$N = 2$ SUSY QED and nonlinear/linear SUSY relation

Kazunari Shima, Motomu Tsuda°
(Saitama Institute of Technology)

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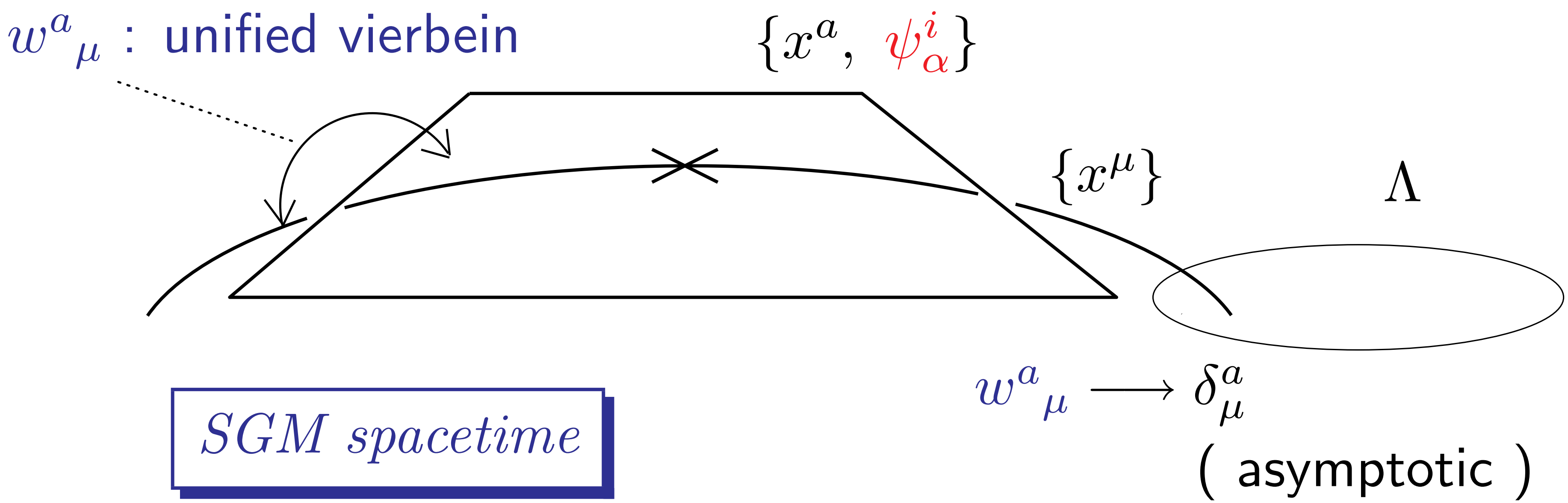
1. Nonlinear Supersymmetric General Relativity (NLSUSY GR)

NLSUSY GR action

is defined in *SGM spacetime*, where the tangent spacetime is specified not only $SO(3,1)$ Minkowski coordinates x^a but also

$SL(2,C)$ Grassmanian coordinates ψ_α^i

$\sim$ as the coset parameters of $\frac{superGL(4R)}{GL(4R)}$.



NLSUSY GR action is given by **the Einstein-Hilbert (EH) form**,

$$L_{\rm NLSUSY\ GR}(w) = \frac{c^4}{16\pi G}|w|(\ \Omega(w) - \Lambda\ ), \tag{1}$$

where

$$|w| = \det w^a{}_\mu = \det\{e^a{}_\mu + t^a{}_\mu(\psi^i)\}, \quad t^a{}_\mu(\psi^i) = \frac{\kappa^2}{2i}(\bar\psi^i\gamma^a\partial_\mu\psi^i - \partial_\mu\bar\psi^i\gamma^a\psi^i),$$
$$\Omega(w)\ \text{... the scalar curvature in terms of}\ w^a{}_\mu. \tag{2}$$

In $L_{\rm NLSUSY\ GR}$

- ψ^i_α can be interpreted as Nambu-Goldstone (NG) fermions associated with the spontaneous breaking of super-$GL(4R)$ down to $GL(4R)$.
- SUSY is broken spontaneously from the beginning due to NLSUSY structure of spacetime (as explaines after).

