Holography of non-relativistic string on $AdS_5 \times S^5$

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based on collaborations with Kentaroh Yoshida (KITP, UCSB)

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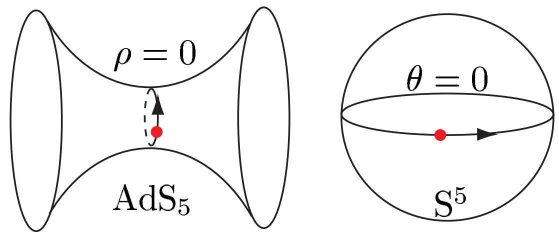
JHEP 0802 (2008) 092 [arXiv:0712.4112 [hep-th]].

NPB 798 (2008) 72 [arXiv:0709.4187 [hep-th]],
JHEP 05 (2007) 051 [arXiv:hep-th/0703061],
JHEP 10 (2006) 078 [arXiv:hep-th/0605124].

Introduction

IIB superstrings in $\mathrm{AdS}_5 \times \mathrm{S}^5$ $\Leftrightarrow$ $\mathcal{N} = 4$ super-Yang-Mills [Maldacena'9711]

- supergravity modes [Gubser-Klebanov-Polyakov'98, Witten'98]
- solvable limits



(1) Penrose limit pp-wave string [Metsaev'0112]

– 8 massive scalars & 8 massive fermions on flat worldsheet : solvable

– gauge theory side : BMN operators [Berenstein-Maldacena-Nastase'0202]

– pp-wave algebra from $psu(2, 2|4)$ [Hatsuda-Kaminura-MS'0202]

– as fluctuations around a 1/2 BPS rotating particle [Gubser-Klebanov-Polyakov'0204]

(2) non-relativistic limit non-relativistic (NR) string [Gomis-Gomis-Kamimura'0507]

– 3 massive and 5 massless scalars & 8 massive fermions on AdS_2 worldsheet : solvable

[Sakai-Tanii'84'85]

– Newton-Hooke algebra from $psu(2,2|4)$ [GGK'0507, Yoshida-MS'0605]

NH algebra for F1 $\supset$ osp(4^*|4) $\quad$ ($\supset$ so(1,2)$\times$so(3)$\times$so(5))

– as fluctuations around 1/2 BPS AdS_2 [Drukker-Gross-Tseytlin'00] [Yoshida-MS'0703,'0709]

– gauge theory side : AdS/CFT dictionary in the non-relativistic limit

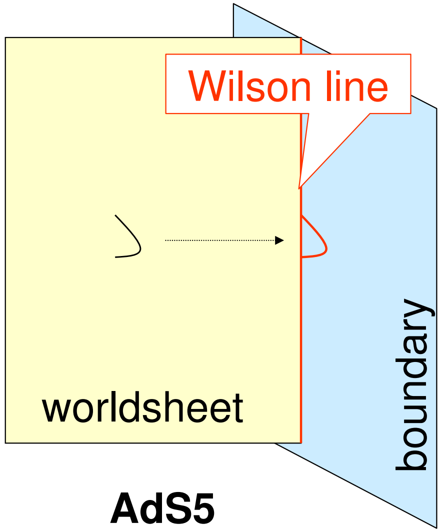
semiclassical modes **operator insertions**

(Non-normalizable mode)

3 massive bosons $\overset{\Delta=2}{\Longleftrightarrow}$ $D_a\phi_6 + iF_{a0}\ (a=1,2,3)$ SO(3)

5 massless bosons $\overset{\Delta=1}{\Longleftrightarrow}$ $\phi_{a'}\ (a'=1,\cdots,5)$ SO(5)

8 massive fermions $\overset{\Delta=3/2}{\Longleftrightarrow}$ $\Gamma_0 h_- \Psi \qquad h_\pm = \frac{1}{2}(1 \pm i\Gamma_{04})$



Non-relativistic string in $\mathrm{AdS}_5 \times \mathrm{S}^5$

NR limit can be viewed as taking a close-up of AdS_2 [Gomis-Gomis-Kamimura'0507]

NR string from semiclassical limit [Yoshida-MS'0703,'0712]

cf) pp-wave string is a semiclassical limit about BPS particle rotating in S^5 [GKP'0204]

Nambu-Goto action $\quad S_{\mathrm{NG}} = \frac{\sqrt{\lambda}}{2\pi} \int d^2\xi \sqrt{\det g}\,, \quad g_{\alpha\beta} = \partial_\alpha X^M \partial_\beta X^N G_{MN}$

$$\text{in } \mathrm{EAdS}_5 \times \mathrm{S}^5 : \; ds^2 = \frac{1}{z^2}\left((dX^\mu)^2 + dz^2\right) + d\Omega_5^2 \qquad (\mu = 0,1,2,3)$$

