Update of axion CDM energy density

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5. Phase transition effect
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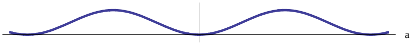
Introduction : strong CP problem

- In the SM, generically, there can be $\frac{\theta}{64\pi^2} G\tilde{G}$ term which breaks CP.
- Neutron EDM ; $\bar{\theta} \equiv \theta + \mathrm{arg}\det M_q < 10^{-9}$.
- Another fine tuning problem
- Solution?

Introduction : axionic solution

- Introduce a dynamical field a(axion) with the coupling $\frac{1}{64\pi^2}\frac{a}{F_a}G\tilde{G}$ and without the tree level potential.(nonlinearly realized $U(1)_{PQ}$)
- By the Vafa-Witten theorem, CP symmetric point, i.e. $a + \bar{\theta} + \mathrm{argdet} M_q = 0$, becomes a minimum.
- $V(\theta) = -C(T)\cos(\theta)$, where $\theta \equiv a/F_a$.



Peccei & Quinn(1977), Weinberg(1978), Wilczek, (1978), J. E. Kim(1979),Shifman, Vainstein & Zakharov, (1980),Dine, Fischler & Srednicki(1981),Zhitnitskii(1980).

Introduction : cosmological implication

- Since it's very weakly interacting, it's hard to detect.
- However, by the same reason, it can affect the evolution of the Universe.(CDM)
- Preskill, Wise & Wilczek(1983), Abbott & Sikivie(1983), Dine & Fischler(1983).
- More precisely by Turner(1986)

Estimation of axion energy density : initial conditions

- Unlike the ordinary DM candidates (ex, WIMP), the relic density of the coherently oscillating bosonic DM are determined by the field equation rather than the Boltzmann's equation. So it crucially depends on the initial condition.
- $\ddot{\theta} + 3H(t)\dot{\theta} + m_a^2(t)\theta + \mathcal{O}(\theta^3) = 0$, where $H \sim T^2/M_{pl}$ and $m_a^2(T) = C(T)/F_a^2 = \frac{C_0}{F_a^2}T^{-n}$.
- In the case of $T_{RH} < F_a$, the inflation determines initial condition $a = F_a\theta_1$.
- The determination of θ_1 is totally stochastic

Estimation of axion energy density : T_1 and Ω_a

- After the inflation, it remains constant until $3H(T_1) \sim m_a(T_1)$.
- At $T = T_1$ or $t = t_1$, it begins to roll down the potential and starts to oscillate.
- $T_1 = \left(C_0 M_{pl}/F_a\right)^{\frac{1}{4+n}}$
- The total number $R^3\rho_a/m_a$ is conserved. We can estimate the present axion energy density.

$$\rho_a(T_\gamma) \simeq m_a(T_\gamma) m_a(T_1) \left(\frac{R^3(T_1)}{R^3(T_\gamma)}\right)\theta_1^2 \tag{1}$$

Estimation of axion energy density : an adiabatic invariant

- The coherent state is not the number eigenstate, rather it's the superposition of the number eigenstates, $|\phi\rangle = |1\rangle + |2\rangle + |3\rangle + \cdots$.
- What is number density of the coherently oscillating field?
- If the parameters are changed adiabatically, there is a *a*diabatic invariant I. In the case of harmonic potential it is the comoving number density. ($I = R^3\rho_a/m_a$)
- Therefore, the interpretation of the number conservation cannot be used in the general case.

Estimation of axion energy density : possible corrections

In this calculation, we assumed several things.

- $\dot{m}_a/m_a \ll m_a$.
- $H \ll m_a$.
- $\theta \ll 1$.
- $\gamma \equiv \frac{S_0}{S_1} = 1$.

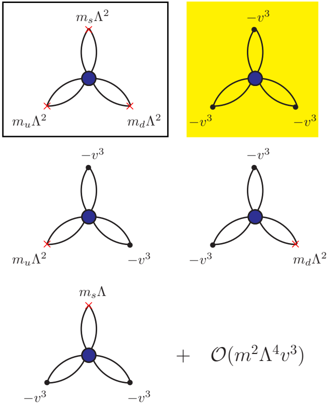
Estimation of axion energy density : possible corrections

In this calculation, we assumed several things.