Symmetries in NLSUSY GR

$L_{\rm NLSUSY\ GR}$ possesses promising large symmetries isomorphic to

$SO(N)$ $(SO(10))$ SP group

through an invariance under the following NLSUSY transformations,

$$\delta_\zeta \psi^i = \frac{1}{\kappa}\zeta^i - i\kappa\bar{\zeta}^j\gamma^\mu\psi^j\partial_\mu\psi^i, \quad \delta_\zeta e^a{}_\mu = 2i\kappa\bar{\zeta}^i\gamma^\rho\psi^i\partial_{[\mu}e^a{}_{\rho]}, \tag{3}$$

and a (generalized) local Lorentz invariance, etc.

Therefore, the no-go theorem is overcome (circumvented) in a sense that

" the nontivial N-extended SUSY gravity theory with $N > 8$ "

has been constructed in the NLSUSY invariant way.

NLSUSY GR action in Riemann spacetime

{ SGM spacetime }

is unstable due to the NLSUSY structure of spacetime,
and decays spontaneously (called *"Big Decay"* of spacetime) to

{ *Riemann spacetime* $\oplus$ matter } - system

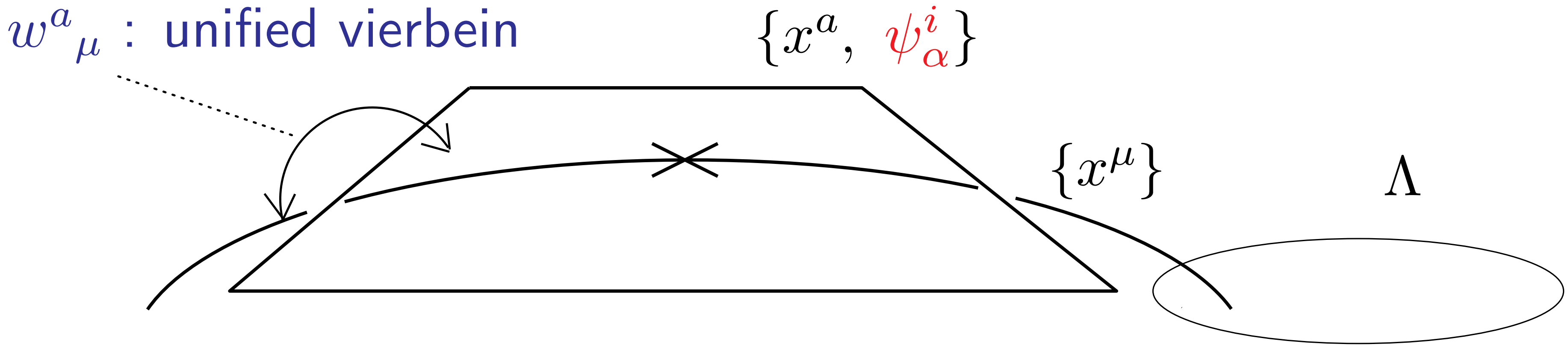
described by the EH action coupled with NG-fermion (superon) matter.
Then, the action called the SGM action expanded as follows;

$$L_{\rm SGM}(e, \psi^i) = \frac{c^4}{16\pi G}|e|\{R(e) - \Lambda + \tilde{T}(e, \psi^i)\}, \tag{4}$$

where

$R(e)$ ... the scalar curvature of ordinary EH action,

$\tilde{T}(e, \psi^i)$ ... the kinetic term and the gravitational interactions of ψ^i. (5)

