

Moduli Stabilization & the Pattern of Sparticle Spectra

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(SUSY 08 , Seoul)

KC & H.P. Nilles , JHEP 04 (2007) 006 [hep-ph/0702146]

KC , Jeong , Nakamura , Okumura & Yamaguchi , in preparation

W. Cho , KC , Y. Kim & C. Park , PRL (2008) [arXiv : 0709.0288]
JHEP (2008) [arXiv : 0711.4526]

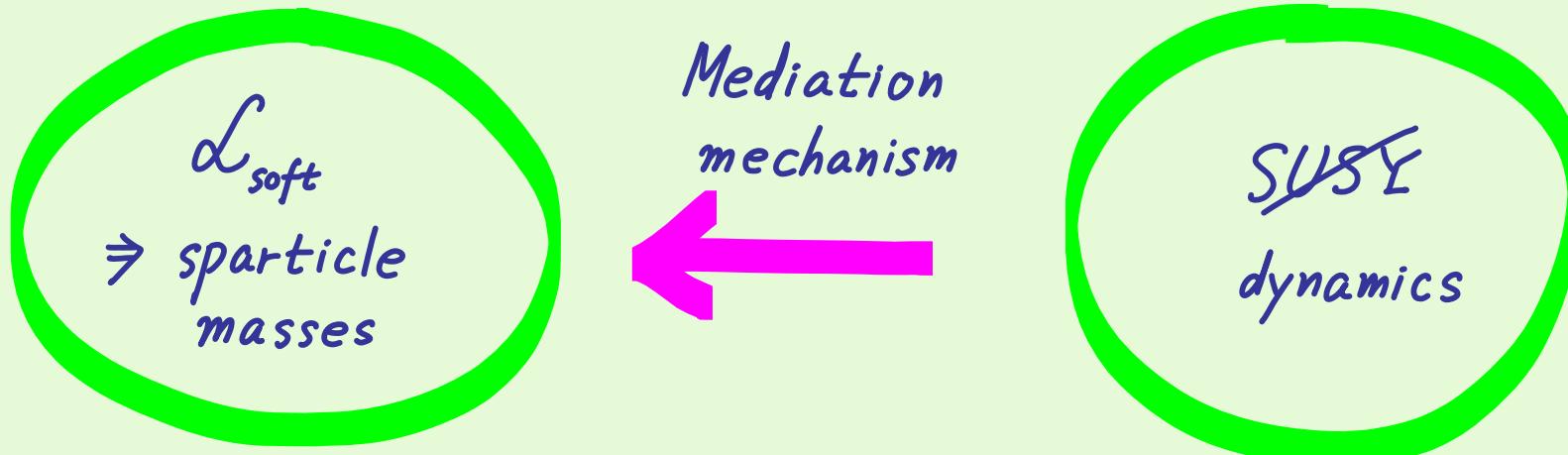
- ◆ *Introduction*
- ◆ *Stabilization of gauge coupling modulus and messenger mass modulus , and the pattern of sparticle masses*
- ◆ *Determination of gaugino mass ratios with M_{T2} kink.*

◆ Introduction

Weak scale SUSY is perhaps the leading candidate for new physics at TeV :

- Protect the weak scale from quadratic divergence
- Gauge coupling unification (within the MSSM)
- Good CDM candidate (under R-parity)
- Easily pass the precision EW test

If the idea of weak scale SUSY is correct,
LHC will be able to discover (some) superparticles
and information on sparticle spectra :



Constraints on $\mathcal{L}_{\text{soft}}$ from FCNC & CP

\Rightarrow Gauge mediation, Anomaly mediation,
Dilaton / Modulus mediation , ...

In typical 4D effective theories of string model, anomaly, dilaton/modulus and gauge mediations exist together, although the relative importance of these mediations is model-dependent.

* Anomaly mediation

Randall & Sundrum ;

Giudice, Luty, Murayama & Rattazzi

Mediation by 4D SUGRA multiplet ($\Rightarrow g_{\mu\nu}, \psi_\mu$) which is most conveniently parameterized by

$$C = C_0 + \theta^2 F^C.$$

$$\left(g_{\mu\nu}^{(E)} = C_0^* C_0 g_{\mu\nu}^{(SC)}, \quad \frac{F^C}{C_0} = m_{3/2} + \frac{1}{3} \partial_x K F^I \right)$$

RG point μ appears through $C^* C / \mu^2$.

$$\Rightarrow \delta m_{\text{gaugino}} \sim \delta m_{\text{sfermion}} \sim \frac{1}{8\pi^2} \frac{F^C}{C_0}$$

* Dilaton / Modulus mediation \leftarrow particular type
of gravity mediation

There can be dilaton/modulus mediation in any theory
with $\frac{1}{g_a^2} = \langle \overline{T} \rangle$ gauge coupling modulus
(= string dilaton or volume modulus)

$$\int d^2\theta \frac{1}{4} T W^{a\alpha} W_\alpha^a = R(T_0) \left(-\frac{1}{4} F_{\mu\nu}^a F^{a\mu\nu} + \bar{\lambda}^a \not{D} \lambda^a \right) \\ - \frac{1}{2} F^T \lambda^a \lambda^a + \dots$$

$$\Rightarrow \delta m_{\text{gaugino}} \sim \delta m_{\text{sfermion}} \sim \frac{F^T}{T}$$

At leading order in $\frac{g_{\text{GUT}}^2}{8\pi^2} \sim \frac{1}{8\pi^2(T+T^*)}$, T -mediated
soft terms preserve flavor & CP.

(dilaton mediation in heterotic string , volume modulus
mediation in Type IIB string)

Flavor & CP conserving modulus mediation
in flux compactification : KC, Jeong & Okumura,
arXiv : 0804.4283

- Gauge coupling modulus : $\frac{1}{g_{\text{GUT}}^2} = \langle T \rangle$
 - * $\frac{1}{\text{Re}(T)} \propto g_{st}^n$ or α'^n , and defines controllable expansion of 4D effective action which is expected to be as good as the loop expansion in 4D gauge theory.
 - * At leading order in this expansion, T couples to matters in flavor universal way. \downarrow real constant
 - * Axionic shift symmetry $T \rightarrow T + i\alpha$ assures T couplings to matters are CP conserving.