- $\dot{m}_a/m_a \ll m_a$. phase transition
- $H \ll m_a$. anharmonic effect 2
- $\theta \ll 1$. anharmonic effect 1
- $\gamma \equiv \frac{S_0}{S_1} = 1$. phase transition

Temperature dependence of axion mass : t'Hooft interaction

- $V = -K^{-5}\bar{u}u\bar{d}d\bar{s}s\, e^{i\theta} + \text{h.c.}$
- By closing the light quark loops,
  $V = -K^{-5}(m_u m_d m_s/\bar{\rho}^6)\, e^{i\theta} + \text{h.c.}$.



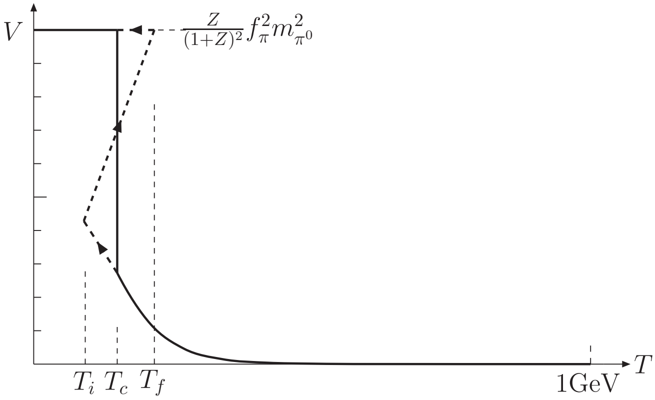
Temperature dependence of axion mass : instanton size integration

- $C(T) = \int d\rho\, n(\rho, T)$.
  (tHooft(1976), Gross, Yaffe & Pisarski(1981))
- For the sufficiently high temperature(1GeV), there is no IR ambiguity.
- $n(\rho, T) = n(\rho, 0) e^{\left\{-\frac{1}{3}\lambda^2(2N+N_f) - 12A(\lambda)\left[1+\frac{1}{6}(N-N_f)\right]\right\}}$.
- $n(\rho, 0) = m_u m_d m_s C_N (\xi\rho)^3 \frac{1}{\rho^5} \left(\frac{4\pi^2}{g^2}\right)^{2N} e^{-8\pi^2/g^2(\Lambda)}$.
- Using 3-loop beta function.

$$\begin{aligned}\alpha_c(\mu) &= \frac{g_c^2(\mu)}{4\pi} \simeq \frac{4\pi}{\beta_0 \ln(\mu^2/\Lambda_{\rm QCD}^2)} \left[1 - \frac{2\beta_1}{\beta_0^2} \frac{\ln[\ln(\mu^2/\Lambda_{\rm QCD}^2)]}{\ln(\mu^2/\Lambda_{\rm QCD}^2)} \right. \\ &+ \frac{4\beta_1^2}{\beta_0^4 \ln^2(\mu^2/\Lambda_{\rm QCD}^2)} \times \left(\left(\ln\left[\ln(\mu^2/\Lambda_{\rm QCD}^2)\right] - \frac{1}{2}\right)^2 \right. \\ &\left.\left. + \frac{\beta_2\beta_0}{8\beta_1^2} - \frac{5}{4}\right)\right] \qquad (2)\end{aligned}$$

Temperature dependence of axion mass : numerical result

- $V = -C(T)\cos\theta$ where $C(T) = C_0\left(\frac{T}{1\mathrm{GeV}}\right)^{-n}$.
- For $\Lambda_{\mathrm{QCD}} = 380\ (440,\ 320)$ MeV, we obtain $C_0 = 3.964 \times 10^{-12}$ $(1.274 \times 10^{-11}, 9.967 \times 10^{-13})$, and $n = 6.878 (6.789,\ 6.967)$.
- From $3H(T_1) = m(T_1)$, we obtain $T_1 = 0.808 \times \left(\frac{F_a}{10^{12}\mathrm{GeV}}\right)^{-0.182} \left(0.916 \times \left(\frac{F_a}{10^{12}\mathrm{GeV}}\right)^{-0.184}, 1.020 \times \left(\frac{F_a}{10^{12}\mathrm{GeV}}\right)^{-0.185}\right)$.