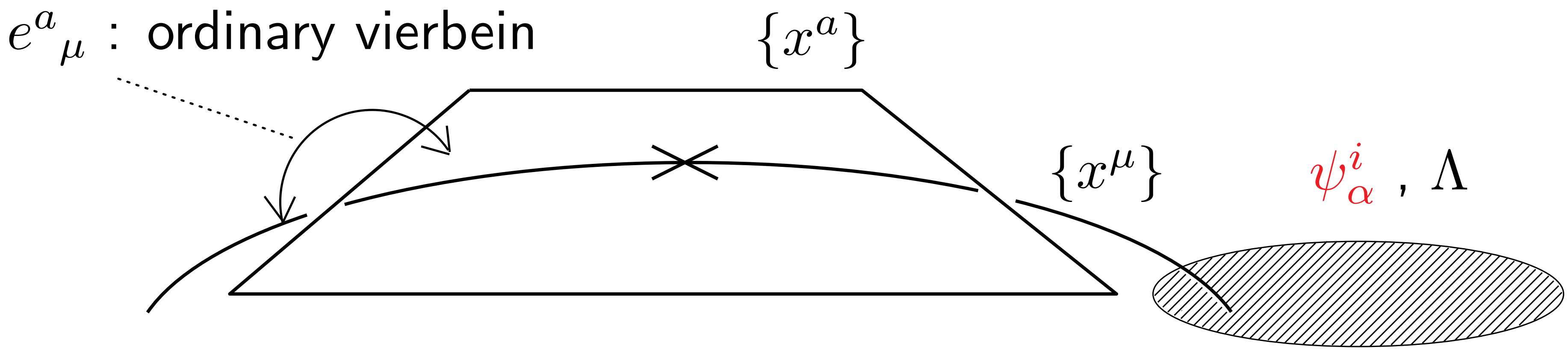


$w^a{}_\mu \longrightarrow \delta^a_\mu$

(asymptotic)

SGM spacetime

Spontaneous SUSY Breaking (*Big Decay*)



$e^a{}_\mu \longrightarrow \delta^a_\mu$

(asymptotic)

Riemann spacetime $\oplus$ matter

SGM (NLSUSY GR) action in Riemann-flat spacetime

In *the asymptotic region (local frame in the Riemann-flat* ${e^a}_\mu \longrightarrow \delta^a_\mu$*)*,

$L_{\rm SGM}(e,\psi^i)$ ($\sim$ *corresponding to* [**cosm. const.** $\mathbf{\Lambda}$ + $\tilde{\mathbf{T}}(\mathbf{e},\psi^{\mathbf{i}})$] terms)

$$
\begin{aligned}
\longrightarrow \quad L_{\rm NLSUSY}(\psi^i) &= -\frac{1}{2\kappa^2}|w| \ (\Leftrightarrow \textbf{spacetime volume form}) + \cdots \\
&= -\frac{1}{2\kappa^2}\left(1 \ (\Leftrightarrow \textbf{b.g. energy density}) + {t^a}_a \ (\Leftrightarrow \textbf{kin.}) + \frac{1}{2!}{t^a}_a{t^b}_b + \cdots\right) + \cdots \\
&= -\frac{1}{2\kappa^2}\left(1 - i\kappa^2\bar{\psi}^i\partial\!\!\!/\psi^i - \frac{1}{2}\kappa^4\bar{\psi}^i\partial\!\!\!/\psi^i\bar{\psi}^j\partial\!\!\!/\psi^j + \cdots\right) + \cdots,
\end{aligned}
\tag{6}
$$

where

$$
\kappa^2 = \left(\frac{c^4\Lambda}{8\pi G}\right)^{-1}. \tag{7}
$$

$\Longrightarrow$ Low energy physics for the NLSUSY model $L_{\rm NLSUSY}(\psi^i)$?

2. Cosmology and low energy particle physics in NLSUSY GR

In order to study the low energy particle physics in NLSUSY GR,

NL/L SUSY relation

is very important.

Essential points in NL/L SUSY relation

(1) Component fields φ^I in LSUSY theories are uniquely represented as composites of ψ^i in a SUSY invariant way.

$\Longrightarrow$ SUSY invariant relations

Then,

(2) the NLSUSY action ($\sim$ the massless theory with the NG fermions) is related to the ($interacting$) LSUSY actions.

As a significant example of the NL/L SUSY relation,
we consider SUSY QED

for $\underline{N = 2 \text{ SUSY}}$ (realistic in the SGM scenario)

and in $\underline{d = 2}$ (for simplicity).

