Tri-bimaximal Mixing from Cascades

SUSY08

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Collaborator

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(arXiv:0804.4055 [hep-ph])

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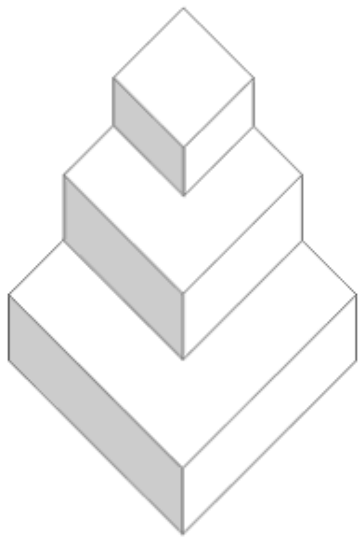


cascade

1. Introduction

Neutrino Oscillation

Maltoni *et al*, [hep-ph/0405172] ver.6 (Sep 2007) 3σ

Best Fit

$\Downarrow$

$$\sin^2\theta_{12} = 0.26 - 0.32 - 0.40 \quad : \text{Large}$$
$$\sin^2\theta_{23} = 0.34 - 0.50 - 0.67 \quad : \text{Maximal}$$
$$\sin^2\theta_{13} \leq 0.050 \quad : \text{Small}$$

$\Downarrow$

$$\sin^2\theta_{12} \simeq \frac{1}{3}$$
$$\sin^2\theta_{23} \simeq \frac{1}{2}$$
$$\sin^2\theta_{13} \sim 0$$

Tri-bimaximal Generation Mixing

Harrison, Perkins, Scott (2002)

$$V_{\rm TB} = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{2}} \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} \Rightarrow \begin{matrix} \sin^2\theta_{12} = \frac{1}{3} \\ \sin^2\theta_{23} = \frac{1}{2} \\ \sin^2\theta_{13} = 0 \end{matrix} \Leftrightarrow \begin{matrix} \textbf{Experiments} \\ \sin^2\theta_{12} \simeq \frac{1}{3} \\ \sin^2\theta_{23} \simeq \frac{1}{2} \\ \sin^2\theta_{13} \leq 0.050 \end{matrix}$$

- ν_2 : Tri-maximal Mixture of $\nu_e,\ \nu_\mu,\ \nu_\tau$
- ν_3 : Bi -maximal Mixture of $\nu_\mu,\ \nu_\tau$

$$\boxed{V_{\rm TB} \simeq V_{\rm MNS}^{\rm exp}}$$

$$\Downarrow$$

- **Good theoretical motivation to look for flavor structure**

  - Flavor symmetry

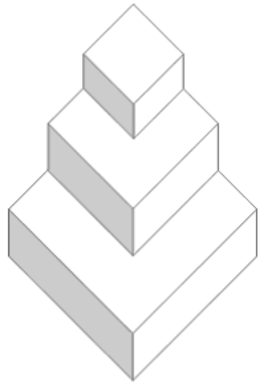
    e.g.) Discrete symmetry : S_3, A_4,,,

  - Mass texture

    $\Rightarrow$ "Cascade" matrix

2. Cascade Matrix

$$M_{\text{cas}} = \begin{pmatrix} \delta & \delta & \delta \\ \delta & \lambda & \lambda \\ \delta & \lambda & 1 \end{pmatrix} v, \quad \delta \ll \lambda \ll 1$$



("cascade hierarchy" by Dorsner and Barr, (2001))

(Mass) Eigenvalues

$$m_1 : m_2 : m_3 \sim \delta : \lambda : 1$$

Mixing angles

$$\theta_{12} \sim \frac{\delta}{\lambda}, \quad \theta_{23} \sim \lambda, \quad \theta_{13} \sim \delta, \quad \left[\theta_{ij} \sim \frac{m_i}{m_j} \right]$$

- The fermion mass matrices of the cascade form naturally lead to the tri-bimaximal generation mixing in the lepton sector.