V
$\frac{Z}{(1+Z)^2} f_\pi^2 m_{\pi^0}^2$
T_i T_c T_f
1GeV
T

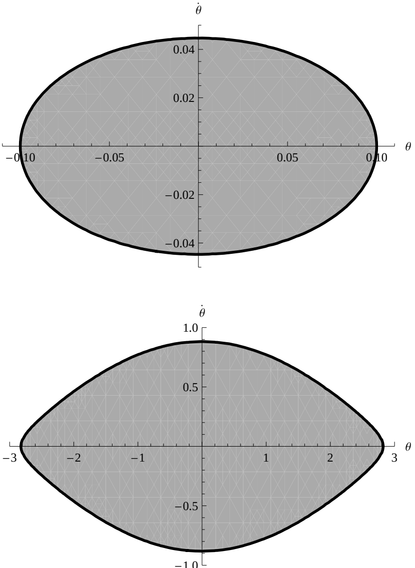
Anharmonic effect

- 1. Deformation of path in phase space
- 2. Initial adjustment

Anharmonic effect 1: deformation of path in phase space

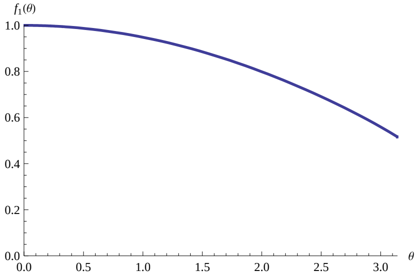
- $I \equiv \frac{1}{2\pi} \oint pdq$. The area swept by one period.
- For the harmonic potential, it's the area of the ellipse.
- For the anharmonic potential, it's deformed.

Anharmonic effect 1: deformation of path in phase space



Anharmonic effect 1: deformation of path in phase space

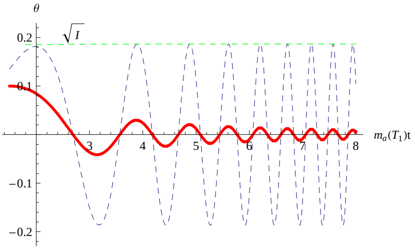
$$f_1(\theta) = \frac{2\sqrt{2}}{\pi\theta^2}\int_{-\theta}^{+\theta}\sqrt{\cos\theta' - \cos\theta}\, d\theta'$$

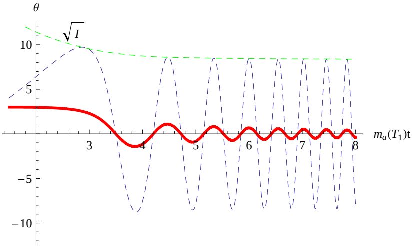


Anharmonic effect 2: initial adjustment

- Since we start when $3H(T_1) = m_a(T_1)$ is satisfied, for the first few oscillations, the expansion rate is not so small. So the second adiabaticity condition $H \ll m_a$ is broken.
- If θ_1 is larger, this duration is longer.(Anharmonic correction)
- To correct it, use T_2 and θ_2 rather than T_1 and θ_1.
- Even in the harmonic case, there is the finite correction.($\sim 1.8\times$)

