New $\mathcal{N} \geq 4$ Superconformal Chern-Simons Theories

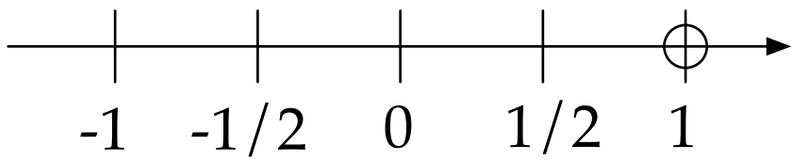
Sangmin Lee
Seoul National University

Kazuo Hosomichi, Ki-Myeong Lee, SL, Sungjay Lee, Jaemo Park
[arXiv:0805.3662] + work in progress

20 June 2008, SUSY 2008

Super-Chern-Simons Theories

- Super-YM-CS cannot go beyond $\mathcal{N} = 3$. [Kao-K.Lee 92]



- CS without YM can go beyond $\mathcal{N} = 3$!
  --- useful for $\mathcal{N} = 8$ M2-brane theory ? [Schwarz 04]

- Bagger-Lambert theory [Bagger-Lambert 06-07][Gustavsson 07]
  Super-conformal $\mathcal{N} = 8$ CS-matter based on a Lie-3-algebra.

- Gaiotto-Witten construction [Gaiotto-Witten 0804.2907]
  Large class of new $\mathcal{N} = 4$ super-conformal CS theories.

$\mathcal{N}$ = 4 and twisted multiplets

$\mathcal{N} = 4$ super-algebra includes $SO(4) = SU(2)_L \times SU(2)_R$ R-symmetry

	(ordinary)	$SU(2)_L$	$SU(2)_R$	$SU(2)_L$	$SU(2)_R$	(twisted)
(hyper)	q_α	2	1	1	2	$\tilde{q}_{\dot\alpha}$
	$\psi_{\dot\alpha}$	1	2	2	1	$\tilde{\psi}_\alpha$
(vector)	A_μ	1	1	1	1	$\tilde{A}_\mu$
	$\chi_{\alpha\dot\alpha}$	2	2	2	2	$\tilde{\chi}_{\alpha\dot\alpha}$
	$s_{\dot\alpha\dot\beta}$	1	3	3	1	$\tilde{s}_{\alpha\beta}$

Gaiotto-Witten : most general $\mathcal{N} = 4$ with ordinary hypers only.

We add twisted hypers to Gaiotto-Witten.

Gaiotto-Witten construction [Gaiotto-Witten 0804.2907]

Key steps :

- Begin with $\mathcal{N}=1$ SUSY theory with $SU(2)_{\text{diag}} \subset SU(2)_L \times SU(2)_R$ global symmetry.

- Adjust the $\mathcal{N}=1$ super-potential to restore $SU(2)_L \times SU(2)_R$.

- The SO(4) R-symmetry together with $\mathcal{N}=1$ SUSY generate a full $\mathcal{N}=4$ structure.

- Rewrite the action and SUSY transformation rules in a manifestly $\mathcal{N}=4$ covariant way.

Field content

Matter : $q^A_\alpha, \psi^A_{\dot\alpha}, F^A_\alpha$ $\quad Sp(2n)$ with ω_{AB}

reality condition : $\bar q^\alpha_A = (q^A_\alpha)^\dagger = \epsilon^{\alpha\beta}\omega_{AB} q^B_\beta$

Gauge group $G \subset Sp(2n)$

$(t^m)^A{}_B$ satisfy $[t^m, t^n] = f^{mn}{}_p t^p, \quad \mathrm{Tr}(t^m t^n) = k^{mn}$

$t^m_{AB} \equiv \omega_{AC}(t^m)^C{}_B = t^m_{BA}$

Gauge : $(A_m)_\mu, \chi_m$

Conditions for $\mathcal{N}$ = 4 SUSY

$$k_{mn} t^m_{(AB} t^n_{C)D} = 0 \quad \text{(“fundamental identity”)}$$

Classification in terms of Lie super-algebra

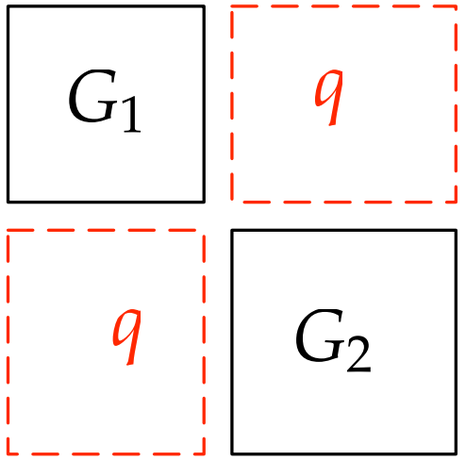
Fundamental identity implies existence of a Lie super-algebra :