3. Tri-bimaximal Generarion Mixing

Neutrino Sector (Charged lepton sector: Diagonal)

- **What is the form of M_ν for the tri-bimaximal mixing?**

$$M_\nu = V_{\rm TB}^* \cdot {\rm Diag}\{m_1, m_2, m_3\} \cdot V_{\rm TB}^\dagger$$
$$= \frac{m_1}{6}\begin{pmatrix} 4 & -2 & -2 \\ -2 & 1 & 1 \\ -2 & 1 & 1 \end{pmatrix} + \frac{m_2}{3}\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + \frac{m_3}{2}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{pmatrix}$$

Cascade Matrix (See-saw machanism : $M_\nu \simeq M_N M_R^{-1} M_N^T$)

Dirac Mass Matrix

$$M_N = \begin{pmatrix} \delta & \delta & \delta \\ \delta & \lambda & -\lambda \\ \delta & -\lambda & 1 \end{pmatrix} v, \quad (\delta \ll \lambda \ll 1)$$

Majorana Mass Matrix

$$M_R = \begin{pmatrix} M_1 & & \\ & M_2 & \\ & & M_3 \end{pmatrix}$$

$$M_\nu \simeq \frac{v^2}{M_3}\begin{pmatrix} \delta^2 & \delta\lambda & \delta \\ \delta\lambda & \lambda^2 & -\lambda \\ \delta & -\lambda & 1 \end{pmatrix} + \frac{v^2}{M_1}\delta^2\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + \frac{v^2}{M_2}\lambda^2\begin{pmatrix} \delta^2/\lambda^2 & \delta/\lambda & -\delta/\lambda \\ \delta/\lambda & 1 & -1 \\ -\delta/\lambda & -1 & 1 \end{pmatrix}$$

$$\Rightarrow |m_1| \ll |m_{2,3}| \text{ (Normal Hierarchy)}, \quad |\delta m_3| \ll |\lambda m_2|$$

Cascade Matrix (See-saw mechanism)

Mass Eigenvalues

$$(m_1,\ m_2,\ m_3) = \left(0 + \frac{v^2}{6M_3},\ \frac{3\delta^2 v^2}{M_1} + \frac{v^2}{3M_3},\ \frac{2\lambda^2 v^2}{M_2} + \frac{v^2}{2M_3}\right)$$

