APPARENT HORIZON IN
HOLOGRAPHIC DUAL OF
QUARK-GLUON
HYDRODYNAMICS

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What we do

◇ Apply AdS/CFT on rapidly expanding system: Quark-gluon plasma at RHIC and LHC. Precisely: 5D AdS BH/4D finite T SYM.

◇ Why we do?

◇ Outstandingly most perfect fluid for human.

◇ First possibility of dynamical (?) BH in AdS/CFT context.

◇ Q: Can one prove that such a time-dependent thermal system is really a BH?

◇ A: Yes we can!
Introduction
RHIC fireball

http://www.bnl.gov/RHIC/heavy_ion.htm
Most perfect fluid ever

* Experiment suggests that the system saturates theoretical lower bound on viscosity.
  Adare et al. 06

* **Most perfect** fluid for human. Even less viscous than liquid helium.

* Extremely low viscosity:
  * Created particles are **strongly coupled**. We cannot rely on perturbative QCD.

* Why not apply AdS/CFT?
  * Need a **non-static** generalization of AdS/CFT.
Predictions from AdS

* Janik & Peschanski (PRD, 06) have shown that,
  * if one assumes that dual 5D metric does not have singularity at (would-be) horizon.
  * viscosity (to entropy ratio) does take the lowest possible value $\eta/s = 1/4\pi$.
* Other quantities too, determined by micro. (cf. $T_H$)
* Very nice!! However...
  * First of all, is such rapidly expanding system really a BH? Really horizon, not mere a coord. sing.?
  * Why want to avoid singularity?
Problem

* The (would-be) horizon is **moving** fast. Is it really a BH? (Cf. Dual plasma expands with speed of light.)

* **Fefferman-Graham**-like metric (employed by Janik & Peschanski) covers only **outside** horizon.

  * The metric is not proper to discuss the **very existence** of the horizon

  * and hence the **regularity** at the (would-be) horizon.

  * Want a coordinate that **penetrates** through the horizon.
Our achievement

* We propose to use Eddington-Finkelstein-like metric that can penetrate through EH.

* We have proven the existence of horizon in this time-dependent 5D spacetime (dual to the QGP expanding by speed of light).

* Dual 5D computation now on solid ground!
Setup
AdS/CFT correspondence

observer outside

Flat spacetime

FF limit

Black 3-brane metric

NH limit

“Spatial infinity” of $\text{AdS}_5$ ~ UV boundary of 4D theory

Open strings on 3-brane in flat spacetime

4D gauge theory (strongly coupled)

Closed strings in $\text{AdS}_5$

5D gravity (weakly coupled)
4D side
RHIC fireball again

http://www.bnl.gov/RHIC/heavy-ion.htm
Bjorken flow

- Boost symmetric one dimensional expansion
- Write in terms of proper time $\tau$ and rapidity $y$

$$ds_4^2 = -d\tau^2 + \tau^2 dy^2 + d\vec{x}_\perp^2$$

(Bjorken 1983)
4D energy-momentum

* Recent development in (2nd order) relativistic hydrodynamics (arXiv:0712...) by
  * Baier, Romatschke, Son, Starinets, & Stephanov
  * Bhattacharyya, Hubeny, Minwalla, & Rangamani

* 4D energy momentum tensor given in terms of
  * energy $E$, pressure $P$ (with state equation $E=3P$),
  * viscosity $\eta$, relaxation time $\tau_{\pi}$,
  * and another transport coefficient $\lambda_1$. 
5D side
Holographic reconstruction

\[ ds^2 = r^2 \left( -A(r, \tau) d\tau^2 + 2 d\tau dr + \tau^2 B(r, \tau) dy^2 + C(r, \tau) d\vec{x}_\perp^2 \right) \]

- We propose Eddington-Finkelstein-like metric (w/o coordinate singularity at horizon)
- Solve 5D bulk (vacuum) Einstein equation in late-time expansions by \( \tau^{-2/3} \).
- Boundary conditions at spatial infinity \( r \to \infty \):
  - \( g_{\mu \nu} \to r^{-2} \text{diag}(-1, \tau^2, 1, 1) \)
  - Extrinsic curvature \( \delta S / \delta g_{\mu \nu} \sim r^2 T^{(4)}_{\mu \nu} \)
**Apparent horizon**

\[ l_-: \text{inward} \quad l_+: \text{outward} \]

Expansion along double null direction:

\[ \theta \pm \equiv L\pm[\text{vol}(\perp)] \]

* Product of expansions: \( \Theta = \theta + \theta^- \)
  
  * \( \Theta \leq 0 \): ordinary situation
  
  * \( \Theta > 0 \): both null directions are shrinking (\(~\text{inside horizon}\) )
  
  * \( \Theta = 0 \): apparent horizon

* Thm: There exists event horizon outside AH.
Our result

\[ \Theta = -\frac{9}{2} \frac{u^4 - w^4}{u^2} + O(\tau^{-2/3}) \]

(We have explicitly computed up to \(O(\tau^{-2})\).)

\[ O(\tau^{-2/3}) = \frac{w^4}{u^5} \times \text{const. can be absorbed by} \]

\[ r \rightarrow r + \frac{\text{const.}}{\tau} \]

- \(w \sim \) initial temperature
- Certainly \(\Theta\) becomes positive for \(u < w\).
- Horizon exists and can cover **singularity**!
- Dual computation now on **solid** ground: We got
  - Energy density, viscosity, relaxation time, etc.
Summary & discussion

* **QGP** at heavy ion collider has turned out to be:

  * least viscous (most strongly coupled) liquid that human being has ever observed.

* **5D BH** as its dual description:

  * We, for the first time, have **proven** that it is **really a BH** in this **time dependent** system.

* There remains ambiguity for AH due to \( r \rightarrow r + \frac{\text{const.}}{\tau} \)

* Where is the event horizon?

* BH thermodynamics? How to show \( dM = TdA \)?
Message

* Now the setup is put on firm ground:

  * It **is** a BH with (seemingly) moving horizon.
  * It’s a topic directly connected to **experiment:** RHIC&LHC.

  * **String theory, general relativity, and nuclear physics** are tightly intertwined in this field.

  * Especially **BH** has been a bridge that connects **gravity** and **quantum mechanics.**

* Let us discuss/work together!