$N = 2$ SUSY QED from $N = 2$ NL/L SUSY relation

(1) $\sim$ SUSY invariant relations

As for $N = 2$ vector supermultiplet which has the following multiplet's structure,

$$\begin{pmatrix} +1 \\ +\frac{1}{2}, \ +\frac{1}{2} \\ 0 \end{pmatrix} \Leftrightarrow \begin{pmatrix} v^a \\ \lambda^i \\ A, \phi \end{pmatrix} + [\ \text{auxiliary fields } (D, \cdots)\ ],$$

the SUSY invariant relations are written as the composite forms of ψ^i in all orders as follows;

$$
\begin{aligned}
C &= -\frac{1}{8}\xi\kappa^3\bar{\psi}^i\psi^i\bar{\psi}^j\psi^j, \\
\Lambda^i &= -\frac{1}{2}\xi\kappa^2\psi^i\bar{\psi}^j\psi^j(1 - i\kappa^2\bar{\psi}^k\partial\!\!\!/\psi^k), \\
M^{ij} &= \frac{1}{2}\xi\kappa\bar{\psi}^i\psi^j\left(1 - i\kappa^2\bar{\psi}^k\partial\!\!\!/\psi^k - \frac{1}{2}\kappa^4\epsilon^{ab}\bar{\psi}^k\psi^l\partial_a\bar{\psi}^k\gamma_5\partial_b\psi^l\right), \\
\phi &= -\frac{1}{2}\xi\kappa\epsilon^{ij}\bar{\psi}^i\gamma_5\psi^j\left(1 - i\kappa^2\bar{\psi}^k\partial\!\!\!/\psi^k - \frac{1}{2}\kappa^4\epsilon^{ab}\bar{\psi}^k\gamma_5\psi^l\partial_a\bar{\psi}^k\partial_b\psi^l\right), \\
v^a &= -\frac{i}{2}\xi\kappa\epsilon^{ij}\bar{\psi}^i\gamma^a\psi^j(1 - i\kappa^2\bar{\psi}^k\partial\!\!\!/\psi^k), \\
\lambda^i &= \xi\psi^i|w|, \\
D &= \frac{\xi}{\kappa}|w|, \qquad (8)
\end{aligned}
$$

where $A = M^{ii}$.

Also, for $N = 2$ scalar supermultiplet for matter fields which has the following multiplet's structure,

$$\begin{pmatrix} +\frac{1}{2} \\ 0,\ \ 0 \\ -\frac{1}{2} \end{pmatrix} \Leftrightarrow \begin{pmatrix} \chi \\ B^i \\ \nu \end{pmatrix} + [\ \text{auxiliary fields } F^i\ ],$$

the SUSY invariant relations become

$$\begin{aligned} \chi &= \xi^i \left[\psi^i |w| + \frac{i}{2}\kappa^2 \partial_a \{ \gamma^a \psi^i \bar{\psi}^j \psi^j (1 - i\kappa^2 \bar{\psi}^k \partial\!\!\!/ \psi^k) \} \right], \\ B^i &= -\kappa \left(\frac{1}{2} \xi^i \bar{\psi}^j \psi^j - \xi^j \bar{\psi}^i \psi^j \right) |w|, \\ \nu &= \xi^i \epsilon^{ij} \left[\psi^j |w| + \frac{i}{2}\kappa^2 \partial_a \{ \gamma^a \psi^j \bar{\psi}^k \psi^k (1 - i\kappa^2 \bar{\psi}^l \partial\!\!\!/ \psi^l) \} \right], \\ F^i &= \frac{1}{\kappa} \xi^i \left\{ |w| + \frac{1}{8}\kappa^4 \partial_a \partial^a (\bar{\psi}^j \psi^j \bar{\psi}^k \psi^k) \right\} - i\kappa \xi^j \partial_a (\bar{\psi}^i \gamma^a \psi^j |w|). \end{aligned} \tag{9}$$