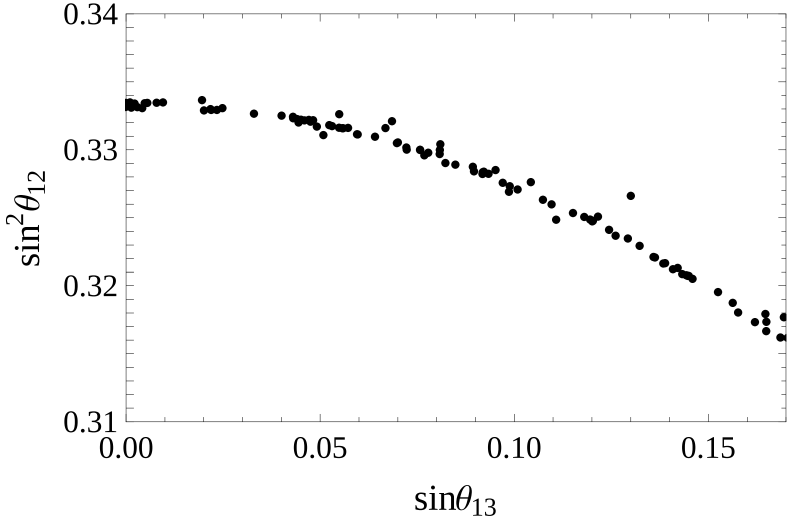
Mixing Angles

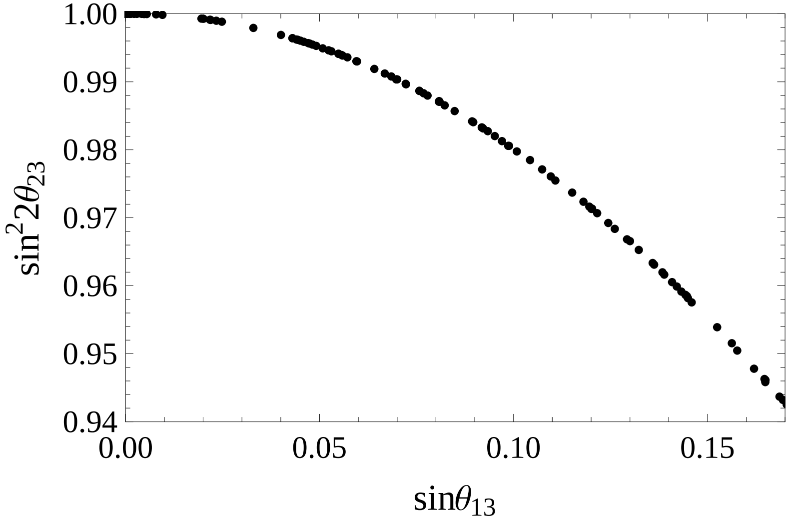
$$\sin^2\theta_{12} = \left|\frac{1}{\sqrt{3}} - \mathcal{O}\left(\frac{m_1}{m_2}\right)\right|^2$$

$$\sin^2\theta_{23} = \left|\frac{-1}{\sqrt{2}} + \mathcal{O}\left(\frac{m_1}{m_3}\right) + \mathcal{O}\left(\frac{\delta}{\lambda}\frac{m_2}{m_3}\right)\right|^2$$

$$\sin^2\theta_{13} = \left|0 + \mathcal{O}\left(\frac{\delta}{\lambda}\right) + \mathcal{O}\left(\frac{m_1 m_2}{m_3^2}\right)\right|^2$$

Numerical Analyses





Mixing Angles

$$\sin^2\theta_{12} = \left|\frac{1}{\sqrt{3}} - \mathcal{O}\left(\frac{m_1}{m_2}\right)\right|^2$$

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$$\sin^2\theta_{13} = \left|0 + \mathcal{O}\left(\frac{\delta}{\lambda}\right) + \mathcal{O}\left(\frac{m_1 m_2}{m_3^2}\right)\right|^2$$

Mass Squared Diferrences

$$\Delta m_{21}^2 \equiv |m_2|^2 - |m_1|^2, \;\; \Delta m_{31}^2 \equiv |m_3|^2 - |m_1|^2$$

Mixing Angles

$$\sin^2\theta_{12} = \left|\frac{1}{\sqrt{3}} - \mathcal{O}\left(\frac{m_1}{m_2}\right)\right|^2$$

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$$\sin^2\theta_{13} = \left|0 + \mathcal{O}\left(\frac{\delta}{\lambda}\right) + \mathcal{O}\left(\frac{m_1 m_2}{m_3^2}\right)\right|^2$$

Mass Squared Diferrences

$$\Delta m_{21}^2 \equiv |m_2|^2 - |m_1|^2, \quad \Delta m_{31}^2 \equiv |m_3|^2 - |m_1|^2$$

Parameter Independent Relation

$$\frac{1}{9}\left(\sin^2\theta_{23} - \frac{1}{2}\right) - \frac{r}{4}\left(\sin^2\theta_{12} - \frac{1}{3}\right) - \frac{\sqrt{2}\,r}{27}\sin\theta_{13} = 0$$

$$\left(r \equiv \Delta m_{21}^2/\Delta m_{31}^2 \ll 1\right)$$

$$\Rightarrow \theta_{23} \simeq \pi/4 \quad (\because r \ll 1)$$

Charged Lepton Sector

$$M_E = \begin{pmatrix} \delta_e & \delta_e & \delta_e \\ \delta_e & \lambda_e & \lambda_e \\ \delta_e & \lambda_e & 1 \end{pmatrix} v$$

$$|\lambda_e| \simeq \frac{|m_\mu|}{|m_\tau|} \simeq 6 \times 10^{-2}$$

$$|\delta_e| \simeq \frac{|m_e|}{|m_\tau|} \simeq 3 \times 10^{-4}$$

Effects on Mixing Angles

$$\sin^2\theta_{12} \simeq \left| \frac{1}{\sqrt{3}} - \mathcal{O}\left(\frac{m_1}{m_2}\right) - \frac{1}{\sqrt{3}}\frac{m_e}{m_\mu} \right|^2$$