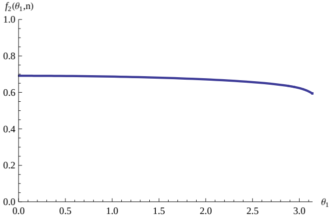
Anharmonic effect 2: initial adjustment



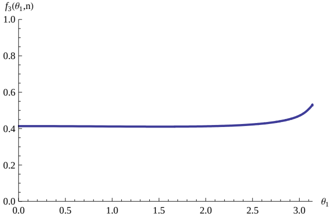


Anharmonic effect 2: initial adjustment

$$f_2(\theta_1, n) = T_2/T_1$$

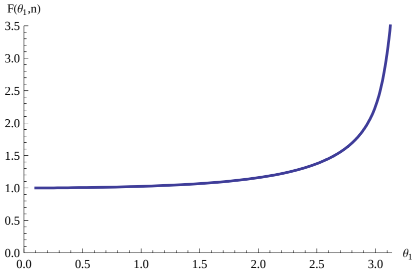


$$f_3(\theta_1, n) = \theta_2/\theta_1$$



Anharmonic effect 2: Full anharmonic correction

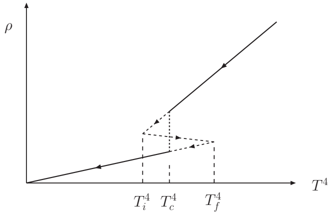
$$
\begin{aligned}
\Omega_a &\propto \frac{m_a(T_2)\theta_2^2}{T_2^3} f_1(\theta_2) \\
&\propto \frac{\theta_1^2 c_3^2 \tilde{f}_3^2(\theta_1, n) f_1(\theta_1 c_3 \tilde{f}_3(\theta_1, n))}{c_2^{3+n/2} \tilde{f}_2^{3+n/2}(\theta_1, n)} \\
&= 1.846 \times \theta_1^2 \times F(\theta_1, n)
\end{aligned} \tag{3}
$$



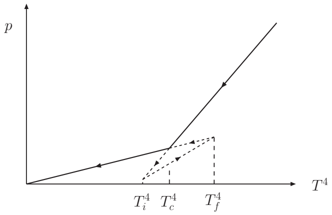
Phase transition effect : first adiabaticity condition and entropy production

- We don't know much about the QCD phase transition
- Parametrizing the phase transition with T_i and T_f.
- If super cooling occurs significantly, $\Delta t \lesssim m_a^{-1}$ is possible.(axion number change) But it's parameter space is very small.
- DeGrand(1985), DeGrand, Kephart & Weiler(1986).

Phase transition effect : super cooling

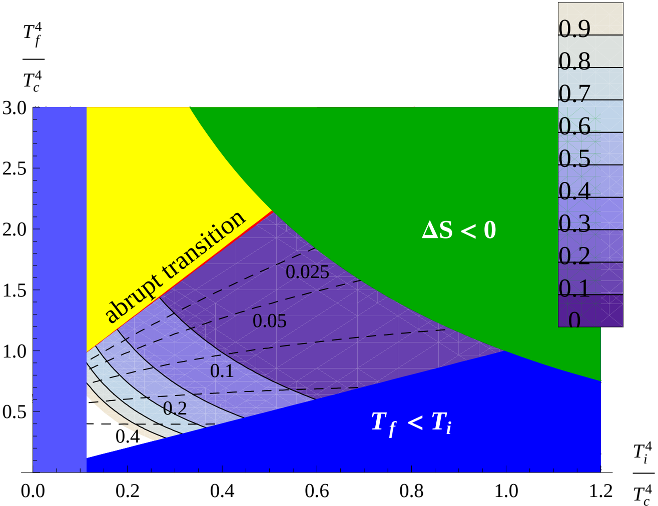


(a)

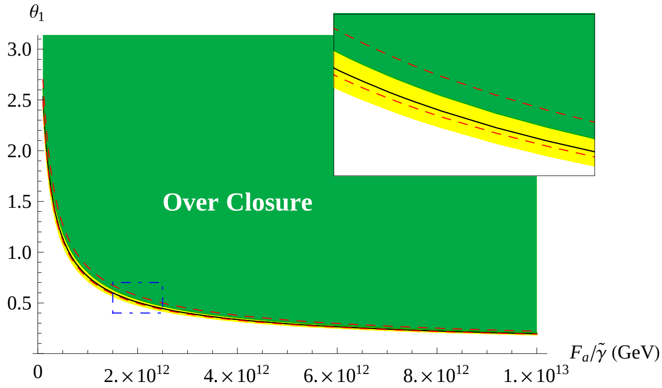


(b)

Phase transition effect : $T_i - T_f$



Result



θ_1
3.0
2.5
2.0
1.5
1.0
0.5
Over Closure
0
$2.\times10^{12}$
$4.\times10^{12}$
$6.\times10^{12}$
$8.\times10^{12}$
$1.\times10^{13}$
$F_a/\tilde{\gamma}$ (GeV)

Summary

- More refined numbers of QCD parameters
- More accurate instanton size integral(3-loop)
- Explicitly showing anharmonic effects
- The corrections from initial adjustment
- The effects from the phase transition (abrupt transition and entropy production)