1-Loop Effective Action on Orbifolds

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Introduction

- Orbifolds are a very popular choice for the internal geometry for String Theory and Field Theory with Extra Dimensions
- Simplest Geometry allowing 4d chiral fermions, and $N = 1$ SUSY
- Plethora of Symmetry Breaking Possibilities
- Particular limits of more general spaces (CY)
- High degree of calculability!

Introduction

- Special Interest in perturbative corrections
- Moduli stabilization problem: Many 4d scalars with no potential at tree level
  - No-Scale SUSY breaking
- Often: Interested in Large Volume → Perturbation theory both reliable and dominant over nonperturbative physics....
- Want to go beyond eff. potential approximation

The covariant background method

[DeWitt 1965; Gilkey 1975]

- Split all fields in background and fluctuations

$$\phi(x) = \varphi(x) + \varphi_{dyn}(x)$$

$$\mathcal{A}_M(x) = A_M(x) + A_{dyn,M}(x) \quad \text{etc} \ \ldots$$

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$$D_M = \nabla_M - iA_M - i\omega_M$$

$$E = E(\varphi, A_M, g_{MN})$$

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$$- \tfrac{1}{180}(R_{MN})^2 + \tfrac{1}{180}(R_{MNST})^2 + \tfrac{1}{12}(F_{MN})^2] + \dots\Big)$$

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- Describes effective action including operators with derivatives
- Effective potential is obtained by setting to zero all derivatives...

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- (Will also be needed for orbifold)

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- Nontrivial λ gives nonlocal renorm. (finite)

...Torus...

- Basic idea: Further expand heat kernel in powers of λ: [GG 08]

$$K(x,x,T) = \frac{\sqrt{g}}{(4\pi T)^{d/2}}\left(\sum_r a_r T^r\right)$$

...Torus...

- Basic idea: Further expand heat kernel in powers of λ:

[GG 08]

$$K(x,x-\lambda,T)=\frac{\sqrt{g}}{(4\pi T)^{d/2}}\exp\left(\frac{-\lambda^2}{4T}\right)\left(\sum_{r,s}a_{r,j_1\cdots j_s}T^r\lambda^{j_1}\cdots\lambda^{j_s}\right)\exp(i\lambda^j[A_j+\omega_j])$$

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- Exponential prefactor: Suppression for nonzero λ, finite (nonlocal) contributions
- Wilson line → partial resummation
- Has been calculated up to dim 4 operators

Result up to dim 4 operators

$$K(x,x-\lambda,T) = \frac{\sqrt{g}}{(4\pi T)^{d/2}}\exp(-\frac{\lambda^2}{4T})$$

[GG 08]

$$\Big\{1+T\left[\tfrac{1}{6}R-E\right]+T\lambda^j\left[-\tfrac{1}{12}R_{;j}+\tfrac{1}{2}E_{;j}-\tfrac{1}{6}F^M{}_{j;M}\right]$$

$$+T\lambda^j\lambda^k\Big[\tfrac{1}{40}R_{;jk}+\tfrac{1}{120}R_{jk;}{}^M{}_M-\tfrac{1}{90}R^M{}_jR_{Mk}$$

$$+\tfrac{1}{180}R^{MN}R_{MjNk}+\tfrac{1}{180}R^{MNL}{}_jR_{MNLk}-\tfrac{1}{6}E_{;jk}$$

$$+\tfrac{1}{24}R^M{}_jF_{Mk}+\tfrac{1}{12}F_{Mj}F^M{}_k+\tfrac{1}{12}F_{Mj;}{}^M{}_k\Big]$$

$$+T^2[\tfrac{1}{2}(\tfrac{1}{6}R-E)^2+\tfrac{1}{6}(\tfrac{1}{5}R-E)_{;}{}^M{}_M-\tfrac{1}{180}R_{MN}R^{MN}$$

$$+\tfrac{1}{180}R^{MNLS}R_{MNLS}+\tfrac{1}{12}F_{MN}F^{MN}]\Big\}\exp(i\lambda^j[A_j+\omega_j])$$

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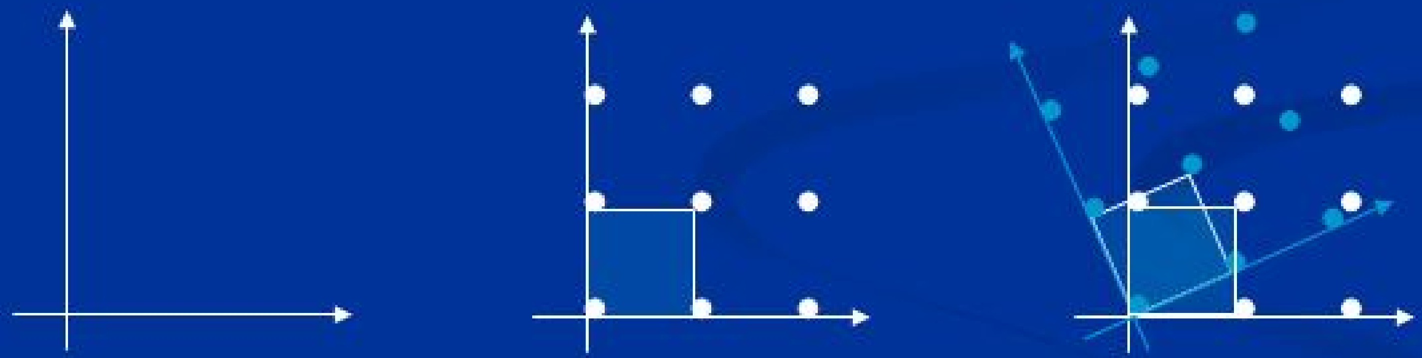
- Contributions with $\lambda = 0$ now correspond to renormalization at fixed points of P^k
- Remains to evaluate $K(x, Px - \lambda)$

...Orbifold...