$< 1\%$

$$\sin^2\theta_{23} \simeq \left| \frac{-1}{\sqrt{2}} + \mathcal{O}\left(\frac{m_1}{m_3}\right) + \mathcal{O}\left(\frac{\delta}{\lambda}\frac{m_2}{m_3}\right) - \frac{1}{\sqrt{2}}\frac{m_\mu}{m_\tau} \right|^2$$

$\sim 6\%$

$$\sin^2\theta_{13} \simeq \left| \mathcal{O}\left(\frac{\delta}{\lambda}\right) + \mathcal{O}\left(\frac{m_1 m_2}{m_3^2}\right) + \frac{1}{\sqrt{2}}\frac{m_e}{m_\mu} \right|^2$$

$\lesssim 1\%$

4. Flavor Violation, Leptogenesis

Lepton Flavor Violation

$$\mathrm{Br}(l_i \to l_j \gamma) \sim \frac{3\alpha \left|(m_{\tilde{l}}^2)_{ij}\right|^2}{2\pi m_{\mathrm{SUSY}}^4} \left(\frac{M_W^4}{m_{\mathrm{SUSY}}^4} \tan^2\beta \right) \equiv \frac{3\alpha \left|(m_{\tilde{l}}^2)_{ij}\right|^2}{2\pi m_{\mathrm{SUSY}}^4} \cdot B$$