Semiclassical limit $\quad$ Classical solution : $\boxed{X^0 = \tau\,, \quad z = \sigma}$

$$X^a = 0 + \sqrt{2\pi}\lambda^{-1/4}\,\tilde{x}^a\,, \quad Y^{a'} = 0 + \sqrt{2\pi}\lambda^{-1/4}\,\tilde{y}^{a'}\,, \qquad \lambda \equiv N g_{\mathrm{YM}}^2 = \frac{R^4}{\alpha'^2}.$$

$$S_{\mathrm{NG}} = \sqrt{\lambda} S_{(0)} + \lambda^{1/4} S_{(1)} + S_{(2)} + \cdots$$

◇ classical contribution $S_{(0)} = \dfrac{1}{2\pi}\displaystyle\int d\tau \frac{1}{\epsilon}$

– NS-NS flux $B_{0z} = -\frac{1}{z^2}$: $S_B = \frac{\sqrt{\lambda}}{2\pi}\int d\tau \frac{1}{\sigma}\Big|_{\sigma=\epsilon}^{\sigma=\infty} = -\frac{\sqrt{\lambda}}{2\pi}\int d\tau \frac{1}{\epsilon}$. [GGK]

– Legendre transformation :

$S_L = -\int d\tau P_i^\sigma Y^i|_{\sigma=0} = -\frac{\sqrt{\lambda}}{2\pi}\int d\tau \frac{1}{\epsilon}$ where $P_M^\sigma = \frac{\partial \mathcal{L}}{\partial(\partial_\sigma X^M)}$. [Drukker-Gross-Ooguri'9904]

◇ only $S_{(2)}$ remains under the semiclassical limit $\lambda \to \infty$.

$$S_{(2)} = \frac{1}{2}\int d^2\xi \sqrt{g} g^{\alpha\beta}\left(\frac{1}{\sigma^2}\partial_\alpha \tilde{x}^a \partial_\beta \tilde{x}^a + \partial_\alpha \tilde{y}^{a'} \partial_\beta \tilde{y}^{a'}\right)$$

Under $\boxed{x = \frac{1}{\sigma}\tilde{x}\ ,\quad y = \tilde{y}}$, it reduces to $S_{\rm NR} + \dfrac{1}{2}\displaystyle\int d^2\xi \partial_\sigma \left(\frac{x^2}{\sigma}\right)$ with

$$S_{\rm NR} = \frac{1}{2}\int d^2\xi \sqrt{g}\left[g^{\alpha\beta}\left(\partial_\alpha x^a \partial_\beta x^a + \partial_\alpha y^{a'} \partial_\beta y^{a'}\right) + 2(x^a)^2\right]$$

where 2-dim induced metric is $g_{\alpha\beta} = \frac{1}{\sigma^2}\delta_{\alpha\beta}$ with $R^{(2)} = -2$.

Summarizing, we have obtained the NR string action from the semiclassical limit.

3 scalars x^a $(a = 1, 2, 3)$	with $m^2 = 2$	$SO(3)$
5 scalars $y^{a'}$ $(a' = 1, \cdots, 5)$	with $m^2 = 0$	$SO(5)$
8 fermions $\vartheta^\alpha (\alpha = 1, \cdots, 8)$	with $m^2 = 1$	

Holography of NR string

<u>non-normalizable mode</u>

$\Phi^I = (x^a, y^{a'})$ propagating in Euclidean AdS$_2$: $ds^2 = \frac{1}{\sigma^2}(d\tau^2 + d\sigma^2)$ is determined by the boundary value Φ_0^I uniquely [Witten'9805,Freedman et al'9804]

$$\Phi(\sigma,\tau) \to \sigma^{1-\Delta}\Phi_0(\tau) \quad (\sigma \to 0)\ , \qquad \Delta = \frac{1}{2}\left(1 + \sqrt{1+4m^2}\right)$$

Φ is called a non-normalizable (NN) mode.

We note that the convergence of $S_{(2)}$ leads to the precise unitarity bound on the dimension of a scalar.

$S_{(2)}$ evaluated with Φ_0 is [Klebanov-Witten'9905]

$$S_{(2)}[\Phi_0] = -\sum_{I=1}^{8}(\Delta - 1/2)\pi^{-1/2}\frac{\Gamma(\Delta)}{\Gamma(\Delta - (1/2))}\int d\tau d\tau' \frac{\Phi_0^I(\tau)\Phi_0^I(\tau')}{(\tau - \tau')^{2\Delta}}$$

NN mode corresponds to a source Φ_0 that couples to the operator $\mathcal{O}$ with dimension Δ.

Though $x^a \to \frac{1}{\epsilon}$ $(\sigma \to \epsilon)$, the fluctuation $\tilde{x}^a$ does not diverge $\tilde{x} \to \epsilon^0$ because $x = \frac{1}{\sigma}\tilde{x}$. NN mode corresponds to a fluctuation reaching the boundary.