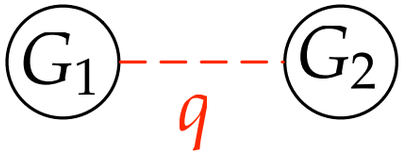
$$[M^m, M^n] = f^{mn}{}_p M^p, \quad [M^m, Q_A] = Q_B (t^m)^B{}_A, \quad \{Q_A, Q_B\} = t^m_{AB} M_m.$$

$$[\{Q_A, Q_B\}, Q_C] + (\text{cyclic}) = 0 \quad \Longleftrightarrow \quad k_{mn} t^m_{(AB} t^n_{C)D} = 0$$

Lie super-algebras classifies Gaiotto-Witten theories completely!

Typical examples : $U(N|M)$, $OSp(N|M)$





Summary

$$
\begin{aligned}
\mathcal{L} \;=\; & \frac{\varepsilon^{\mu\nu\lambda}}{4\pi}\left(k_{mn}A^m_\mu\partial_\nu A^n_\lambda + \frac{1}{3}f_{mnp}A^m_\mu A^n_\nu A^p_\lambda\right) \\
& + \frac{1}{2}\omega_{AB}\left(-\epsilon^{\alpha\beta}Dq^A_\alpha Dq^B_\beta + i\epsilon^{\dot\alpha\dot\beta}\psi^A_{\dot\alpha}\not{D}\psi^B_{\dot\beta}\right) \\
& - i\pi k_{mn}\epsilon^{\alpha\beta}\epsilon^{\dot\gamma\dot\delta}J^m_{\alpha\dot\gamma}J^n_{\beta\dot\delta} - \frac{\pi^2}{6}f_{mnp}(\mu^m)^\alpha{}_\beta(\mu^n)^\beta{}_\gamma(\mu^p)^\gamma{}_\alpha,
\end{aligned}
$$

$$
\delta q^A_\alpha = i\eta_\alpha{}^{\dot\alpha}\psi^A_{\dot\alpha}, \quad \delta A^m_\mu = 2\pi i\eta^{\alpha\dot\alpha}\gamma_\mu J^m_{\alpha\dot\alpha},
$$

$$
\delta\psi^A_{\dot\alpha} = \left[\not{D}q^A_\alpha + \frac{2\pi}{3}(t_m)^A{}_B q^B_\beta(\mu^m)^\beta{}_\alpha\right]\eta^\alpha{}_{\dot\alpha}.
$$

$$
\mu^m_{\alpha\beta} \equiv t^m_{AB}q^A_\alpha q^B_\beta, \quad J^m_{\alpha\dot\gamma} \equiv q^A_\alpha t^m_{AC}\psi^C_{\dot\gamma}
$$

(moment map multiplet)

Adding Twisted Hypers [Hosomichi-Lee^3-Park 0805.3662]

Matter : $(q^A_\alpha, \psi^A_{\dot\alpha}, F^A_\alpha)$, $\mu^m_{\alpha\beta} \equiv t^m_{AB} q^A_\alpha q^B_\beta$, $j^m_{\alpha\dot\gamma} \equiv q^A_\alpha t^m_{AC} \psi^C_{\dot\gamma}$

$(\tilde q^A_{\dot\alpha}, \tilde\psi^A_\alpha, \tilde F^A_\alpha)$, $\tilde\mu^m_{\dot\alpha\dot\beta} \equiv \tilde t^m_{AB} \tilde q^A_{\dot\alpha} \tilde q^B_{\dot\beta}$, $\tilde j^m_{\dot\alpha\alpha} \equiv \tilde q^A_{\dot\alpha} \tilde t^m_{AB} \tilde\psi^B_\alpha$

Gauge symmetry is shared.

Super-algebra structure should still be maintained.

No additional constraint for $\mathcal{N} = 4$ SUSY.