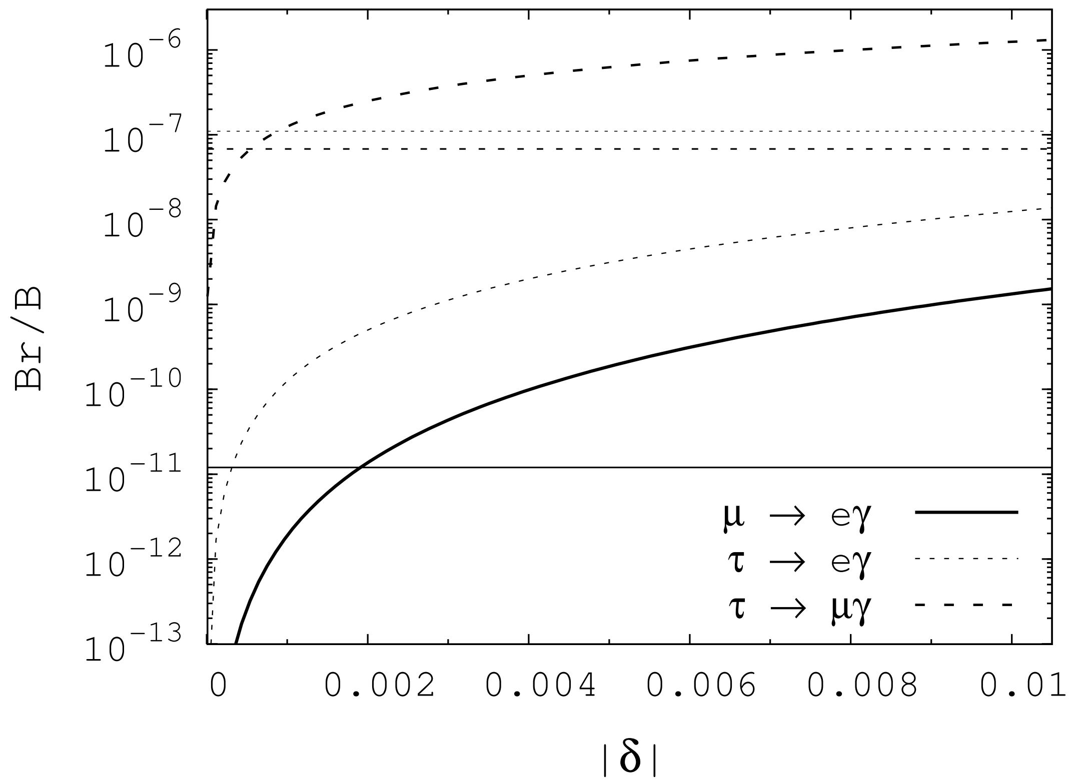
(e.g.) $\delta = \lambda^2 = 10^{-4}$

Cascade Model

$$\mathrm{Br}(\mu \to e\gamma) \sim 10^{-15} B$$
$$\mathrm{Br}(\tau \to e\gamma) \sim 10^{-12} B$$
$$\mathrm{Br}(\tau \to \mu\gamma) \sim 10^{-8} B$$

Exp. Bounds(90 % C.L.)

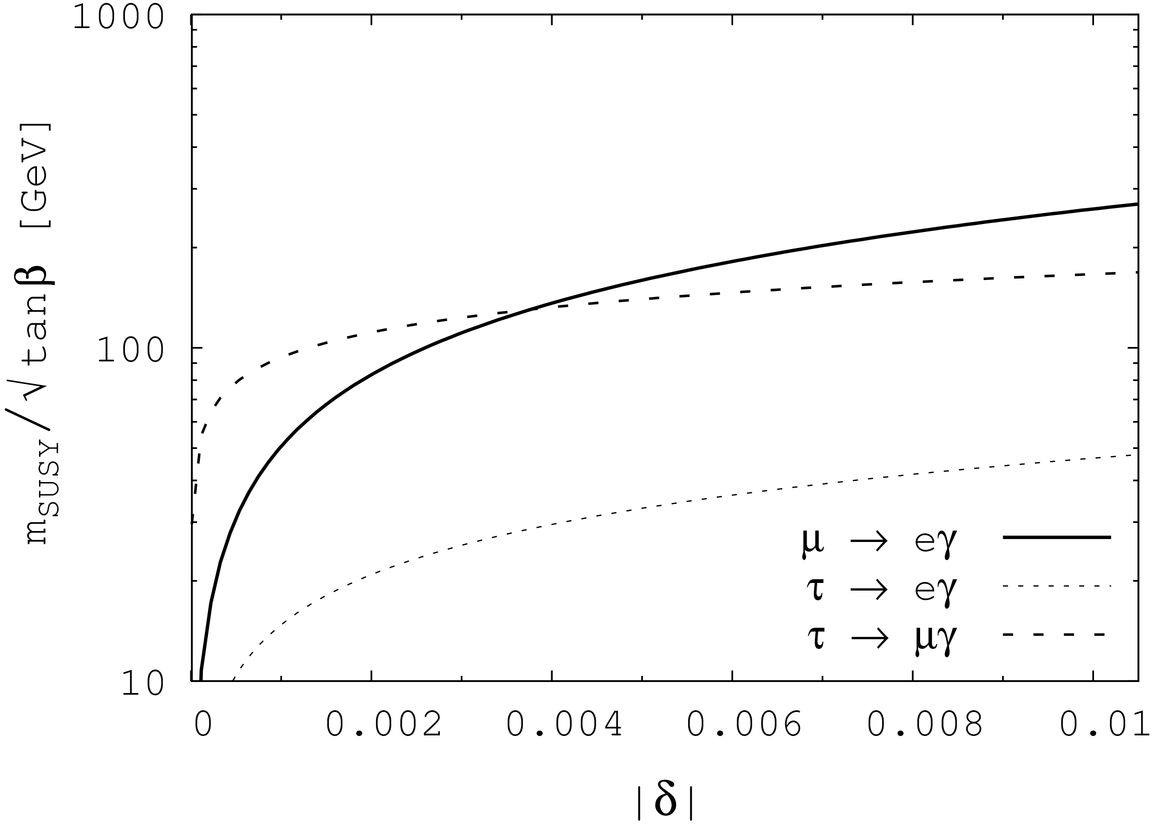
$$\mathrm{Br}(\mu \to e\gamma) < 1.2 \times 10^{-11}$$
$$\mathrm{Br}(\tau \to e\gamma) < 1.1 \times 10^{-7}$$
$$\mathrm{Br}(\tau \to \mu\gamma) < 6.8 \times 10^{-8}$$



