New $\mathcal{N} \geq 4$ Superconformal Chern-Simons Theories

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[arXiv:0805.3662] + work in progress

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Super-Chern-Simons Theories

- Super-YM-CS cannot go beyond $\mathcal{N} = 3$. [Kao-K.Lee 92]

- CS without YM can go beyond $\mathcal{N} = 3$!
  --- useful for $\mathcal{N} = 8$ M2-brane theory? [Schwarz 04]

- Bagger-Lambert theory [Bagger-Lambert 06-07][Gustavsson 07]
  Super-conformal $\mathcal{N} = 8$ CS-matter based on a Lie-3-algebra.

- Gaiotto-Witten construction [Gaiotto-Witten 0804.2907]
  Large class of new $\mathcal{N} = 4$ super-conformal CS theories.
$\mathcal{N} = 4$ and twisted multiplets

$\mathcal{N} = 4$ super-algebra includes $SO(4) = SU(2)_L \times SU(2)_R$ R-symmetry

<table>
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<tr>
<th>(hyper)</th>
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<th>(ordinary)</th>
<th>$SU(2)_L$</th>
<th>$SU(2)_R$</th>
<th>(twisted)</th>
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<tr>
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<td>$\tilde{\psi}_\alpha$</td>
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<td>$\tilde{A}_\mu$</td>
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<td>$\chi_{a\dot{\alpha}}$</td>
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<tr>
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Gaiotto-Witten: most general $\mathcal{N} = 4$ with ordinary hypers only.

We add twisted hypers to Gaiotto-Witten.
Gaiotto-Witten construction [Gaiotto-Witten 0804.2907]

Key steps:

- Begin with $\mathcal{N} = 1$ SUSY theory with $SU(2)_{\text{diag}} \subset SU(2)_L \times SU(2)_R$ global symmetry.

- Adjust the $\mathcal{N} = 1$ super-potential to restore $SU(2)_L \times SU(2)_R$.

- The SO(4) R-symmetry together with $\mathcal{N} = 1$ SUSY generate a full $\mathcal{N} = 4$ structure.

- Rewrite the action and SUSY transformation rules in a manifestly $\mathcal{N} = 4$ covariant way.
Field content

Matter: \( q^A_{\alpha}, \psi^A_{\dot{\alpha}}, F^A_{\alpha} \) \( Sp(2n) \) with \( \omega_{AB} \)

reality condition: \( \bar{q}^\alpha_A = (q^A_\alpha)^\dagger = \epsilon^{\alpha\beta} \omega_{AB} q^B_\beta \)

Gauge group \( G \subset Sp(2n) \)

\( (t^m)^A_B \) satisfy \( [t^m, t^n] = f^{mn}_p t^p \), \( \text{Tr}(t^m t^n) = k^{mn} \)

\( t^m_{AB} \equiv \omega_{AC}(t^m)^C_B = t^m_{BA} \)

Gauge: \( (A_m)_\mu, \chi_m \)

Conditions for \( \mathcal{N} = 4 \) SUSY

\[ k_{mn} t^m_{(AB} t^n_{C)D} = 0 \quad (\text{"fundamental identity"}) \]
Classification in terms of Lie super-algebra

Fundamental identity implies existence of a Lie super-algebra:

\[ [M^m, M^n] = f^{mn}_p M^p, \quad [M^m, Q_A] = Q_B (t^m)^B_A, \quad \{Q_A, Q_B\} = t^m_{AB} M_m. \]

\[ \{\{Q_A, Q_B\}, Q_C\} + \text{(cyclic)} = 0 \iff k_{mn} t^m_{(AB} t^n_{C)D} = 0 \]

Lie super-algebras classifies Gaiotto-Witten theories completely!

Typical examples: \( U(N|M), OSp(N|M) \)
\[ \mathcal{L} = \frac{\varepsilon^{\mu\nu\lambda}}{4\pi} \left( k_{mn} A^m_\mu \partial_\nu A^n_\lambda + \frac{1}{3} f_{mnp} A^m_\mu A^n_\nu A^p_\lambda \right) \\
+ \frac{1}{2} \omega_{AB} \left( -\varepsilon^{\alpha\beta} Dq^A_\alpha Dq^B_\beta + i\varepsilon^{\hat{\alpha}\hat{\beta}} \psi^A_{\hat{\alpha}} \mathcal{D} \psi^B_{\hat{\beta}} \right) \\
- i\pi k_{mn} \varepsilon^{\alpha\beta} \varepsilon^{\gamma\delta} j^m_{\alpha\gamma} j^n_{\beta\delta} - \frac{\pi^2}{6} f_{mnp} (\mu^m)^\alpha_\beta (\mu^n)^\beta_\gamma (\mu^p)^\gamma_\alpha, \]

\[ \delta q^A_\alpha = i\eta_\alpha \dot{\psi}^A_{\hat{\alpha}}, \quad \delta A^m_\mu = 2\pi i\eta^{\alpha\hat{\alpha}} (\mu^m)^\gamma_\alpha \gamma_{\mu j}^m_{\alpha\hat{\alpha}}, \]

\[ \delta \psi^A_{\hat{\alpha}} = \left[ \mathcal{D} q^A_\alpha + \frac{2\pi}{3} (t^A_B)_{\beta} (\mu^m)^\beta_\alpha \right] \eta^{\alpha\hat{\alpha}}. \]

\[ \mu^m_{\alpha\beta} \equiv t^m_{AB} q^A_\alpha q^B_\beta, \quad j^m_{\alpha\gamma} \equiv q^A_\alpha t^m_{AC} \psi^C_{\gamma} \]