- Flavon moduli U which have highly non-universal couplings to matters to generate hierarchical Yukawa couplings :

$$y_{ij} \sim e^{-K_{ij} U}$$

- Very often, T & U have different topological origins , so that U can be fixed by flux with $m_U \sim M_{st}$, while T remains light with $m_T \sim m_{3/2}$, e.g. $T = 4\text{-cycle volume}$, $U = 3\text{-cycle volume}$.

$3\text{-form flux} \Rightarrow m_U \gg m_T \Rightarrow F^U \ll F^T$
 \swarrow topological moduli mass hierarchy

\Rightarrow Flavor & CP conserving modulus mediation.

* Gauge mediation (H. Murayama's talk)

SM gauge charged vector-like exotic matter fields

$$\underbrace{\Xi^c + \bar{\Xi}}_{\text{messenger}} \quad \text{with} \quad \int d^2\theta \ X \ \Xi^c \bar{\Xi} .$$

↑ messenger mass modulus
 (= flat direction in supersymmetric
 limit with $M_{pe} \rightarrow \infty$)

Integrating out $\Xi^c + \bar{\Xi}$ at $\langle X \rangle = X_0 + \theta^2 F^X$

$$\Rightarrow \delta m_{\text{gaugino}} \sim \delta m_{\text{sfermion}} \sim \frac{1}{8\pi^2} \frac{F^X}{X_0} .$$

Most of known semi-realistic string models have
 exotic vector-like matter fields.

In (string-motivated) models in which anomaly, dilaton/modulus and gauge mediations coexist, the resulting pattern of sparticle masses is determined by

Anomaly : Dilaton/Modulus : Gauge

$$\sim \frac{1}{8\pi^2} \frac{F^C}{C} : \frac{F^T}{T} : \frac{1}{8\pi^2} \frac{F^X}{X}$$

which are determined by the stabilization mechanism of T & X .

Everett, Kim, Ouyang & Zurek,
arXiv: 0806.2330 ; 0806.0592

◆ Stabilization of T (= gauge coupling modulus)

- All couplings of T are of the gravitational strength.

- $\langle V \rangle = K_{i\bar{j}} F^i F^{\bar{j}*} - 3 m_{3/2}^2 \approx 0 \quad (M_{Pl}=1)$

$$\Rightarrow K_{T\bar{T}} |F^T|^2 \sim \left| \frac{F^T}{T} \right|^2 \lesssim \Theta(m_{3/2}^2)$$

* Three different schemes of T -stabilization

A) $\frac{F^T}{T} \sim m_{3/2} \gg \frac{1}{8\pi^2} \frac{F^C}{C}$

Gersdorff & Hebecker ; Berg, Haack, Körs
Stabilization by perturbative Kähler corrections :

$$K = -3 \ln(T + T^*) + \frac{\xi_{\alpha'} (>0)}{(T + T^*)^{3/2}} + \frac{\xi_s (<0)}{(T + T^*)^2}$$

$W = W_0 =$ (flux-induced) T -independent superpotential

$$B) \quad \frac{F^T}{T} \sim \frac{m_{3/2}}{\ln(M_{Pl}/m_{3/2})} \sim \frac{1}{8\pi^2} \frac{F^C}{C}$$

Stabilization by nonperturbative dynamics

$$W = W_0 + A e^{-aT} \quad (KKLT)$$

KC, Falkowski, Nilles, Olechowski

$$\Rightarrow \frac{F^T}{T} \sim \frac{m_{3/2}}{aT} \sim \frac{m_{3/2}}{\ln(M_{Pl}/m_{3/2})}$$

$$C) \quad \frac{F^T}{T} \ll \frac{m_{3/2}}{8\pi^2} \sim \frac{1}{8\pi^2} \frac{F^C}{C}$$

$$\text{Flux stabilization : } m_T \sim \left\langle \frac{\partial^2 W_{\text{flux}}}{\partial T^2} \right\rangle \sim \frac{M_{Pl}}{(T+T^*)^n}$$

$$\Rightarrow \frac{F^T}{T} \sim \frac{m_{3/2}^2}{m_T} \sim \frac{m_{3/2}^2}{M_{Pl}}$$

◆ Stabilization of X (= messenger mass modulus)

$$\langle V \rangle = K_{\bar{x}\bar{x}} F^{\bar{x}} F^{*\bar{x}} - 3 m_{3/2}^2 \approx 0 \quad (M_{Pl} = 1)$$

$\Rightarrow K_{x\bar{x}} |F^x|^2 \sim |F^x|^2 \lesssim \mathcal{O}(m_{3/2}^2)$, however
it is allowed that $\frac{F^x}{X} \gg m_{3/2}$.

* Three different schemes of X -stabilization

I) $\frac{F^x}{X} \gg 8\pi^2 M_{3/2}$

Conventional gauge mediation

$$\text{II) } \frac{F^X}{X} \sim m_{3/2} \sim \frac{F^C}{C}$$

Pomarol & Rattazzi

Stabilization by SUGRA effects

$$\int d^2\theta \ C^3 \frac{X^n}{M_{Pl}^{n-3}} \Rightarrow \frac{F^X}{X_0} = - \frac{2}{n-1} \frac{F^C}{C}$$