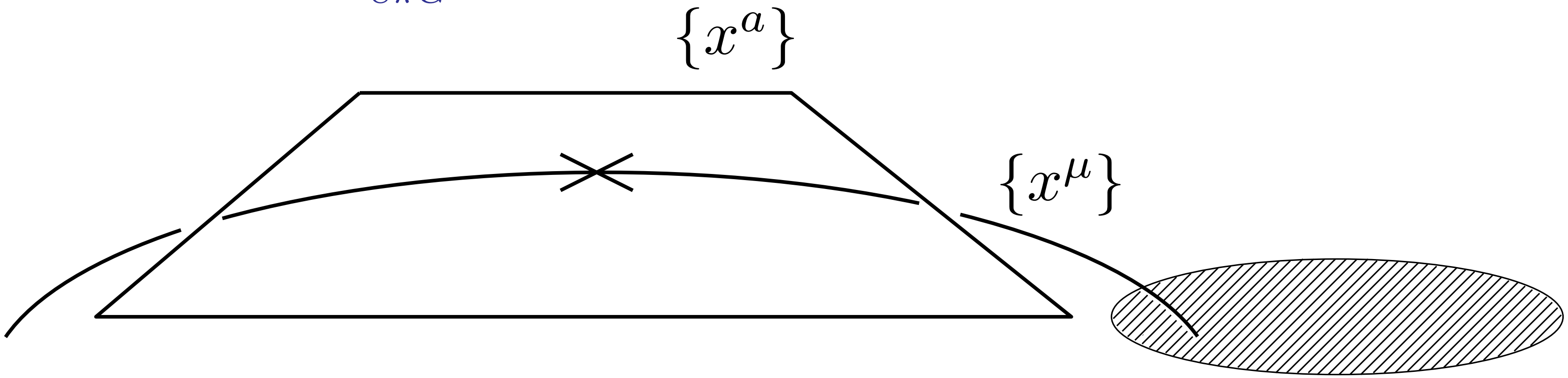
Then,
(2) the relation between $N=2$ NLSUSY action
and $N=2$ SUSY QED action becomes

$$
\begin{aligned}
L_{N=2\mathrm{NLSUSY}}(\psi^i) &= -\frac{1}{2\kappa^2}|w| + [\text{ tot. der. }] \\
&= L_{N=2\mathrm{SUSYQED}} \quad \textbf{with the SUSY invariant relations} \\
&= -\frac{1}{4}(F_{ab})^2 + \frac{i}{2}\bar{\lambda}^i \partial\!\!\!/ \lambda^i + \frac{1}{2}(\partial_a A)^2 + \frac{1}{2}(\partial_a \phi)^2 + \frac{1}{2}D^2 - \frac{\xi}{\kappa}D \qquad (\Leftrightarrow \mathcal{V}_{\mathrm{kin}}) \\
&\quad + \frac{i}{2}\bar{\chi}\partial\!\!\!/ \chi + \frac{1}{2}(\partial_a B^i)^2 + \frac{i}{2}\bar{\nu}\partial\!\!\!/ \nu + \frac{1}{2}(F^i)^2 \qquad (\Leftrightarrow \Phi^i_{\mathrm{kin}}) \\
&\quad + f(A\bar{\lambda}^i\lambda^i + \epsilon^{ij}\phi\bar{\lambda}^i\gamma_5\lambda^j - A^2 D + \phi^2 D + \epsilon^{ab}A\phi F_{ab}) \\
&\quad + e\Big\{ i v_a \bar{\chi}\gamma^a \nu - \epsilon^{ij} v^a B^i \partial_a B^j + \bar{\lambda}^i \chi B^i + \epsilon^{ij}\bar{\lambda}^i \nu B^j \\
&\quad - \frac{1}{2}D(B^i)^2 + \frac{1}{2}(\bar{\chi}\chi + \bar{\nu}\nu)A - \bar{\chi}\gamma_5 \nu\phi \Big\} + \frac{1}{2}e^2({v_a}^2 - A^2 - \phi^2)(B^i)^2 \\
&\quad + \cdots . \qquad (10)
\end{aligned}
$$

Cosmology and particle physics from NL/L SUSY relation

$$L_{\rm SGM}(e,\psi^i) \longrightarrow G_{\mu\nu}(e) = \frac{8\pi G}{c^4}\left\{\tilde{T}_{\mu\nu}(e,\psi^i) - g_{\mu\nu}\frac{c^4\Lambda}{8\pi G}\right\}$$

$$\longrightarrow \rho_D \sim \frac{c^4\Lambda}{8\pi G}$$



$$e^a{}_\mu \longrightarrow \delta^a_\mu$$

$$L_{N=2{\rm NLSUSY}}(\psi^i) + [\text{ tot. der. }] = L_{N=2{\rm SUSYQED}}(\varphi^I)$$

$$\longrightarrow {M_{\rm SUSY}}^2 \sim < D > \sim \sqrt{\frac{c^4\Lambda}{8\pi G}}$$

Physical vacuum with $U(1)$ ... stable and the lightest massive fermion $\lambda^i \Leftrightarrow \nu$

$$\longrightarrow {m_\nu}^2 \sim \sqrt{\frac{c^4\Lambda}{8\pi G}} \quad (\text{at } -f\xi \sim \mathcal{O}(1))$$

Therefore, SGM (NLSUSY GR)

predicts

the observed value of the (dark) energy density of the universe $\rho_D^{\rm obs}$

and

naturally explains

the mysterious numerical relations between m_ν and $\rho_D^{\rm obs}$:

$$\rho_D^{\rm obs} \sim (10^{-12} GeV)^4 \sim {m_\nu}^4. \tag{11}$$

3. Systematics of NL/L SUSY relation and $N=2$ SUSY QED

The SUSY invariant relations

$\Longrightarrow$ are systematically obtained in the superfield formulation.

