

Higgs Boson Mass, New Physics and Inflation

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OUTLINE

- Higgs Boson / Seesaw Mechanisms
- Gauge-Higgs Unification
- Inflation
- Conclusion



WHERE is the SM Higgs Boson?

- From arguments based on vacuum stability and perturbativity, and with no new physics between M_Z and M_{Planck} , one finds

$$130 \text{ GeV} \lesssim m_h \lesssim 170 \text{ GeV}$$



SM Higgs boson mass bounds

From the Higgs potential in the SM,

Higgs quartic coupling determines $M_H^2 = \lambda v^2$ ($v = 246\text{GeV}$)

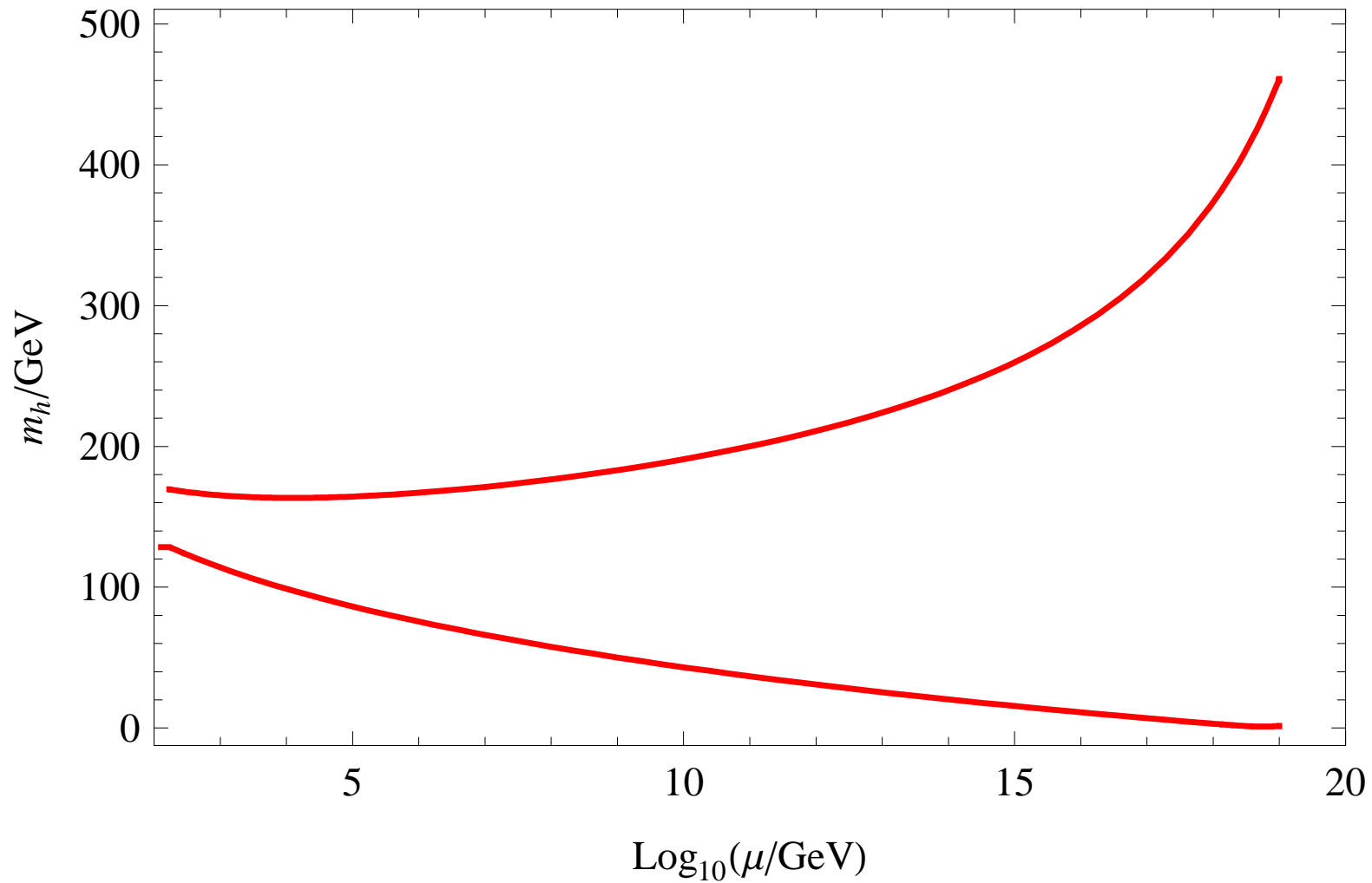
Once Higgs mass is measured, its high energy behavior can be understood via RGE running of $\lambda(\mu)$

$$\frac{d\lambda}{d\ln\mu} = \frac{1}{16\pi^2}\beta_\lambda^{(1)} + \frac{1}{(16\pi^2)^2}\beta_\lambda^{(2)},$$