$$\text{III) } \frac{F^X}{X} \sim \frac{F^T}{T}$$

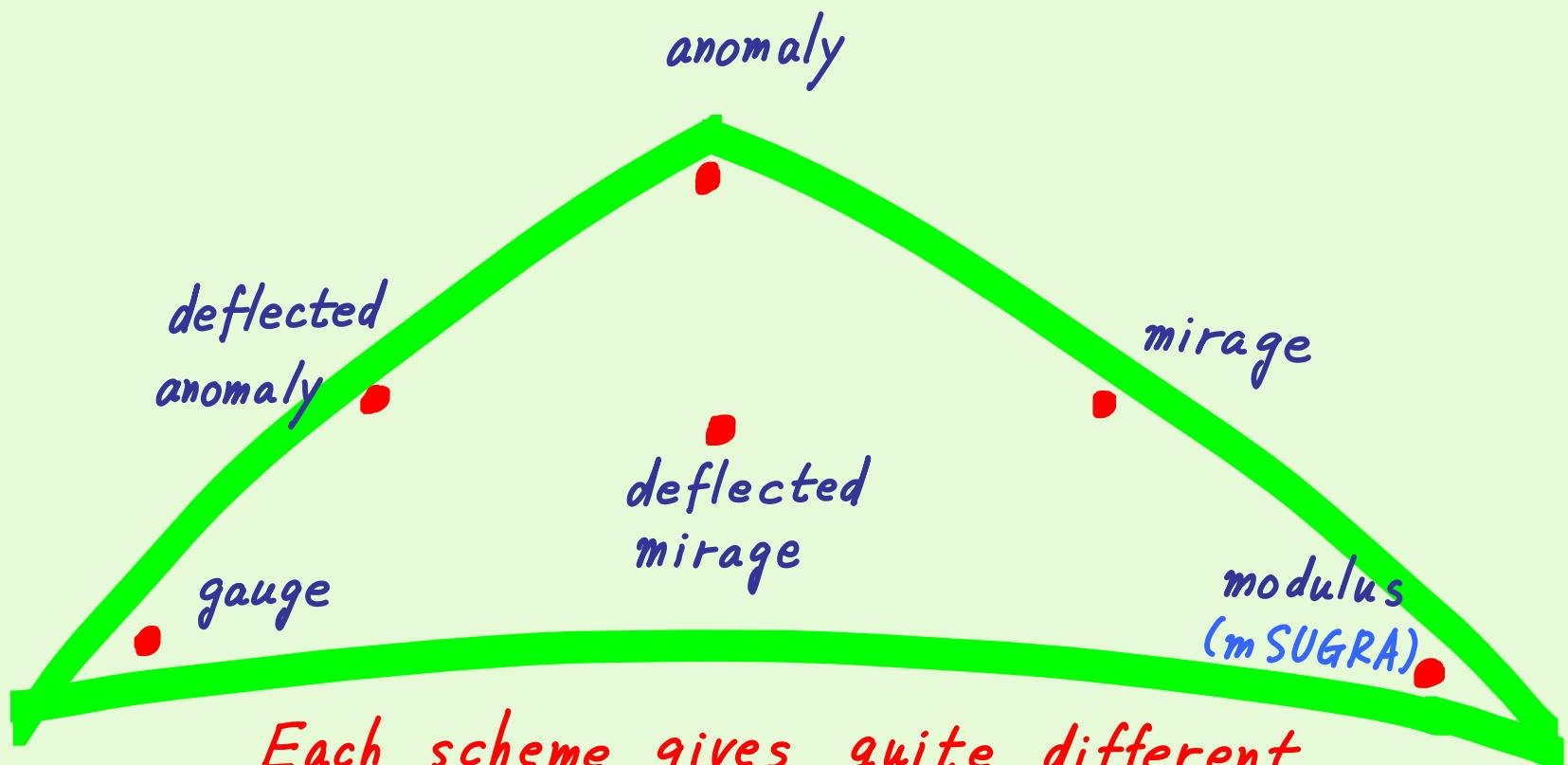
Stabilization by the D-term of anomalous $U(1)$:

$$T \rightarrow T + i \delta_{GS} \alpha(x), \quad X \rightarrow e^{-i q_x \alpha(x)} X$$

$$\Rightarrow \left\{ \begin{array}{l} D_A = \underbrace{\delta_{GS} \frac{\partial K}{\partial T}}_{\text{T-dependent FI term}} - q_x |X|^2 \sim \left| \frac{F^T}{T} \right|^2 \\ \frac{F^X}{X} \sim \frac{F^T}{T} \end{array} \right.$$

KC & Jeong

Consideration of moduli stabilization suggests that mixed mediations are equally probable as the simpler mediation dominated by one of the anomaly, dilaton/modulus & gauge mediations.



Each scheme gives quite different pattern of sparticle masses.

* Gaugino Mass Pattern KC & Nilles

Consideration of moduli stabilization leads to three distinct patterns of gaugino masses:

- Assume the high scale gauge coupling unification.

$$\Rightarrow \left(\frac{M_a}{g_a^2} \right)_{\text{Tev}} = \underbrace{\frac{1}{2} F^T - \frac{N}{8\pi^2} \frac{F^X}{X}}_{\text{universal}} + \frac{b_a(\text{Tev})}{16\pi^2} \frac{F^C}{C}$$

↙ # of messengers

$$\bullet g_1^2 : g_2^2 : g_3^2 \Big|_{\text{Tev}} \approx 1 : 2 : 6$$

- m SUGRA pattern

Modulus or gauge domination :

$$\frac{F^T}{T} \sim m_{3/2} \quad \text{or} \quad \frac{F^X}{X} \gg 8\pi^2 m_{3/2}$$

$$\Rightarrow \left(\frac{M_a}{g_a^2} \right)_{TeV} \approx \text{universal}$$

$$\Rightarrow M_{\tilde{B}} : M_{\tilde{W}} : M_{\tilde{g}} \Big|_{TeV} \approx 1 : 2 : 6$$

- Anomaly pattern

$$\frac{F^T}{T} \ll \frac{m_{3/2}}{8\pi^2}, \quad N \frac{F^X}{X} \ll m_{3/2}$$

$$\Rightarrow \left(\frac{M_a}{g_a^2} \right)_{TeV} \propto b_a(TeV)$$

$$\Rightarrow M_{\tilde{B}} : M_{\tilde{W}} : M_{\tilde{g}} \Big|_{TeV} \approx 3.3 : 1 : 9$$