Lower Bound of $m_{\rm SUSY}$

$$\mathrm{Br}(\mu \to e\gamma) \sim 10^{-15} B < 1.2 \times 10^{-11}$$
$$\mathrm{Br}(\tau \to e\gamma) \sim 10^{-12} B < 1.1 \times 10^{-7}$$
$$\mathrm{Br}(\tau \to \mu\gamma) \sim 10^{-8} B < 6.8 \times 10^{-8}$$

$$B \equiv \frac{M_W^4}{m_{\rm SUSY}^4} \tan^2\beta \Rightarrow M_W \cdot \left(\frac{1}{B}\right)^{1/4} = \frac{m_{\rm SUSY}}{\sqrt{\tan\beta}}$$



Leptogenesis

$$\varepsilon_1 = \frac{1}{8\pi} \sum_{j\neq 1} \frac{\mathrm{Im}\{(D M_N M_N^\dagger D^\dagger)_{j1}\}^2}{|(D M_N M_N^\dagger D^\dagger)_{11}|} F(r_j) \qquad \left(r_j \equiv \frac{|M_j|^2}{|M_1|^2} \right)$$

$$\Downarrow \; F(r_j) \equiv \sqrt{r_j} \left[\frac{2}{1 - r_j} - \ln\left(1 + \frac{1}{r_j}\right) \right] \qquad \text{(SUSY SM)}$$

$$\simeq \frac{-1}{16\pi |\delta|^2} \left[\mathrm{Im}[\delta^2 e^{i(\theta_3 - \theta_1)}] \left| \frac{M_1}{M_3} \right| \right]$$

$$\eta_{\mathrm{L}} = \frac{135\zeta(3)}{4\pi^4} \cdot \frac{\kappa s}{g_* n_\gamma} \cdot \varepsilon_1, \quad \eta_{\mathrm{B}} = -\frac{8}{23} \eta_{\mathrm{L}}$$

WMAP 3-years

$$\eta_{\mathrm{B}} = (5.6 - 6.5) \times 10^{-10}$$

Cas. Hierarchy

$$|\delta| \gtrsim (7.5 - 8.8) \times 10^{-3} \; \Rightarrow m_{\mathrm{SUSY}} \gtrsim 450 \sqrt{\tan\beta} \text{ GeV}$$

5. Model

	L_1	L_2	L_3	R_1	R_2	R_3	ϕ_1	ϕ_2	ϕ_3
$U(1)_f$	$2m+1$	1	0	$2m+1$	1	0	$-2m-3$	-2	-1

(m : Positive Integer)

$$M_D = \begin{pmatrix} \frac{\phi_1\phi_2^{m-1}\phi_3}{\Lambda^{m+1}} & \frac{\phi_2^{m+1}}{\Lambda^{m+1}} & \frac{\phi_2^{m}\phi_3}{\Lambda^{m+1}} \\ \frac{\phi_2^{m+1}}{\Lambda^{m+1}} & \frac{\phi_2}{\Lambda} & \frac{\phi_3}{\Lambda} \\ \frac{\phi_2^{m}\phi_3}{\Lambda^{m+1}} & \frac{\phi_3}{\Lambda} & 1 \end{pmatrix} v$$

$$\Downarrow$$

$$\Downarrow \ \langle\phi_1\rangle \simeq \langle\phi_2\rangle \simeq \langle\phi_3\rangle \equiv \lambda\Lambda$$

$$\Downarrow$$

$$M_D \simeq \begin{pmatrix} \lambda^{m+1} & \lambda^{m+1} & \lambda^{m+1} \\ \lambda^{m+1} & \lambda & \lambda \\ \lambda^{m+1} & \lambda & 1 \end{pmatrix} v, \qquad \delta = \lambda^{m+1}$$