(moment map multiplet)
Adding Twisted Hypers [Hosomichi-Lee³-Park 0805.3662]

Matter : \((q^A_\alpha, \psi^A_\dot{\alpha}, F^A_\dot{\alpha})\), \[ \mu^m_{\alpha\beta} \equiv t^m_{AB} q^A_\alpha q^B_\beta, \quad j^m_{\alpha\dot{\gamma}} \equiv q^A_\alpha t^m_{AC} \psi^C_\dot{\gamma} \]

\((\bar{q}^A_\dot{\alpha}, \bar{\psi}^A_\alpha, \bar{F}^A_\alpha)\), \[ \bar{\mu}^m_{\dot{\alpha}\dot{\beta}} \equiv \bar{t}^m_{AB} \bar{q}^A_\dot{\alpha} \bar{q}^B_\dot{\beta}, \quad \bar{j}^m_{\dot{\alpha}\dot{\alpha}} \equiv \bar{q}^A_\dot{\alpha} \bar{t}^m_{AB} \bar{\psi}^B_\alpha \]

Gauge symmetry is shared. ★

Super-algebra structure should still be maintained. ★

No additional constraint for \(\mathcal{N} = 4\) SUSY. ★
\[ \mathcal{L} = \frac{\varepsilon^{\mu \nu \lambda}}{4\pi} \left( k_{mn} A^m_{\mu} \partial_{\nu} A^n_{\lambda} + \frac{1}{3} f_{mnp} A^m_{\mu} A^n_{\nu} A^p_{\lambda} \right) \]

\[ + \frac{1}{2} \omega_{AB} \left( -\epsilon^{\alpha \beta} D q^A_{\alpha} D q^B_{\beta} + i \epsilon^{\hat{\alpha} \hat{\beta}} \psi^A_{\hat{\alpha}} D \psi^B_{\hat{\beta}} \right) + \frac{1}{2} \omega_{AB} \left( -\epsilon^{\hat{\alpha} \hat{\beta}} D \tilde{q}^A_{\hat{\alpha}} D \tilde{q}^B_{\hat{\beta}} + i \epsilon^{\alpha \beta} \tilde{\psi}^A_{\alpha} D \tilde{\psi}^B_{\beta} \right) \]

\[ - i \pi k_{mn} \epsilon^{\alpha \beta} \epsilon^{\gamma \delta} f_{\alpha \gamma}^{m} f_{\beta \delta}^{n} - i \pi k_{mn} \epsilon^{\hat{\alpha} \hat{\beta}} \epsilon^{\gamma \delta} \tilde{f}_{\alpha \gamma}^{m} \tilde{f}_{\beta \delta}^{n} + 4 \pi i k_{mn} \epsilon^{\alpha \gamma} \epsilon^{\beta \delta} f_{m n}^{\mu} \eta^{\mu}_{\alpha \beta \gamma \delta} \]

\[ + i \pi k_{mn} \left( \epsilon^{\hat{\alpha} \hat{\gamma}} \epsilon^{\hat{\beta} \hat{\delta}} \tilde{m}_{\hat{\alpha} \hat{\beta}} \psi^A_{\hat{\gamma}} t_{\hat{\alpha} \hat{\beta}}^{n} \psi^B_{\hat{\delta}} + \epsilon^{\alpha \gamma} \epsilon^{\beta \delta} \mu_{\alpha \beta} \tilde{\psi}^A_{\gamma} \tilde{t}_{\hat{\alpha} \hat{\beta}}^{n} \tilde{\psi}^B_{\delta} \right) \]

\[ - \frac{\pi^2}{6} f_{mnp} (\mu^m)_{\alpha}^{\beta} (\mu^n)_{\gamma}^{\beta} (\mu^p)_{\gamma}^{\alpha} - \frac{\pi^2}{6} f_{mnp} (\tilde{\mu}^m)_{\alpha}^{\hat{\beta}} (\tilde{\mu}^n)_{\gamma}^{\hat{\beta}} (\tilde{\mu}^p)_{\gamma}^{\alpha} \]

\[ + \pi^2 (\tilde{\mu}^{mn})_{\hat{\gamma}} (\mu_m)_{\alpha}^{\beta} (\mu_n)_{\beta}^{\alpha} + \pi^2 (\mu^{mn})_{\gamma} (\tilde{\mu}_m)_{\hat{\alpha}}^{\hat{\beta}} (\tilde{\mu}_n)_{\hat{\beta}}^{\hat{\alpha}} \]