- Mirage pattern

$$\frac{F^T}{T} \sim \frac{m_{3/2}}{\ln(M_{pe}/m_{3/2})} \quad \text{or} \quad \frac{F^X}{X} \sim m_{3/2} \Rightarrow$$

$$M_{\tilde{B}} : M_{\tilde{W}} : M_{\tilde{g}} \Big|_{TeV} \approx (1 + 0.66\alpha) : (2 + 0.2\alpha) : (6 - 1.8\alpha)$$

$$\alpha = \frac{m_{3/2}}{\left(\frac{1}{2} \frac{F^T}{T} - \frac{g_{GUT}^2 N}{8\pi^2} \frac{F^X}{X} \right) \ln \left(\frac{M_{pe}}{m_{3/2}} \right)} = \begin{cases} 1 & (KKLT) \\ \frac{4(n-1)}{N} & W = X^n / M_{pe}^{n-3} \\ & (\text{deflected anomaly}) \\ & \vdots \end{cases}$$

$$M_{\tilde{B}} : M_{\tilde{W}} : M_{\tilde{g}} \Big|_{TeV} = \begin{cases} 1 : 1.6 : 3.8 & (\alpha = 0.5) \\ 1 : 1.3 : 2.5 & (\alpha = 1) \\ 1 : 1.2 : 1.7 & (\alpha = 1.5) \end{cases}$$

\Rightarrow Compressed spectrum compared to other patterns

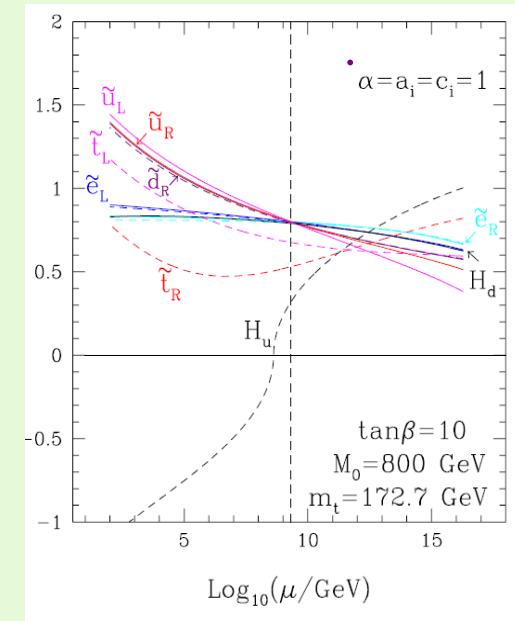
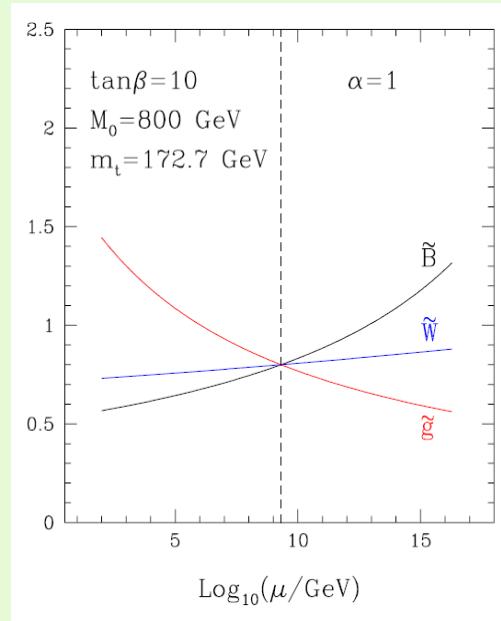
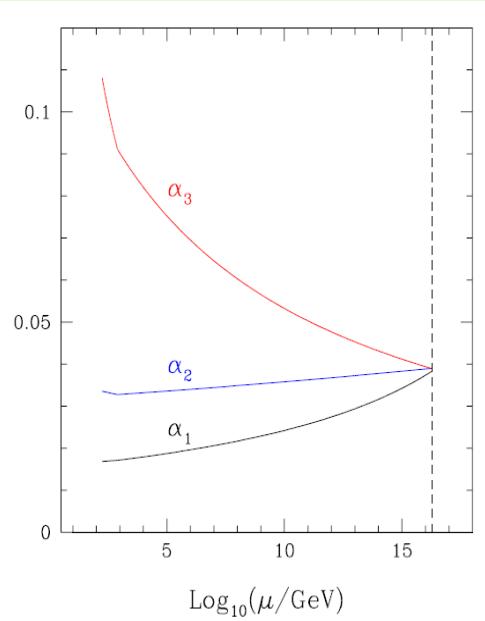
Mirage unification of sparticle masses

KC, Jeong & Okumura

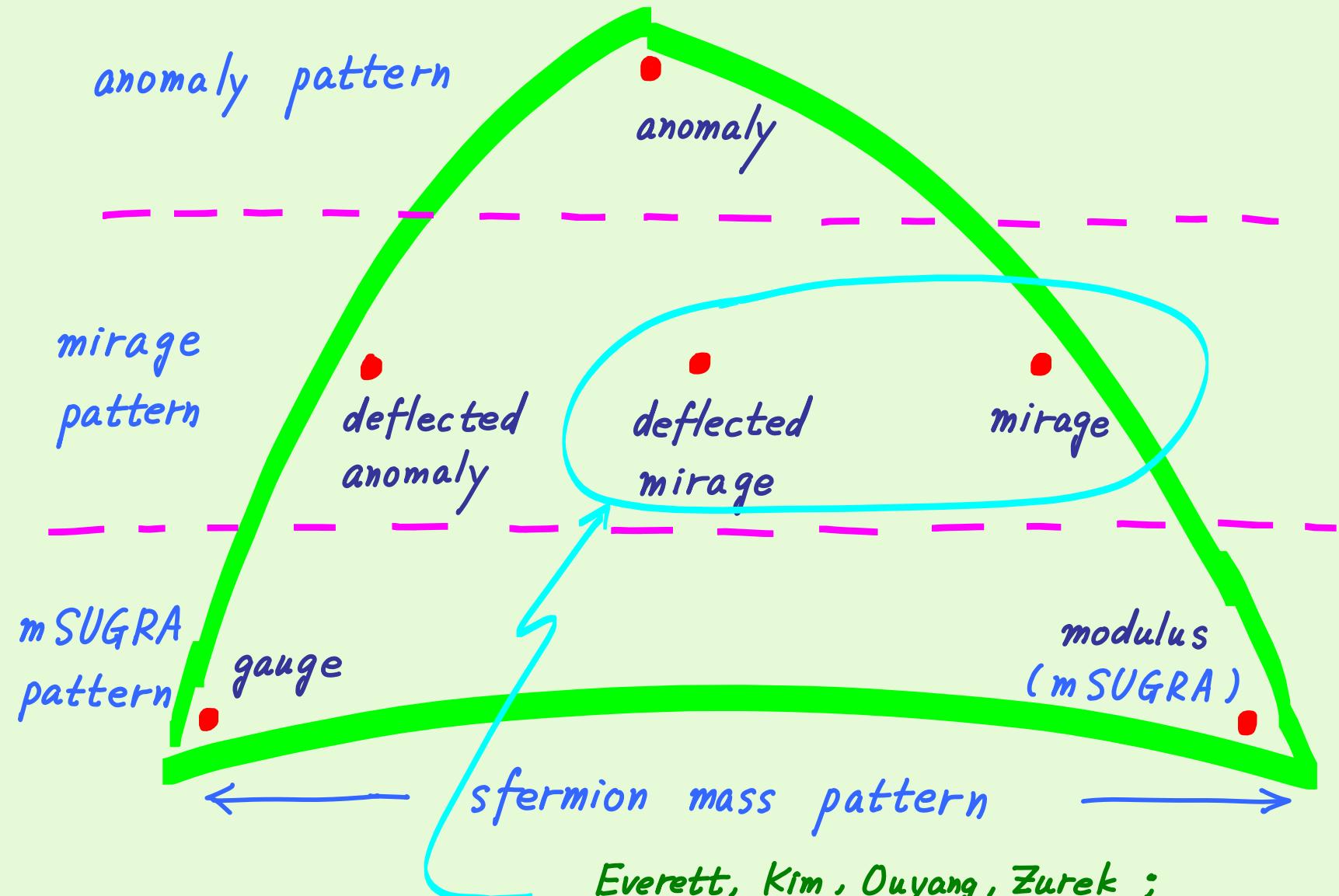
$$M_a(\mu) = M_0 \left[1 - \frac{b_a}{8\pi^2} g_a^2(\mu) \ln \left(\frac{M_{\text{mirage}}}{\mu} \right) \right]$$