6. Summary

- The current experimental data of mixing angles is well approximated by the tri-bimaximal generation mixing.

$$\begin{array}{lcl} \sin^2\theta_{12} & \simeq & 1/3 \\ \sin^2\theta_{23} & \simeq & 1/2 \\ \sin^2\theta_{13} & \lesssim & 0.050 \end{array} \Leftarrow V_{\rm TB} = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{2}} \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

- The cascade matrix leads to the tri-bimaximal generation mixing.

$$M_{\rm cas} = \begin{pmatrix} \delta & \delta & \delta \\ \delta & \lambda & \lambda \\ \delta & \lambda & 1 \end{pmatrix} v, \quad \delta \ll \lambda \ll 1$$

$$\Downarrow$$

Parameter Independent Relation

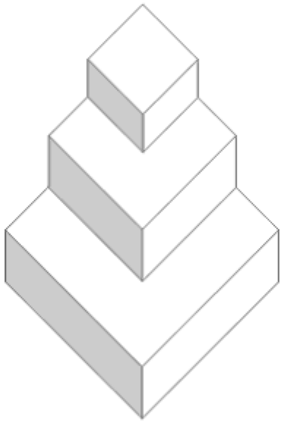
$$\frac{1}{9}\left(\sin^2\theta_{23} - \frac{1}{2}\right) - \frac{r}{4}\left(\sin^2\theta_{12} - \frac{1}{3}\right) - \frac{\sqrt{2}\,r}{27}\sin\theta_{13} = 0$$

$$\left(r \equiv \Delta m^2_{21}/\Delta m^2_{31} \ll 1\right)$$

Quark Sector

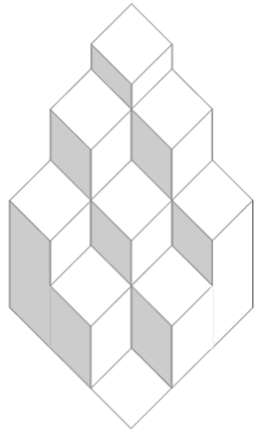
[Cascade]

$$M_{\rm cas} = \begin{pmatrix} \delta & \delta & \delta \\ \delta & \lambda & \lambda \\ \delta & \lambda & 1 \end{pmatrix} v$$



[Waterfall]

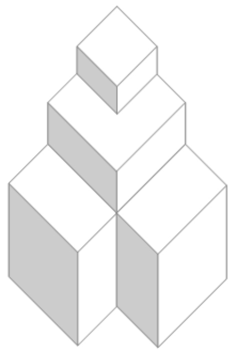
$$M_{\rm wat} = \begin{pmatrix} \delta^2 & \delta\lambda & \delta \\ \delta\lambda & \lambda^2 & \lambda \\ \delta & \lambda & 1 \end{pmatrix} v$$



$\theta_{12} \sim \delta/\lambda, \quad \theta_{23} \sim \lambda, \quad \theta_{13} \sim \delta$ $\left(\theta_{ij} \sim \frac{m_i}{m_j}\right)$	$\theta_{12} \sim \delta/\lambda, \quad \theta_{23} \sim \lambda, \quad \theta_{13} \sim \delta$ $\left(\theta_{ij} \sim \sqrt{\frac{m_i}{m_j}}\right)$

[Hybrid Cascade]

$$M_{\rm hyb} = \begin{pmatrix} 0 & \delta & \delta \\ \delta & \lambda & \lambda \\ \delta & \lambda & 1 \end{pmatrix} v$$



$$\theta_{12} \sim \delta/\lambda, \quad \theta_{23} \sim \lambda, \quad \theta_{13} \sim \delta$$

$$\left(\theta_{12} \sim \sqrt{\frac{m_1}{m_2}}, \quad \theta_{23} \sim \frac{m_2}{m_3}, \quad \theta_{13} \sim \frac{\sqrt{m_1 m_2}}{m_3}\right)$$