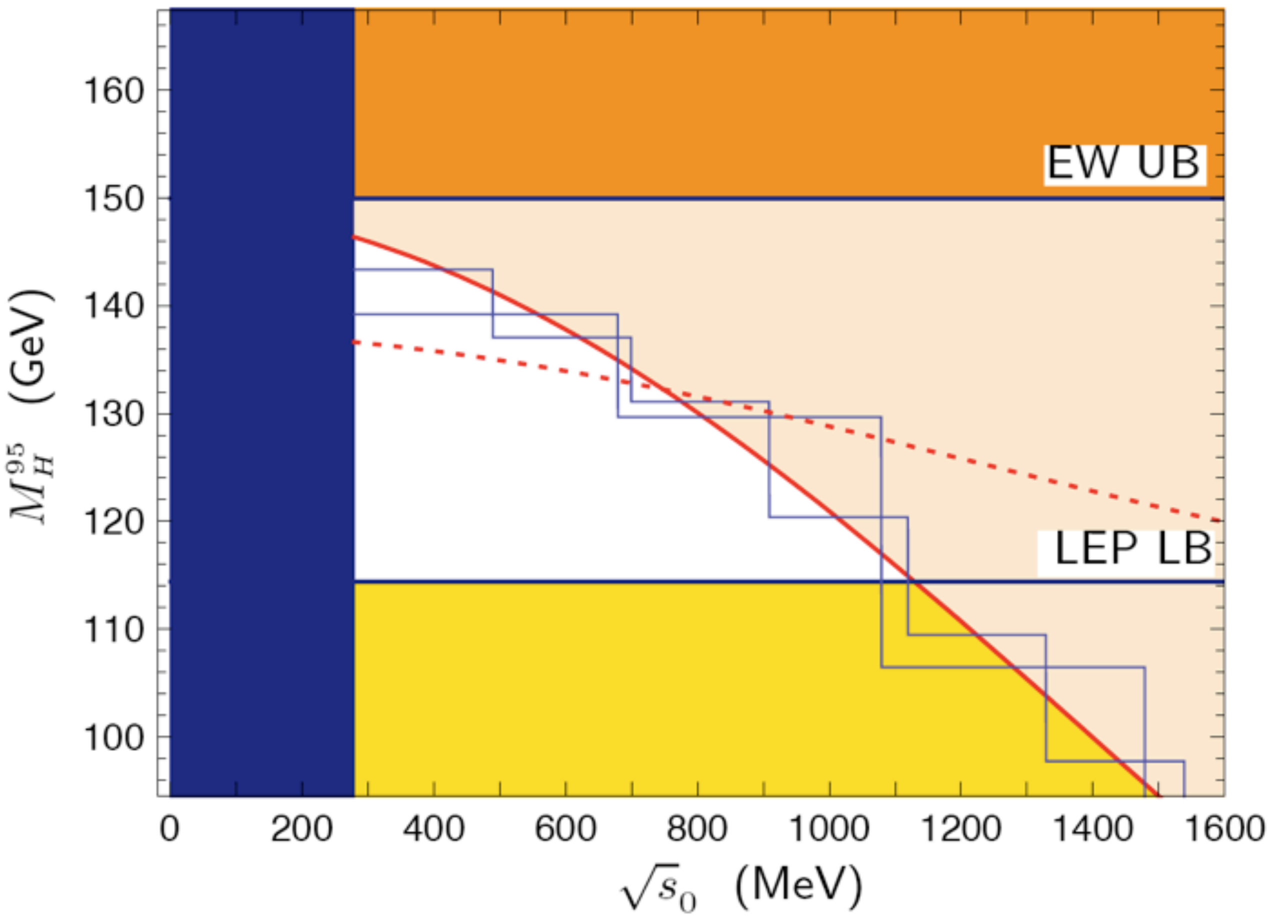
$\Delta\alpha_{had}^{(5)}(M_Z)$	$= 0.02768\ (22)$	[HMNT '07]
M_{top}	$= 172.6\ (1.4)$ GeV	[CDF-D0, Mar '08]
$\alpha_s(M_Z)$	$= 0.118\ (2)$	[PDG '06]

we get

$$M_H = 90\ ^{+33}_{-25}\ \text{GeV} \quad \& \quad M_H < 150\ \text{GeV}\ \ 95\%\text{CL}$$

- The value of $\Delta\alpha_{had}^{(5)}(M_Z)$ is a key input of these EW fits…

- How much does the M_H upper bound change when we shift $\sigma(s)$ by $\Delta\sigma(s)$ [and thus $\Delta\alpha_{had}^{(5)}(M_Z)$ by Δb] to accommodate Δa_μ ?