$$m_i^2(\mu) = m_0^2 + M_0^2 \left\{ Y_i(\mu) - \frac{\dot{Y}_i(\mu)}{16\pi^2} \ln \left(\frac{M_{\text{mirage}}}{\mu} \right) \right\} \frac{\ln \left(\frac{M_{\text{mirage}}}{\mu} \right)}{4\pi^2}$$

$$(M_{\text{mirage}} \sim M_{\text{GUT}} e^{-2\pi^2/\alpha})$$



$$M_{\tilde{B}} : M_{\tilde{W}} : M_{\tilde{g}}$$



Everett, Kim, Ouyang, Zurek ;
KC, Jeong, Nakamura, Okumura, Yamaguchi

◆ Determination of sparticle masses with the M_{T_2} kink

\tilde{g} , χ_2 , χ_1 ← 2nd-lightest neutralino
 χ_1 ← lightest neutralino
assumed to be LSP

(If Higgsinos are heavier than \tilde{W} & \tilde{B} ,)
 χ_1 & χ_2 are gaugino-like .)

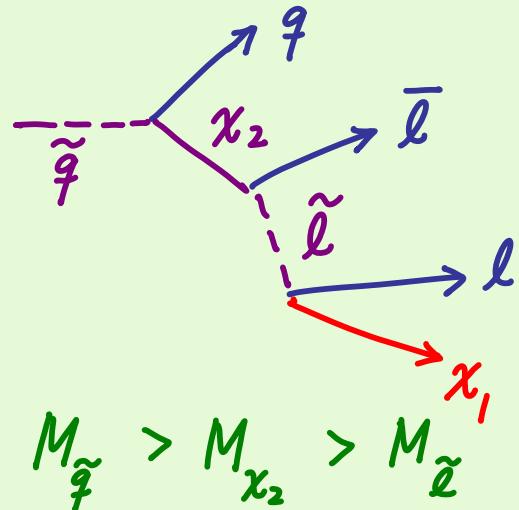
$$M_{\tilde{g}} : M_{\chi_2} : M_{\chi_1} \approx \begin{cases} 1 : 2 : 6 & (\text{mSUGRA}) \\ 1 : 3.3 : 9 & (\text{anomaly}) \\ 1 : 1.3 : 2.5 & \begin{array}{l} (\text{mirage}) \\ \text{with } \alpha = 1 \end{array} \end{cases}$$

KKLT

As the sparticle decay always involves the invisible LSP in the final state (assume R-parity), measuring the sparticle masses is a nontrivial job.

- Invariant mass endpoints in long cascade decays

Hinchliffe et al ; Weiglein et al ; ...



$$m_{q\bar{l}}^{\max} = \frac{\sqrt{(m_{x_2}^2 - m_{\tilde{l}}^2)(m_{\tilde{l}}^2 - m_{x_1}^2)}}{m_{\tilde{l}}}$$

$$m_{q\ell\ell}^{\max} = \frac{\sqrt{(m_{\tilde{q}}^2 - m_{x_2}^2)(m_{x_2}^2 - m_{x_1}^2)}}{m_{x_2}}$$

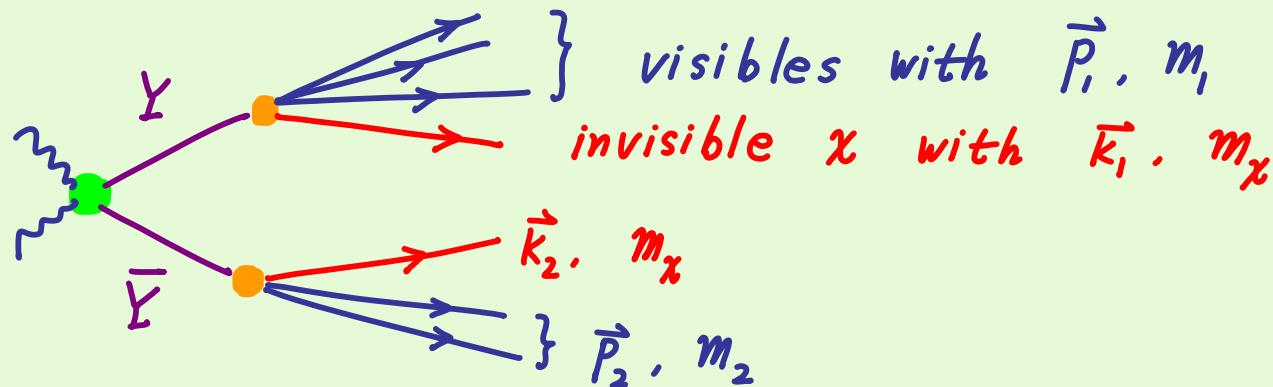
$$m_{q\bar{l}}, \quad m_{q\ell}^{\max}, \quad m_{q\ell\ell}^{\min}$$

- * The accuracy of the overall mass determination is not good.
- * Such a long decay is not available in models with $M_{\tilde{l}} > M_{x_2}$.