Summary

$$
\begin{aligned}
\mathcal{L} \;=\; & \frac{\varepsilon^{\mu\nu\lambda}}{4\pi}\left(k_{mn}A^m_m\partial_\nu A^n_\lambda + \frac{1}{3}f_{mnp}A^m_\mu A^n_\nu A^p_\lambda\right) \\
& +\frac{1}{2}\omega_{AB}\left(-\epsilon^{\alpha\beta}Dq^A_\alpha Dq^B_\beta + i\epsilon^{\dot\alpha\dot\beta}\psi^A_{\dot\alpha}\not{D}\psi^B_{\dot\beta}\right) + \frac{1}{2}\tilde\omega_{AB}\left(-\epsilon^{\dot\alpha\dot\beta}D\tilde q^A_{\dot\alpha}D\tilde q^B_{\dot\beta} + i\epsilon^{\alpha\beta}\tilde\psi^A_\alpha\not{D}\tilde\psi^B_\beta\right) \\
& -i\pi k_{mn}\epsilon^{\alpha\beta}\epsilon^{\dot\gamma\dot\delta}J^m_{\alpha\dot\gamma}J^n_{\beta\dot\delta} - i\pi k_{mn}\epsilon^{\dot\alpha\dot\beta}\epsilon^{\gamma\delta}\tilde J^m_{\dot\alpha\gamma}\tilde J^n_{\dot\beta\delta} + 4\pi i k_{mn}\epsilon^{\alpha\gamma}\epsilon^{\dot\beta\dot\delta}J^m_{\alpha\dot\beta}\tilde J^n_{\dot\delta\gamma} \\
& +i\pi k_{mn}\left(\epsilon^{\dot\alpha\dot\gamma}\epsilon^{\dot\beta\dot\delta}\tilde\mu^m_{\dot\alpha\dot\beta}\psi^A_{\dot\gamma}t^n_{AB}\psi^B_{\dot\delta} + \epsilon^{\alpha\gamma}\epsilon^{\beta\delta}\mu^m_{\alpha\beta}\tilde\psi^A_\gamma\tilde t^n_{AB}\tilde\psi^B_\delta\right) \\
& -\frac{\pi^2}{6}f_{mnp}(\mu^m)^\alpha{}_\beta(\mu^n)^\beta{}_\gamma(\mu^p)^\gamma{}_\alpha - \frac{\pi^2}{6}f_{mnp}(\tilde\mu^m)^\alpha{}_\beta(\tilde\mu^n)^\beta{}_\gamma(\tilde\mu^p)^\gamma{}_\alpha \\
& +\pi^2(\tilde\mu^{mn})^{\dot\gamma}{}_{\dot\gamma}(\mu_m)^\alpha{}_\beta(\mu_n)^\beta{}_\alpha + \pi^2(\mu^{mn})^\gamma{}_\gamma(\tilde\mu_m)^{\dot\alpha}{}_{\dot\beta}(\tilde\mu_n)^{\dot\beta}{}_{\dot\alpha},
\end{aligned}
$$

$$
\begin{aligned}
& \delta q^A_\alpha = +i\eta_\alpha{}^{\dot\alpha}\psi^A_{\dot\alpha}, \quad \delta\tilde q^A_{\dot\alpha} = -i\eta^\alpha{}_{\dot\alpha}\tilde\psi^A_\alpha, \quad \delta A^m_\mu = 2\pi i\eta^{\alpha\dot\alpha}\gamma_\mu(J^m_{\alpha\dot\alpha} - \tilde J^m_{\dot\alpha\alpha}), \\
& \delta\psi^A_{\dot\alpha} = +\left[Dq^A_\alpha + \frac{2\pi}{3}(t_m)^A{}_B q^B_\beta(\mu^m)^\beta{}_\alpha\right]\eta^\alpha{}_{\dot\alpha} - 2\pi(t_m)^A{}_B q^B_\beta(\tilde\mu^m)^{\dot\beta}{}_{\dot\alpha}\eta^\beta{}_{\dot\beta}, \\
& \delta\tilde\psi^A_\alpha = -\left[D\tilde q^A_{\dot\alpha} + \frac{2\pi}{3}(\tilde t_m)^A{}_B\tilde q^B_{\dot\beta}(\tilde\mu^m)^{\dot\beta}{}_{\dot\alpha}\right]\eta_\alpha{}^{\dot\alpha} + 2\pi(\tilde t_m)^A{}_B\tilde q^B_{\dot\beta}(\mu^m)^\beta{}_\alpha\eta_\beta{}^{\dot\beta}.
\end{aligned}
$$