$\Rightarrow$ NN modes may correspond to small deformations of the Wilson loop on the boundary.

Wilson loop $\quad W(C) = \mathrm{Tr} P \exp \int_C ds \left(i A_\mu(x(s)) \dot{x}^\mu(s) + \phi_i(x(s)) \dot{y}^i(s) \right)$

Expand $W(C)$ about the straight Wilson line $W(C_0) = \mathrm{Tr} P \exp \int ds (iA_0 + \phi_6)$ specified by C_0 : $x^\mu_{C_0} = (s, 0, 0, 0)$, $\dot{y}^i_{C_0} = (0, 0, 0, 0, 0, 1)$.

We find (BMN operators are derived in [Miwa'0504])

$$\delta x^\mu(s) \frac{\delta W(C)}{\delta x^\mu}\bigg|_{C_0} = \delta x^\mu(s) \mathrm{Tr} P \, (i F_{\mu 0} + D_\mu \phi_6)_s \mathcal{W}_{C_0} \ , \quad \delta \dot{y}^i \frac{\delta W(C)}{\delta \dot{y}^i}\bigg|_{C_0} = \delta \dot{y}^i \mathrm{Tr} P \, (\phi_i)_s \mathcal{W}_{C_0} \ .$$

Locally supersymmetry condition for small deformations implies $0 = \delta \dot{x}^0 - \delta \dot{y}^6$, while one may set $0 = \delta \dot{x}^0 + \delta \dot{y}^6$ by using reparametrization. That is $\delta x^0 = \delta \dot{y}^6 = 0$.

$iF_{a0} + D_a\phi_6 \quad (a = 1, 2, 3)$,	$\phi_{a'} \quad (a' = 1, \cdots, 5)$,	$\Gamma_0 h_- \Psi \quad (\alpha = 1, \cdots, 8)$
$\Delta = 2$	$\Delta = 1$	$\Delta = 3/2$
3 massive scalars x^a,	5 massless scalars $y^{a'}$,	8 massive fermions ϑ^α

Δ and the symmetry agree with those of fluctuations.

GKPW type relation

Classical

YM: $Z_{\rm YM} \to \int [dA][d\phi]\, W(C_0)\, \exp(-S_{\rm YM}) = \langle W(C_0)\rangle$.

string: $Z_{\rm string} = \int [dX] \exp(-S) \approx \exp(-S_{(0)}[X_0]) = 1$.

$$\Rightarrow \quad \langle W(C_0)\rangle = \exp(-S_{(0)}) = 1$$

Semiclassical (large λ): $\quad Z_{\rm string} = \displaystyle\int [dX] {\rm e}^{-S[X]} \approx {\rm e}^{-S_{(0)}} \int [d\Phi] {\rm e}^{-S_{(2)}[\Phi]} = {\rm e}^{-S_{(2)}[\Phi_0]}$.

Φ_0 corresponds to the source on the boundary so that we propose

$$\left\langle {\rm Tr} P {\rm e}^{\int dt(iA_0+\phi_6)} \cdot {\rm e}^{\int dt \mathcal{O}_I \Phi_0^I} \right\rangle = {\rm e}^{-S_{(2)}[\Phi_0]} \ .$$

- For a circular WL, $S_{(0)} = 0$ is replaced by $S_{(0)} = -\sqrt{\lambda}$ but $S_{(2)}$ is unchanged.
- The higher order fluctuation $S_{(n)}$ $(n > 2)$ gives a n-point coupling in the bulk.

Summary

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- AdS/CFT dictionary in the non-relativistic limit

3 massive scalars	$\leftrightarrow$	$D_a\phi_6 + iF_{a0} \quad (a = 1, 2, 3)$
5 massless scalars	$\leftrightarrow$	$\phi_{a'} \quad (a' = 1, \cdots, 5)$
8 massive fermions	$\leftrightarrow$	$\Gamma_0 h_- \Psi^\alpha \quad (\alpha = 1, \cdots, 8)$

- GKPW type relation

$$\left\langle \mathrm{Tr} P \mathrm{e}^{\int dt(iA_0+\phi_6)} \cdot \mathrm{e}^{\int dt \mathcal{O}_I \Phi_0^I} \right\rangle = \mathrm{e}^{-S_{(2)}[\Phi_0]} \ .$$

◦ Lorentzian AdS$_2$ string

Relation between normalizable modes and conformal quantum mechanics.

Future directions

- Derive CQM from $\mathcal{N} = 4$ SYM
- Normalizable modes on AdS_2 in Poincare disc instead of the strip
- D3-brane ($\mathrm{AdS}_2 \times \mathrm{S}^2$) and D5-brane ($\mathrm{AdS}_2 \times \mathrm{S}^4$)

  works in progress