M-theory on pp-waves with a holomorphic superpotential and its matrix description

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Matrix theory conjecture

- The discovery of D-branes in string theory among other things suggests that Yang-Mills theory can give a nonperturbative formulation of Quantum Gravity.
- Concrete examples are Matrix theory : BFSS for M-theory, Matrix strings for IIA, IKKT for IIB etc. item AdS/CFT correspondence is the most elaborate and well-studied : 4d SCFT and Strings in Curved spacetime.

Examples and Constructions of Matrix Theory

- Take M-theory in 11d, compactify on a circle to get IIA, and give infinite boost along x^{11}.
  - Infinite momentum becomes infinite number of D0-branes in IIA, and the conjecture is that the D0-brane dynamics in the large N limit gives an M-theory description.
  - Alternatively one can consider supermembrane action choose light-cone gauge and discretize the Poisson bracket as matrix commutator.
- The result is maximally supersymmetric $U(N)$ Yang-Mills quantum mechanics.
- Successfully describes M2, M5, scattering amplitudes etc.

Pp-waves

- Unfortunately the Matrix theory conjecture is very much background-dependent but can be applied to any pp-waves.
- Generalized to max. susy pp-wave by Berenstein, Maldacena, Nastase (JHEP 0204, 013 2002) : Shows several attractive features like possibility of perturbative computation of energy spectrum, transverse M5-branes etc.

Inhomogeneous pp-waves

- We are here interested in pp-waves with non-constant flux.

$$ds^2 = 2dx^+dx^- + H(x^+, x^M)(dx^+)^2 + \sum_{M=1}^{9}(dx^M)^2$$

$$G = dx^+ \wedge \phi(x^+, x^M)$$

- Einstein equation becomes $\nabla^2 H = -\phi^2/6$. ϕ is closed and co-closed.
- We implement 6+3 split of transverse space, and allow coordinate dependence on 6d only. (which is to be complexified.)

Pp-wave with holomorphic superpotential

- All the equations are satisfied if we set

$$\phi_{\bar{a}bc} = \partial_{\bar{a}} \partial_{\bar{d}} \bar{W} \epsilon^{\bar{d}}{}_{bc}, \quad H = -|\partial W|^2$$

- It can be verified that there exist in general 4 kinematical and 4 dynamical susy.
- SUSY projection : $\Gamma_{12}\epsilon = \Gamma_{34}\epsilon = \Gamma_{56}\epsilon$: 1/4-BPS

Supermembrane action

Use the 1st order formulation and choose the light-cone gauge, then we have - in terms of Poisson bracket.

$$\mathcal{L} = \frac{1}{2}(D_t X_M)^2 + \frac{1}{2}H + \frac{1}{2}C_{+rs}\{X^r, X^s\} - \frac{1}{4}\{X^M, X^N\}^2 + \text{fermions}$$

Now use the form of our solution, and express in terms of 3 complex coordinates instead of 6 real (3 remains the same)

$$\begin{aligned}\mathcal{L} \;=\; & \frac{1}{2}(D_t X_M)^2 - \frac{1}{4}\{X^M, X^N\}\{X_M, X_N\} + i\theta^\dagger D_t\theta + i\theta^\dagger\Gamma_M\{X^M, \theta\} \\ & -\frac{1}{2}\partial_a W \partial_{\bar{a}}\bar{W} + \frac{1}{2}\epsilon_{dab}\partial_{\bar{d}}\bar{W}\{Z^a, Z^b\} + \frac{1}{2}\epsilon_{\bar{d}\bar{a}\bar{b}}\partial_d \bar{W}\{Z^{\bar{a}}, Z^{\bar{b}}\} \\ & -\frac{i}{8}\epsilon_{\bar{d}\bar{a}\bar{b}}\partial_c\partial_d W\theta^\dagger\Gamma_{\bar{c}ab}\theta - \frac{i}{8}\epsilon_{dab}\partial_{\bar{c}}\partial_{\bar{d}} W\theta^\dagger\Gamma_{c\bar{a}\bar{b}}\theta\end{aligned}$$

Discretization and Gauge Field theory

- Now we can follow the standard discretization procedure and replay the coordinates by $N \times N$ matrices. Poisson bracket becomes matrix commutators.
- The 6+3 split, $SO(3)$ symmetry, and the solution being described in terms of a holomorphic function, all suggest that things are naturally related to $N = 1$ supersymmetric theory in 3+1 dimensions.
- It is indeed quite convenient to view the action as a dimensional reduction of $D = 4$, $N = 1$ theory to quantum mechanics.

Matrix Model Action

$$\mathcal{L} = \mathrm{Tr}\Big(\tfrac{1}{2}D_tX^MD_tX_M + \tfrac{1}{4}[X^M,X^N][X_M,X_N] + i\theta^\dagger D_t\theta - \theta^\dagger\Gamma^M[X_M,\theta]\Big)$$

$$+\tfrac{1}{2}\,\mathrm{Tr}\Big(i\epsilon_{\bar{a}\bar{b}}{}^{c}[\bar{Z}^{\bar{a}},\bar{Z}^{\bar{b}}]\partial_c W + i\epsilon_{ab}{}^{\bar{c}}[Z^a,Z^b]\bar{\partial}_{\bar{c}}\overline{W} - \partial_a W\partial^a\overline{W}\Big)$$

$$-\tfrac{i}{8}\,\mathrm{Tr}\Big(\theta^\dagger\Gamma^a\Gamma^{\bar{1}\bar{2}\bar{3}}\Gamma^b\partial_a\mathrm{Tr}\left(\theta\partial_b W\right) + \theta^\dagger\Gamma^{\bar{a}}\Gamma^{123}\Gamma^{\bar{b}}\bar{\partial}_{\bar{a}}\mathrm{Tr}\left(\theta\bar{\partial}_{\bar{b}}\overline{W}\right)\Big)$$

- Goes over to the membrane action in $N \to \infty$ limit.
- W is now (multi)-trace, holomorphic operator written using 3 complex matrices.
- Exists ambiguity to get a (matrix) W from supergravity W.
- We can use $\mathcal{W} = W - \frac{i}{3}\epsilon_{abc}\mathrm{Tr}\left(Z^a,[Z^b,Z^c]\right)$ for the whole superpotential.

Discussions

- Obtained an interesting class of inhomogeneous pp-waves in 11d and found the matrix action.
- The relation between 11d pp-wave and gauge theory turns out to be very rich.
- Might concentrate on more interesting backgrounds :
  β-deformation of BFSS (addressed by H. Shimada)
- One can try to generalize to other dimensions. (7+2 or 8+1 split).