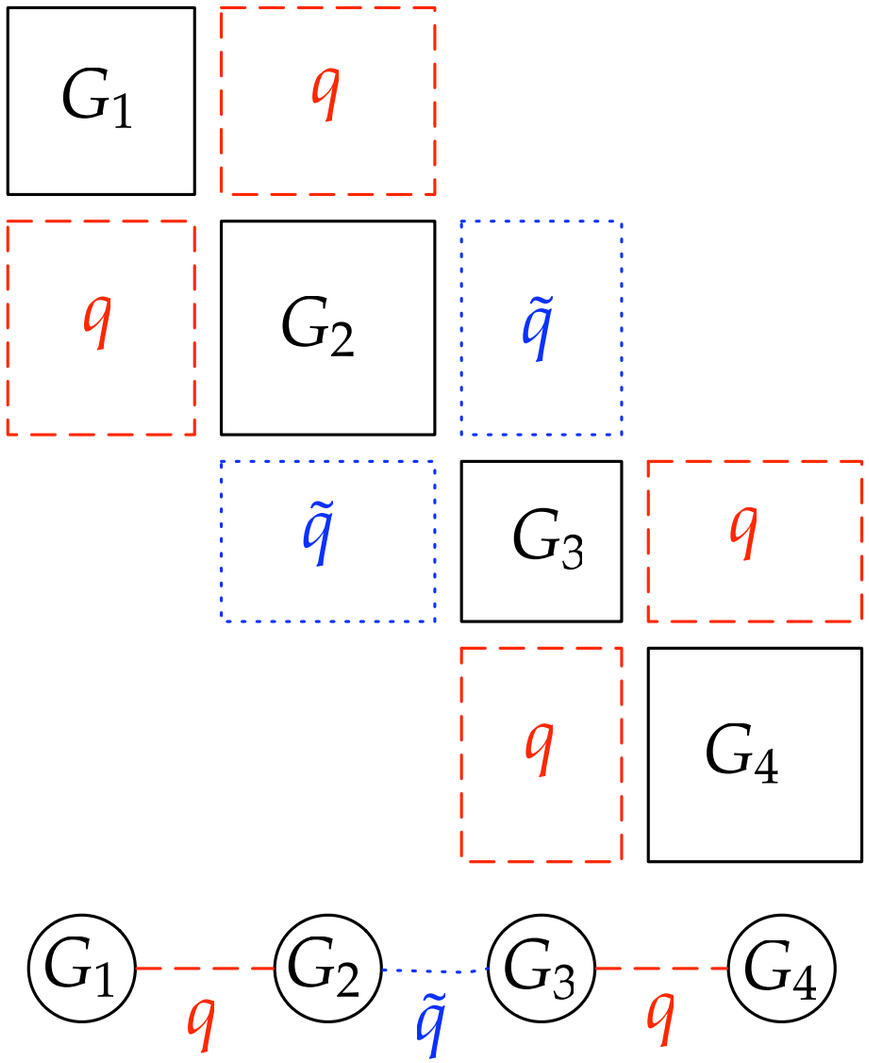
Classification in terms of quivers

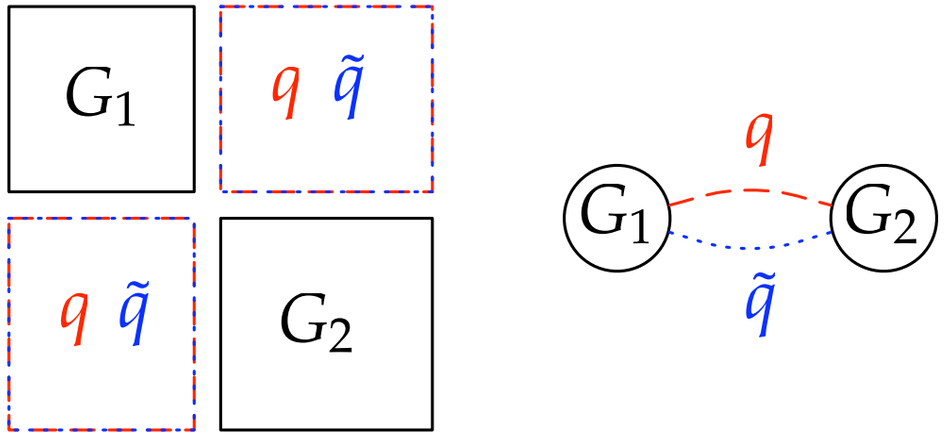
Hypers and twisted hypers should independently form super-algebra structure.

Generically, they form quivers with the two types of hypers alternating between gauge groups.

The quiver can either have open ends or form a closed loop.

Short loops often exhibit enhanced SUSY.