- Kink structure of M_{T2}

Cho, KC, Kim, Park

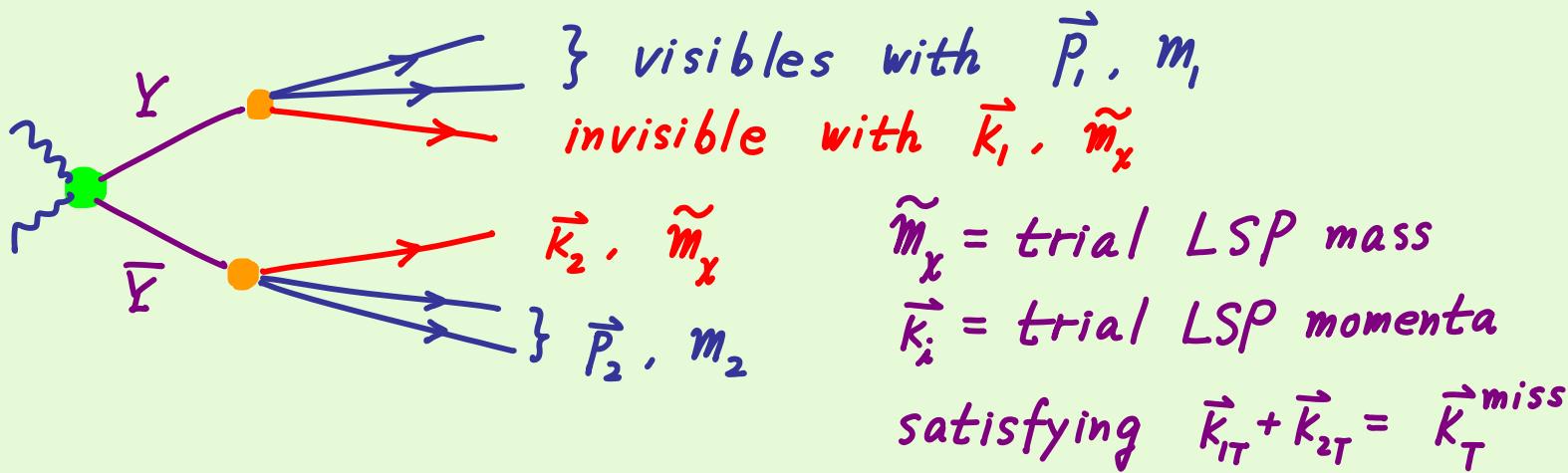
Typical SUSY events :



$\vec{k}_1, \vec{k}_2, m_x$ are unknown except for $\vec{k}_T^{\text{miss}} = \vec{k}_{1T} + \vec{k}_{2T}$

M_{T2} : Generalization of the transverse mass
to the events with two missing particles

Lester, Summers : Barr, Lester, Stephens



$$M_{T2}(\vec{P}_{1T}, m_1, \vec{P}_{2T}, m_2; \tilde{m}_x) \quad \text{Lester, Summers}$$

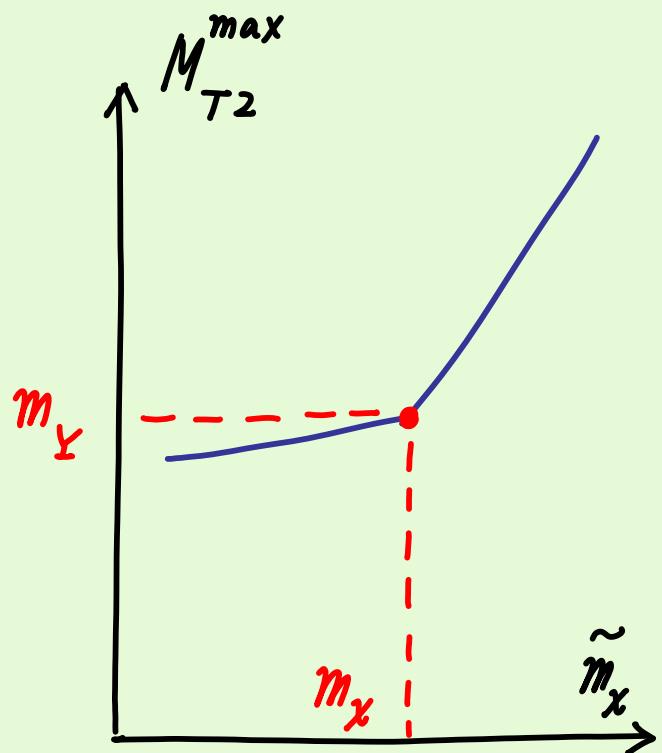
$$= \min_{\{\vec{k}_{iT}\}} \left[\max \left(M_T^{(1)}, M_T^{(2)} \right) \right]$$

$$M_{T2}^{\max}(\tilde{m}_x) = \max_{\{\text{all events}\}} \left\{ M_{T2}(\vec{P}_{1T}, m_1, \vec{P}_{2T}, m_2; \tilde{m}_x) \right\}$$

↙ mother particle mass

$$\left(M_{T2}^{\max}(\tilde{m}_x = m_x) = m_Y \right)$$

The endpoint value of M_{T2} as a function of the trial LSP mass \tilde{m}_x generically can have a kink structure at $\tilde{m}_x = m_x$.