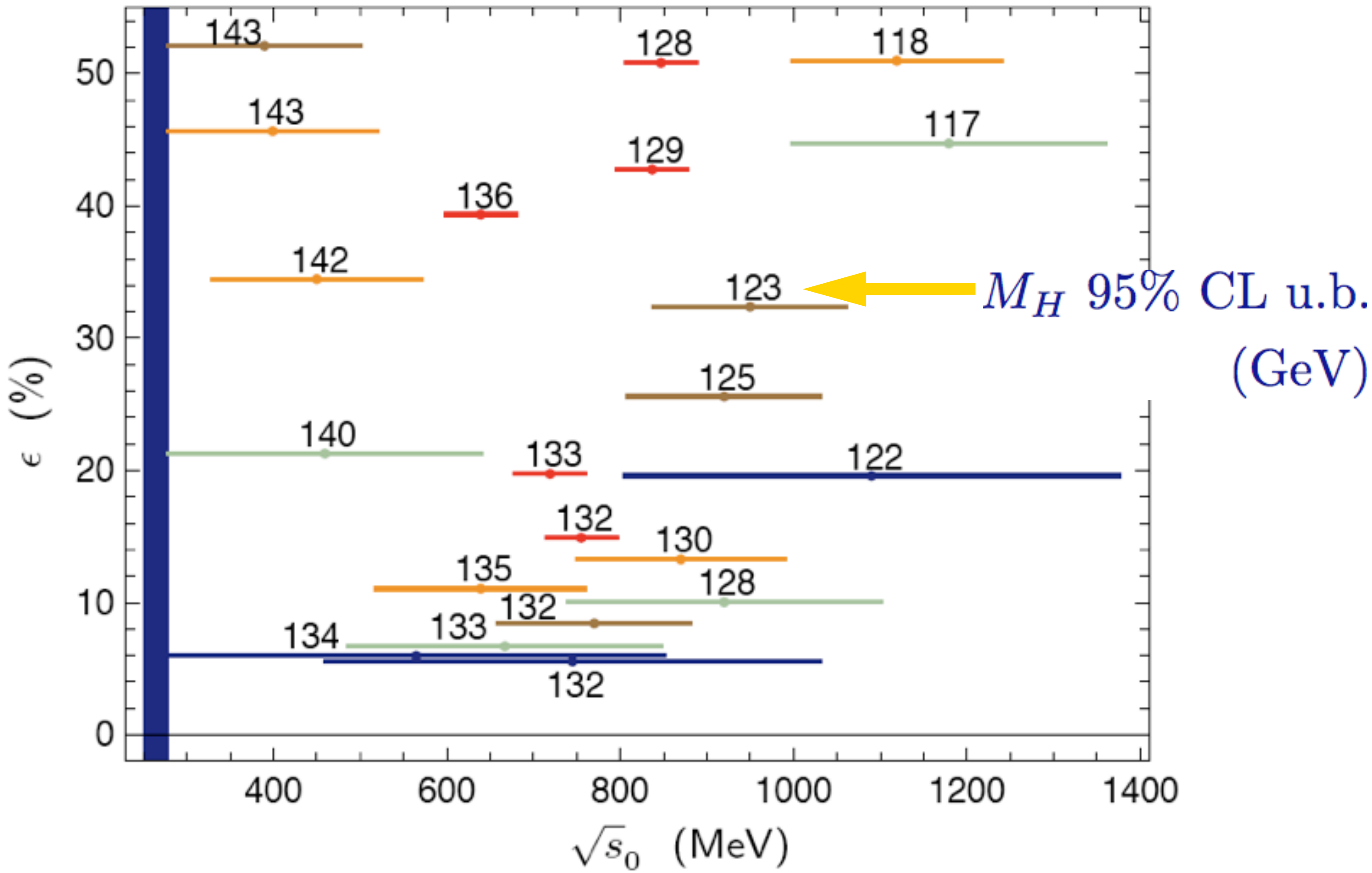


The muon g-2: connection with the SM Higgs mass (2)