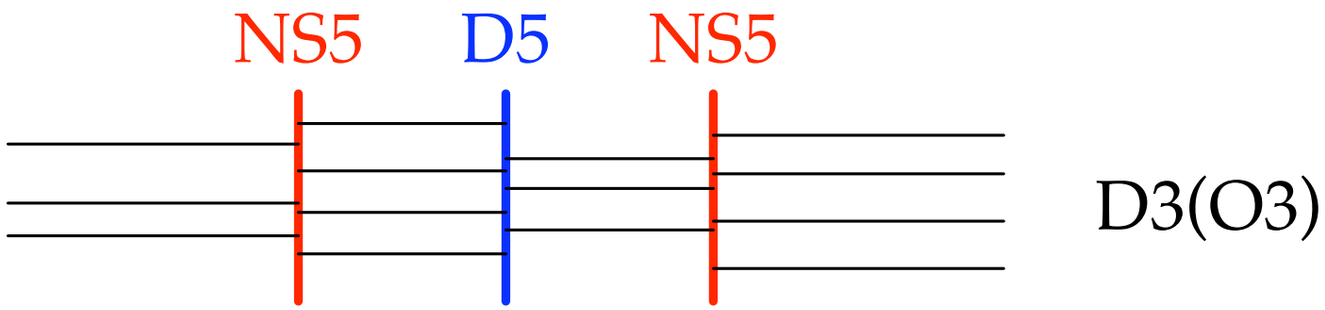




Embedding in String/M-theory

IIB : Janus configuration

	0	1	2	3	4	5	6	7	8	9
D3(O3)	○	○	○	○						
NS5	○	○	○		○	○	○			
D5	○	○	○					○	○	○



"T-dual" !

M : Replace $(C^2/Z_n)^2$ by Taub-NUT$(Z_n)^2$

Bagger-Lambert model ($\mathcal{N}$ = 8)

Based on Lie-3-algebra $\left(f^{abcd}, h^{ab}\right)$ [Bagger-Lambert 06-07][Gustavsson 07]

Positive h^{ab} allows nothing but $f^{abcd} = \varepsilon^{abcd}$ "$SO(4)$" [Papadopoulos] [Gauntlett-Gutowski]

The SO(4) theory is an ordinary CS-matter theory [Raamsdonk]

$$SO(4) \text{ Bagger-Lambert} = \text{Extended } PSU(2|2)$$

From $\mathcal{N} = 8$ to $\mathcal{N} = 4$:

$$\begin{aligned} SO(8) &\supset SO(4)_1 \times SO(4)_2 \\ &\sim (SU(2)_L \times SU(2)_A) \times (SU(2)_B \times SU(2)_R) \end{aligned}$$

$$\begin{aligned} X &\to \tilde{q} : (\mathbf{2},\mathbf{2},\mathbf{1},\mathbf{1}) \oplus q : (\mathbf{1},\mathbf{1},\mathbf{2},\mathbf{2}), \\ \Psi &\to \tilde{\psi} : (\mathbf{2},\mathbf{1},\mathbf{2},\mathbf{1}) \oplus \psi : (\mathbf{1},\mathbf{2},\mathbf{1},\mathbf{2}), \\ \varepsilon &\to \eta : (\mathbf{2},\mathbf{1},\mathbf{1},\mathbf{2}) \oplus \tilde{\eta} : (\mathbf{1},\mathbf{2},\mathbf{2},\mathbf{1}). \end{aligned}$$

ABJM model ($\mathcal{N}$ = 6) [Aharony-Bergman-Jafferis-Maldacena 0806.1218]

$$\text{ABJM} = \text{Extended } U(N|M)$$

R-symmetry enhancement from $SU(2)$ x $SU(2)$ to $SU(4)$

Hypers and twisted hypers together form an $\mathcal{N} = 6$ multiplet.

When $M = N$, at CS level k, the theory is claimed to describe N M2-branes on C^4/Z_k orbifold.

A derivation of the field theory from M-IIB T-duality and brane construction is given.

Another orbifold ($\mathcal{N}$ = 5) [Hosomichi-Lee^3-Park work in progress]

Extended $OSp(N|M)$

R-symmetry enhancement from $SU(2)$ x $SU(2)$ to $USp(4)$

Hypers and twisted hypers together form an $\mathcal{N} = 5$ multiplet.

The theory is claimed to describe N M2-branes on D_k orbifolds.

A derivation of the field theory from M-IIB T-duality
and brane construction can be given.

Discussions

- Mass deformation preserving all SUSY is possible.

- Relation to "M-crystal model" [SL 06][S.Kim-SL-Sj.Lee-J.Park 07]
  (M2-brane counterpart of the brane tiling model)
  has been established in the abelian case.
  It will be useful to uncover a large class of new $\mathcal{N}=2$ theories.

- Physical properties of the new theories are under investigation.
  (moduli space of vacua, BPS states, instanton effects, etc.)

- Large N and solution to the $N^{3/2}$ problem?