\[ \delta q^A_{\alpha} = + i \eta_{\alpha}^{\hat{\alpha}} \psi^A_{\hat{\alpha}} \quad \delta \tilde{q}^A_{\hat{\alpha}} = - i \eta^{\alpha \hat{\alpha}} \tilde{\psi}^A_{\hat{\alpha}} \quad \delta A^m_{\mu} = 2 \pi i \eta^{\alpha \hat{\alpha}} \gamma_{\mu} (j^m_{\alpha \hat{\alpha}} - j^m_{\hat{\alpha} \alpha}) \]

\[ \delta \psi^A_{\hat{\alpha}} = + \left[ D q^A_{\alpha} + \frac{2 \pi}{3} (t_m)^A_{B} q^B_{\beta} (\mu^m)_{\beta}^{\alpha} \right] \eta^{\alpha \hat{\alpha}} - 2 \pi (t_m)^A_{B} q^B_{\beta} (\tilde{\mu}^m)_{\hat{\beta}}^{\alpha} \eta^{\beta \hat{\beta}} \]

\[ \delta \tilde{\psi}^A_{\hat{\alpha}} = - \left[ D \tilde{q}^A_{\hat{\alpha}} + \frac{2 \pi}{3} (\tilde{t}_m)^A_{B} \tilde{q}^B_{\hat{\beta}} (\tilde{\mu}^m)_{\hat{\beta}}^{\hat{\alpha}} \right] \eta_{\alpha \hat{\alpha}} + 2 \pi (\tilde{t}_m)^A_{B} \tilde{q}^B_{\hat{\beta}} (\mu^m)_{\beta}^{\alpha} \eta^{\beta \hat{\beta}} \]
Classification in terms of quivers

Hypers and twisted hypers should independently form super-algebra structure.

Generically, they form quivers with the two types of hypers alternating between gauge groups.

The quiver can either have open ends or form a closed loop.

Short loops often exhibit enhanced SUSY.
Embedding in String/M-theory

IIB : Janus configuration

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NS5    D5    NS5

D3(O3)


M : Replace \((C^2/Z_n)^2\) by Taub-NUT\((Z_n)^2\)

“T-dual”!
Bagger-Lambert model ($\mathcal{N} = 8$)

Based on Lie-3-algebra $\left( f^{abcd}, h^{ab} \right)$ [Bagger-Lambert 06-07][Gustavsson 07]

Positive $h^{ab}$ allows nothing but $f^{abcd} = \varepsilon^{abcd} \text{ “SO}(4)”$ [Papadopoulos] [Gauntlett-Gutowski]

The SO(4) theory is an ordinary CS-matter theory [Raamsdonk]

$\text{SO}(4)$ Bagger-Lambert $= \text{Extended PSU}(2|2)$

From $\mathcal{N} = 8$ to $\mathcal{N} = 4$ :

\[
\begin{align*}
\text{SO}(8) \supset & \quad \text{SO}(4)_1 \times \text{SO}(4)_2 \\
\sim & \quad (\text{SU}(2)_L \times \text{SU}(2)_R) \times (\text{SU}(2)_A \times \text{SU}(2)_B)
\end{align*}
\]

$X \rightarrow \tilde{q} : (2,2,1,1) \oplus q : (1,1,2,2)$,

$\Psi \rightarrow \tilde{\psi} : (2,1,2,1) \oplus \psi : (1,2,1,2)$,

$\varepsilon \rightarrow \eta : (2,1,1,2) \oplus \bar{\eta} : (1,2,2,1)$. 

ABJM model ($\mathcal{N} = 6$) [Aharony-Bergman-Jafferis-Maldacena 0806.1218]

**ABJM = Extended $U(N|M)$**

R-symmetry enhancement from $SU(2) \times SU(2)$ to $SU(4)$

Hypers and twisted hypers together form an $\mathcal{N} = 6$ multiplet.

When $M = N$, at CS level $k$, the theory is claimed to describe $N$ M2-branes on $C^4/\mathbb{Z}_k$ orbifold.

A derivation of the field theory from M-IIB T-duality and brane construction is given.
Another orbifold ($\mathcal{N} = 5$) [Hosomichi-Lee$^3$-Park work in progress]

Extended $OSp(N \mid M)$

R-symmetry enhancement from $SU(2) \times SU(2)$ to $USp(4)$

Hypers and twisted hypers together form an $\mathcal{N} = 5$ multiplet.

The theory is claimed to describe $N$ M2-branes on $D_k$ orbifolds.

A derivation of the field theory from M-IIB T-duality and brane construction can be given.
Mass deformation preserving all SUSY is possible.

Relation to “M-crystal model” [SL 06][S.Kim-SL-Sj.Lee-J.Park 07] (M2-brane counterpart of the brane tiling model) has been established in the abelian case. It will be useful to uncover a large class of new $\mathcal{N}=2$ theories.

Physical properties of the new theories are under investigation. (moduli space of vacua, BPS states, instanton effects, etc.)

Large $N$ and solution to the $N^{3/2}$ problem?