- For each orbifold sector *k*, split into fixed and rotated dimensions

$\mathbb{R}^4$ fixed $\qquad \Lambda_{\parallel}$ fixed $\qquad \Lambda_{\perp}$ rotated

Under P^k :



$$x = x_{\parallel} + x_{\perp}\,, \quad \Lambda = \Lambda_{\parallel} + \Lambda_{\perp}$$

- Either torus can be trivial for certain *k*

...Orbifold

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$$\int dx \sum_{\lambda} K(x, P^k x - \lambda) =$$

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$$\int dx \sum_{\lambda} K(x, P^k x - \lambda) = \int dx_{\parallel} \sum_{\lambda_{\parallel}} K_{\parallel}(x_{\parallel}, x_{\parallel} - \lambda_{\parallel})$$

$$K_{\parallel}(T) = \exp(-T[-D_{\parallel}^2 + E + A_{\perp}^2 + \omega_{\perp}^2])$$

$$\varphi = \varphi(x^{\mu})$$

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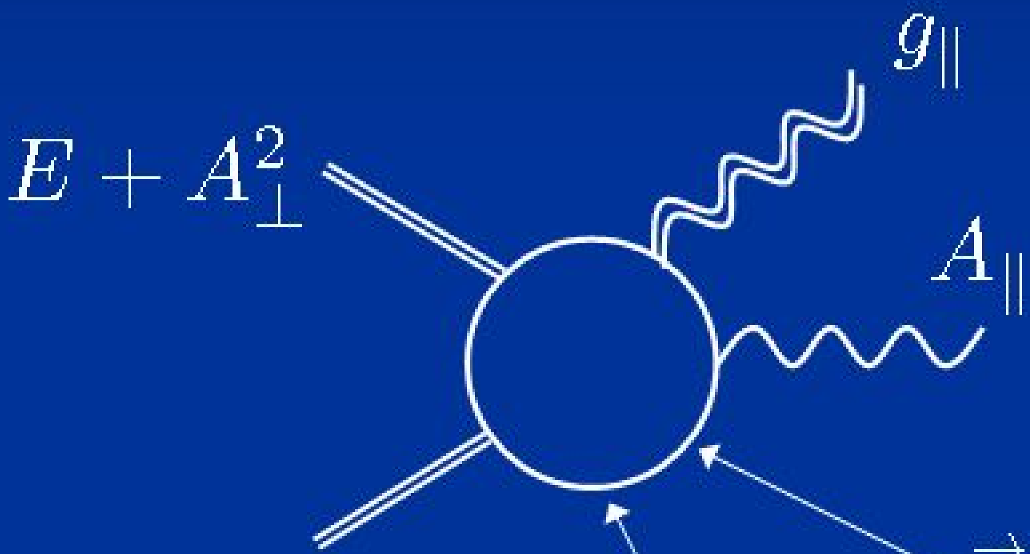
Evaluation as on the $d_{\|}$-dimensional fixed torus with shifted mass matrix

[GG 08]

Diagrammatic Interpretation



$$A_M = A_M(x^\mu)$$
$$E = E(x_\mu)\,,\ \text{etc.}$$

→ All KK modes on $\Lambda_{\|}$ contribute

→ Only zero modes on $\Lambda_{\perp}$ contribute

Matrix $\hat{P}^{-k}$ in the loop

Discrete Wilson lines

- Fields need not be periodic on the torus

$$\phi(x+\lambda) = W_0(\lambda)\phi(x)$$

- Certain discrete WL cannot be transformed into backgrounds for gauge fields: $W_0(\lambda)^N = 1$
- Simple modifications
  - Cts. WL is rescaled: $e^{i\lambda_\parallel \cdot A} \to e^{i\lambda_\parallel \cdot A} W_0(\lambda_\parallel)$
  - Additional matrix in trace: $\mathrm{tr}\ K_\parallel \hat{P}^{-k} \to \mathrm{tr}\ K_\parallel \hat{P}^{-k} Q$

  Q is the projector onto zero modes on the transverse torus $\Lambda_\perp$ carrying the discrete Wilson line $W_0(\lambda_\perp)$

Conclusions

- Applied the covariant background (heat kernel) formalism to calculate 1-loop corrections on arbitrary torus orbifolds
- Result on torus: Covariant expansion in powers of lattice vectors, calculated all dim 4 operators
- Orbifold: Each sector corresponds to the result of its fixed torus with a shifted mass matrix