Linearization of NLSUSY in the $d=2$ superfield formulation

- General superfields are given for the $N=2$ vector supermultiplet by

$$\begin{aligned}\mathcal{V}(x,\theta^i) \ =\ & C(x)+\bar\theta^i\Lambda^i(x)+\frac{1}{2}\bar\theta^i\theta^j M^{ij}(x)-\frac{1}{2}\bar\theta^i\theta^i M^{jj}(x)+\frac{1}{4}\epsilon^{ij}\bar\theta^i\gamma_5\theta^j\phi(x)\\ & -\frac{i}{4}\epsilon^{ij}\bar\theta^i\gamma_a\theta^j v^a(x)-\frac{1}{2}\bar\theta^i\theta^i\bar\theta^j\lambda^j(x)-\frac{1}{8}\bar\theta^i\theta^i\bar\theta^j\theta^j D(x),\end{aligned} \tag{12}$$

and for the $N=2$ scalar supermultiplet by

$$\begin{aligned}\Phi^i(x,\theta^i) \ =\ & B^i(x)+\bar\theta^i\chi(x)-\epsilon^{ij}\bar\theta^j\nu(x)-\frac{1}{2}\bar\theta^j\theta^j F^i(x)+\bar\theta^i\theta^j F^j(x)-i\bar\theta^i\partial\!\!\!/ B^j(x)\theta^j\\ & +\frac{i}{2}\bar\theta^j\theta^j(\bar\theta^i\partial\!\!\!/\chi(x)-\epsilon^{ik}\bar\theta^k\partial\!\!\!/\nu(x))+\frac{1}{8}\bar\theta^j\theta^j\bar\theta^k\theta^k\partial_a\partial^a B^i(x).\end{aligned} \tag{13}$$

• Consider the general superfields on the following ψ^i-dependent specific supertranslations,

$$x'^a = x^a + i\kappa\bar{\theta}^i\gamma^a\psi^i, \quad \theta'^i = \theta^i - \kappa\psi^i, \tag{14}$$

and we denote the general superfields on (x'^a, θ'^i) by

$$\tilde{\mathcal{V}}(x^a, \theta^i; \psi^i(x)) = \mathcal{V}(x'^a, \theta'^i), \quad \tilde{\Phi}(x^a, \theta^i; \psi^i(x)) = \Phi(x'^a, \theta'^i). \tag{15}$$

Then,

• SUSY invariant constraints of the component fields in $\tilde{\mathcal{V}}$ and $\tilde{\Phi}$,

$$\begin{aligned} &\tilde{C} = \tilde{\Lambda}^i = \tilde{M}^{ij} = \tilde{\phi} = \tilde{v}^a = \tilde{\lambda}^i = 0, \quad \tilde{D} = \frac{\xi}{\kappa}, \\ &\tilde{B}^i = \tilde{\chi} = \tilde{\nu} = 0, \quad \tilde{F}^i = \frac{\xi^i}{\kappa}, \end{aligned} \tag{16}$$

give the SUSY invariant relations.

Actions in the $d=2$, $N=2$ NL/L SUSY relation

By changing the integration variables $(x^a, \theta^i) \to (x'^a, \theta'^i)$, we can confirm systematically what LSUSY actions reduce to in NLSUSY representation.