- The LEP direct-search lower bound is M_H^{LB} = 114.4 GeV (95%CL).
- The hypothetical shifts $\Delta\sigma = \varepsilon\sigma(s)$ that bridge the muon g-2 discrepancy conflict with the LEP lower limit when $\sqrt{s_0} >$ ~1.2GeV (for bin widths δ up to several hundreds of MeV).
- While using tau data in the calculation of a_μ^{HLO} almost solves the muon g-2 discrepancy, it increases the value of $\Delta\alpha_{had}^{(5)}(M_Z)$, leading to $M_H <$ 138 GeV (95%CL), in near conflict with M_H^{LB}.
- Recent claim: e^+e^- & tau data consistent below ~1 GeV (after isospin viol. effects & vector meson mixings). We could thus assume that Δa_μ is fixed by hypothetical errors above ~1GeV (where disagreement persists). If so, M_H^{UB} falls below M_H^{LB} !!
- Scenarios where Δa_μ is accommodated without affecting M_H^{UB} are possible, but considerably more unlikely.

How realistic are these shifts $\Delta\sigma(s)$?

- How realistic are these shifts $\Delta\sigma(s)$ when compared with the quoted exp. uncertainties? Study the ratio $\varepsilon=\Delta\sigma(s)/\sigma(s)$:



How realistic are these shifts $\Delta\sigma(s)$? (2)

- The minimum ε is ~ +4%. It occurs if σ is multiplied by $(1+\varepsilon)$ in the whole integration region (!), leading to M_H^{UB} ~ 75 GeV (!!)
- As the quoted exp. uncertainty of $\sigma(s)$ below 1 GeV is ~ a few per cent (or less), the possibility to explain the muon g-2 with these shifts $\Delta\sigma(s)$ appears to be unlikely.
- If, however, we allow variations of $\sigma(s)$ up to ~6% (7%), M_H^{UB} is reduced to less than ~134 GeV (135 GeV). E.g., the ~6% shift in the interval [0.6, 1.2] GeV, required to fix Δa_μ, lowers M_H^{UB} to 130 GeV.
- Reminder: the above M_H upper bounds, like the LEP-EWWG ones, depend on the value of $\sin^2\theta_{\rm eff}^{\rm lept}$. They also depend on M_t & its unc. δM_t. We prepared simple formulae to translate easily M_H upper bounds discussed above into new values corresponding to M_t & δM_t inputs different from those employed here.

Conclusions

- g: Beautiful examples of interplay between theory and experiment: g_e probed at <ppt $\to$ α and extraordinary test of QED's validity; g_μ probed at <ppb $\to$ test of the full SM and great opportunity to unveil (or just constrain) "New Physics" effects!
- The discrepancy Δa_μ is more than 3 σ if e^+e^- data are used. With tau data the deviation is only ~ 1 σ. QED and EW terms solid and ready for E969! HLO will continue improving... LBL??
- Δa_μ can be due to New Physics, or to problems in a_μ^{SM} (or a_μ^{EXP}!). Can it be due to errors in the hadronic $\sigma(s)$? An hypothetical increase $\Delta\sigma(s)$ could bridge Δa_μ, leading however to a decrease on the EW upper bound on the SM Higgs mass M_H ...
- By means of a detailed analysis we conclude that solving Δa_μ via an increase of $\sigma(s)$ is unlikely in view of current exp. error estimates. However, if this turns out to be the solution, then the M_H upper bound drops to about 130 GeV which, in conjunction with the LEP 114 GeV direct lower limit, leaves a rather narrow window for M_H.

The End