Simultaneous determination
of m_x & m_Y

- The sharpness of kink depends on the available data set for $Y \rightarrow x + \text{visible particles}$

- Applications of the M_{T2} kink to the determination of

$m_{\tilde{g}}$, $m_{\tilde{q}}$, m_x :
M. Nojiri's talk

Y.G. Kim's talk
Y. Shimizu's talk

Why kink appears at $\tilde{m}_x = m_x$?

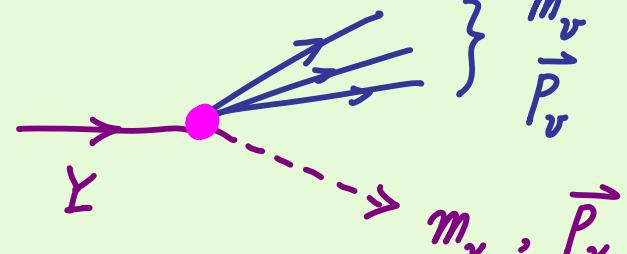


Diagram showing a particle Y decaying into a neutrino ν and a particle X . The neutrino ν has mass m_ν and momentum \vec{P}_ν . The particle X has mass m_x and momentum \vec{P}_x .

$$\begin{aligned} m_\nu^2 + m_x^2 \\ + 2 \left(\sqrt{m_\nu^2 + P_\nu^2} \sqrt{m_x^2 + P_x^2} - \vec{P}_\nu \cdot \vec{P}_x \right) \\ = m_Y^2 \text{ for all events} \end{aligned}$$

Extrapolation to hypothetical decay with same event variables m_ν , \vec{P}_ν & \vec{P}_x , but with arbitrary \tilde{m}_x :

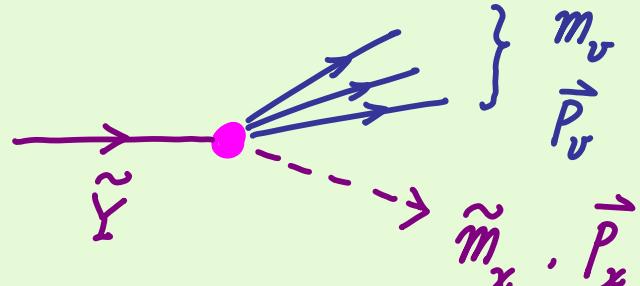
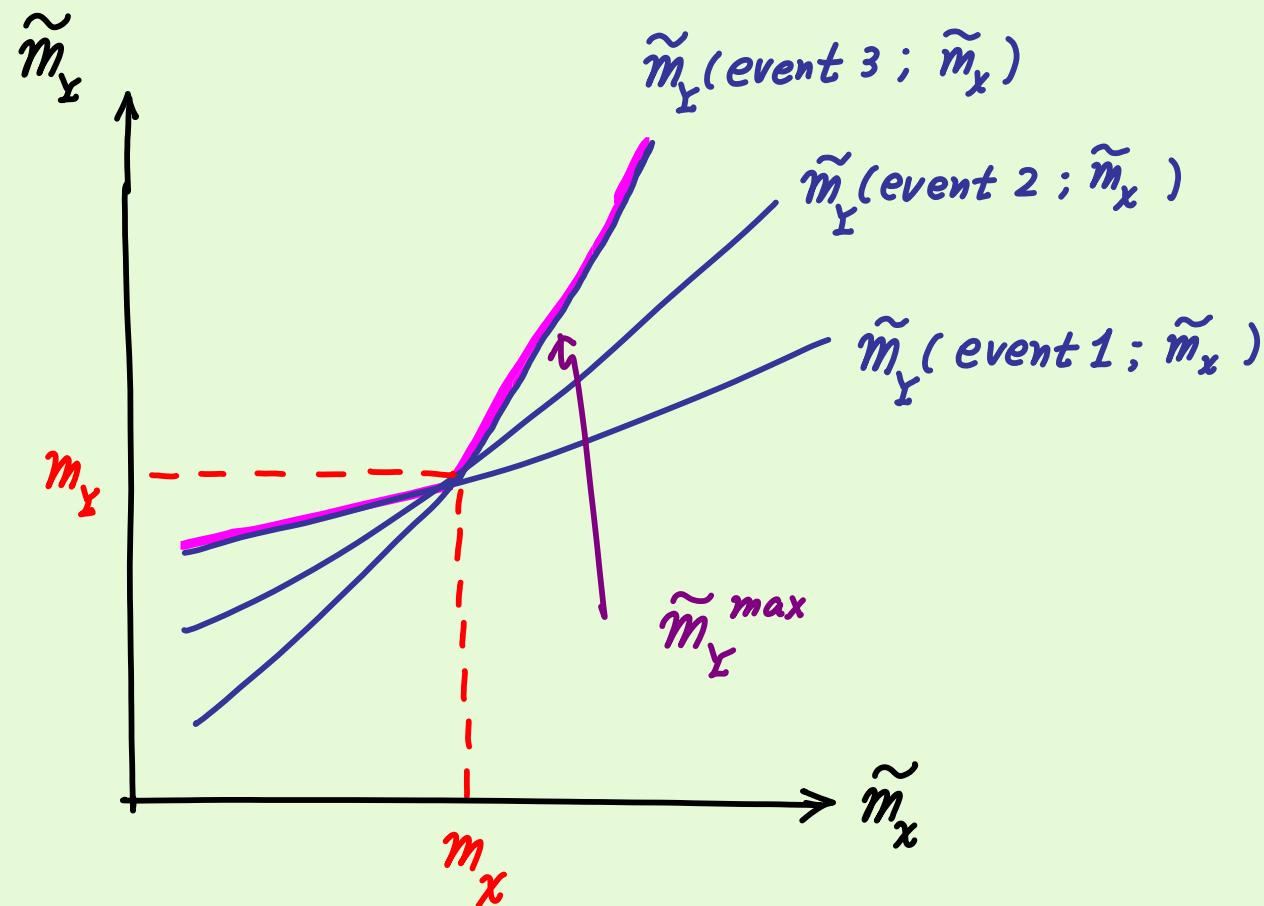


Diagram showing a particle \tilde{Y} decaying into a neutrino ν and a particle \tilde{X} . The neutrino ν has mass m_ν and momentum \vec{P}_ν . The particle \tilde{X} has mass \tilde{m}_x and momentum \vec{P}_x .