(a) The kinetic (free) action with the Fayet-Iliopoulos (FI) D term for the $N=2$ vector supermultiplet $\mathcal{V}$ reduces to $S_{N=2\mathrm{NLSUSY}}$;

$$S_{\mathcal{V}\mathrm{free}} = \int d^2x \left\{ \int d^2\theta^i \, \frac{1}{32} (\overline{D^i \mathcal{W}^{jk}} D^i \mathcal{W}^{jk} + \overline{D^i \mathcal{W}_5^{jk}} D^i \mathcal{W}_5^{jk}) + \int d^4\theta^i \, \frac{\xi}{2\kappa} \mathcal{V} \right\}_{\theta^i=0}$$
$$= \xi^2 S_{N=2\mathrm{NLSUSY}}, \quad (17)$$

where

$$\mathcal{W}^{ij} = \bar{D}^i D^j \mathcal{V}, \quad \mathcal{W}_5^{ij} = \bar{D}^i \gamma_5 D^j \mathcal{V}. \quad (18)$$

(Note) The FI D term gives the correct sign of the NLSUSY action.

(b) Yukawa interaction terms for $\mathcal{V}$ vanish,
i.e.

$$
\begin{aligned}
S_{\mathcal{V}f} \;=\; & \frac{1}{8} \int d^2x \; f \left[\int d^2\theta^i \; \mathcal{W}^{jk} (\mathcal{W}^{jl} \mathcal{W}^{kl} + \mathcal{W}_5^{jl} \mathcal{W}_5^{kl}) \right. \\
& \left. + \int d\bar{\theta}^i d\theta^j \; 2\{ \mathcal{W}^{ij} (\mathcal{W}^{kl} \mathcal{W}^{kl} + \mathcal{W}_5^{kl} \mathcal{W}_5^{kl}) + \mathcal{W}^{ik} (\mathcal{W}^{jl} \mathcal{W}^{kl} + \mathcal{W}_5^{jl} \mathcal{W}_5^{kl}) \} \right]_{\theta^i = 0} \\
= \; & 0,
\end{aligned}
\tag{19}
$$

by means of cancellations among four NG-fermion self-interaction terms.

(c) The *most general* gauge invariant action for $\mathcal{V}$ coupled with Φ^i reduces to $S_{N=2\mathrm{NLSUSY}}$;

$$S_{\mathrm{gauge}} = -\frac{1}{16}\int d^2x \int d^4\theta^i e^{-4e\mathcal{V}}(\Phi^j)^2 = -(\xi^i)^2 S_{N=2\mathrm{NLSUSY}}, \tag{20}$$

Here the $U(1)$ gauge interaction terms proportional to the gauge coupling constant e becomes four NG-fermion self-interaction terms as

$$S_e(\text{for the minimal off shell multiplet}) = \int d^2x \left\{\frac{1}{4}e\kappa\xi(\xi^i)^2\bar{\psi}^j\psi^j\bar{\psi}^k\psi^k\right\}, \tag{21}$$

which are absorbed in the SUSY invariant relation of the auxiliary field $F^i = F^i(\psi)$ by adding four NG-fermion self-interaction terms as

$$F'^i(\psi) = F^i(\psi) - \frac{1}{4}e\kappa^2\xi\xi^i\bar{\psi}^j\psi^j\bar{\psi}^k\psi^k. \tag{22}$$

Therefore,

under the SUSY invariant relations, which are obtained systematically, the $N=2$ NLSUSY action $S_{N=2\mathrm{NLSUSY}}$ is related to $N=2$ SUSY QED action defined by

$$S_{N=2\mathrm{NLSUSY}} = S_{N=2\mathrm{SQED}} \equiv S_{\mathcal{V}\mathrm{free}} + S_{\mathcal{V}f} + S_{\mathrm{gauge}} \tag{23}$$

when $\xi^2 - (\xi^i)^2 = 1$.

$\Longrightarrow$ This NL/L SUSY relation gives the relation between the cosmology and the low energy particle physics in NLSUSY GR as explained before.

4. Discussions

- The similar results are anticipated in $d = 4$ as well.

- The similar arguments in SUSY QCD are also interesting problems in NL/L SUSY relation.

- The extension to large N,

especially to $N = 5$ is important for *superon quintet hypothesis* of SGM scenario with $N = \underline{10} = \underline{5} + \underline{5^*}$ for equipping the $SU(5)$ GUT structure and to $N = 4$ may shed new light on the mathematical structures of the anomaly free non-trivial $d = 4$ field theory.

- Linearizing SGM action $L_{\rm SGM}(e, \psi^i)$ on curved spacetime, which elucidates the topological structure of spacetime, is a challenging problem.