$$\begin{aligned} \tilde{m}_Y^2 &= m_\nu^2 + \tilde{m}_x^2 \\ + 2 \left(\sqrt{m_\nu^2 + P_\nu^2} \sqrt{\tilde{m}_x^2 + P_x^2} - \vec{P}_\nu \cdot \vec{P}_x \right) \\ &= \text{hypothetical mother} \\ &\quad \text{particle mass} \end{aligned}$$

Generically different events give different extrapolations.



$$* \text{Event } A : P_Y = 0, \quad m_v = 0, \quad P_v = P_x = \frac{m_Y^2 - m_x^2}{2m_Y}$$

$$\tilde{m}_Y(A; \tilde{m}_x) = \frac{1}{2m_Y} \left[m_Y^2 - m_x^2 + \left((m_Y^2 + m_x^2)^2 + 4m_Y^2(\tilde{m}_x^2 - m_x^2) \right)^{1/2} \right]$$

$$* \text{Event } B : P_Y = 0, \quad m_v = m_Y - m_x, \quad P_v = P_x = 0$$

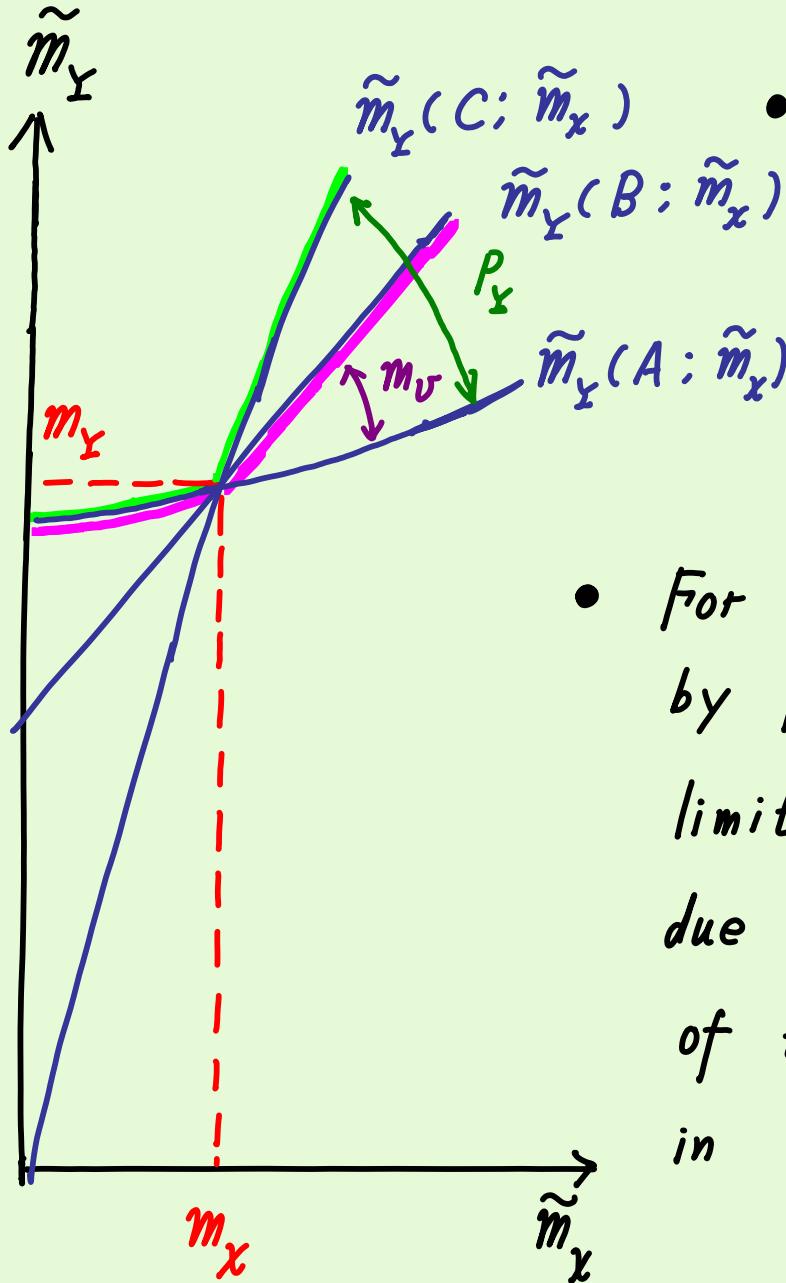
In case that $Y \rightarrow x + \text{multi-visible particles}$, the available event set can contain both A & B.

$$\tilde{m}_Y(B; \tilde{m}_x) = m_Y - m_x + \tilde{m}_x$$

$$* \text{Event } C : P_Y = \frac{m_Y^2 - m_x^2}{2m_X}, \quad m_v = 0, \quad P_x = 0$$

(Lorentz boost of A)

$$\tilde{m}_Y(C; \tilde{m}_x) = \left(\tilde{m}_x^2 + \frac{\tilde{m}_x}{m_x} (m_Y^2 - m_x^2) \right)^{1/2}$$



- Kink becomes sharper if the available data can give a larger range of m_{ν} or P_Y .
- For $Y + \bar{Y}$ pair directly produced by pp , the range of P_Y is limited, so the kink is mostly due to the non-zero range of m_{ν} of the multi-visible particles in $Y \rightarrow \chi + \text{visibles}$.

Conclusion

- One can classify the patterns of sparticle masses based on possible schemes of moduli stabilization.
Known schemes of moduli stabilization suggest that mixed anomaly, modulus & gauge mediations are as probable as the simple mediation dominated by one of the involved mediations.

- Experimental measurement of \tilde{m}_g/m_χ will be useful for discriminating different SUSY-breaking schemes:

m SUGRA pattern : $\tilde{m}_g/m_\chi \sim 6$

anomaly pattern : $\tilde{m}_g/m_\chi \sim 9$

mirage pattern : \tilde{m}_g/m_χ can be much smaller

- M_{T2} -kink might provide a useful tool to determine $\frac{m_{\tilde{g}}}{m_{\chi}}$ (possibly other sparticle masses also) at the LHC.