top Yukawa is important

$$\begin{cases} \beta_\lambda^{(1)} = 12\lambda^2 - \left(\frac{9}{5}g_1^2 + 9g_2^2\right)\lambda + \frac{9}{4}\left(\frac{3}{25}g_1^4 + \frac{2}{5}g_1^2g_2^2 + g_2^4\right) + \underline{12y_t^2\lambda - 12y_t^4}, \\ \beta_\lambda^{(2)} = -78\lambda^3 + 18\left(\frac{3}{5}g_1^2 + 3g_2^2\right)\lambda^2 - \left(\frac{73}{8}g_2^4 - \frac{117}{20}g_1^2g_2^2 + \frac{2661}{100}g_1^4\right)\lambda - 3\lambda y_t^4 \\ + \frac{305}{8}g_2^6 - \frac{289}{40}g_1^2g_2^4 - \frac{1677}{200}g_1^4g_2^2 - \frac{3411}{1000}g_1^6 - 64g_3^2y_t^4 - \frac{16}{5}g_1^2y_t^4 - \frac{9}{2}g_2^4y_t^2 \\ + 10\lambda\left(\frac{17}{20}g_1^2 + \frac{9}{4}g_2^2 + 8g_3^2\right)y_t^2 - \frac{3}{5}g_1^2\left(\frac{57}{10}g_1^2 - 21g_2^2\right)y_t^2 - 72\lambda^2y_t^2 + 60y_t^6. \end{cases}$$

The Standard Model Higgs



Physics beyond the SM required by:

- Neutrino Oscillations

$$\left(\Delta m_{SM}^2 \underset{\text{dim } 5}{\sim} 10^{-10} eV^2 \ll \Delta m_{ATM}^2, \Delta m_{SOL}^2 \right)$$

$$\sum m_{\nu_i} \lesssim 1 \text{ eV}$$

- $\frac{\delta T}{T}$ (Inflation) $\sim 10^{-5}$

- Non-baryonic DM ($\Omega_{CDM} \approx 0.25$)

- Baryon Asymmetry ($n_B/s \sim 10^{-10}$), with $\Omega_B \approx 0.05$

- Dark Energy

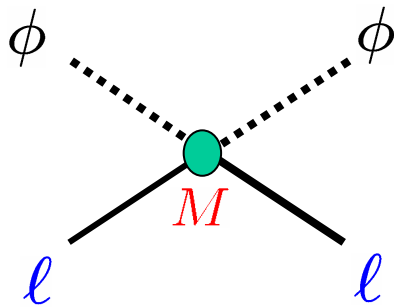


Additional Motivations

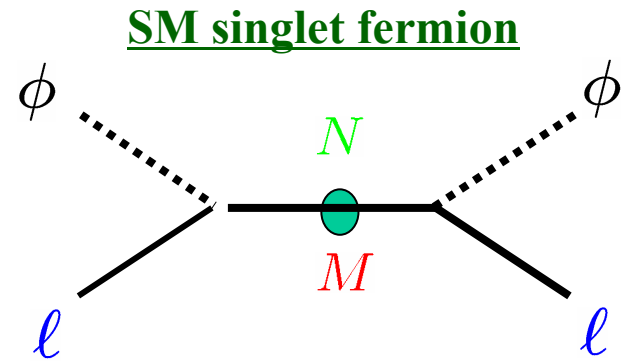
- Gauge Hierarchy Problem; ($M_W \ll M_P$)
- Fermion Masses & Mixings;
- Unification with Gravity (?)
- Family Replication
- Charge Quantization
- Origin of Parity Violation



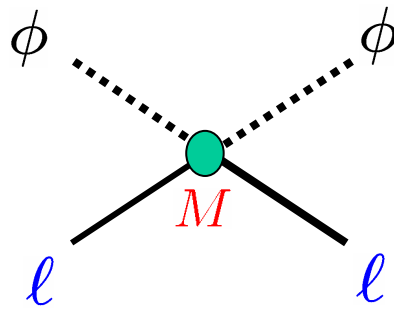
Type I Seesaw



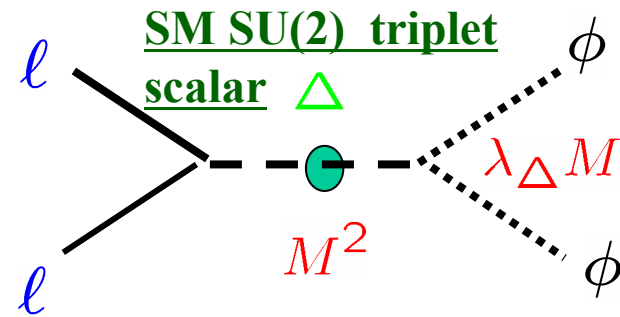
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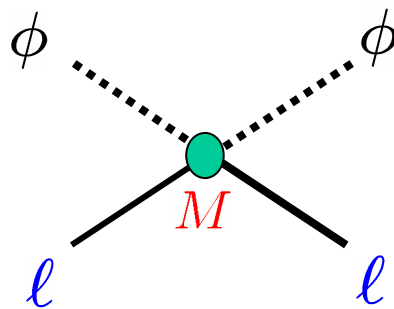
Type II Seesaw



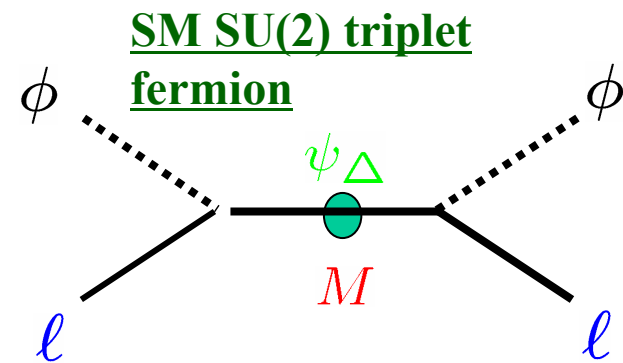
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Type III Seesaw



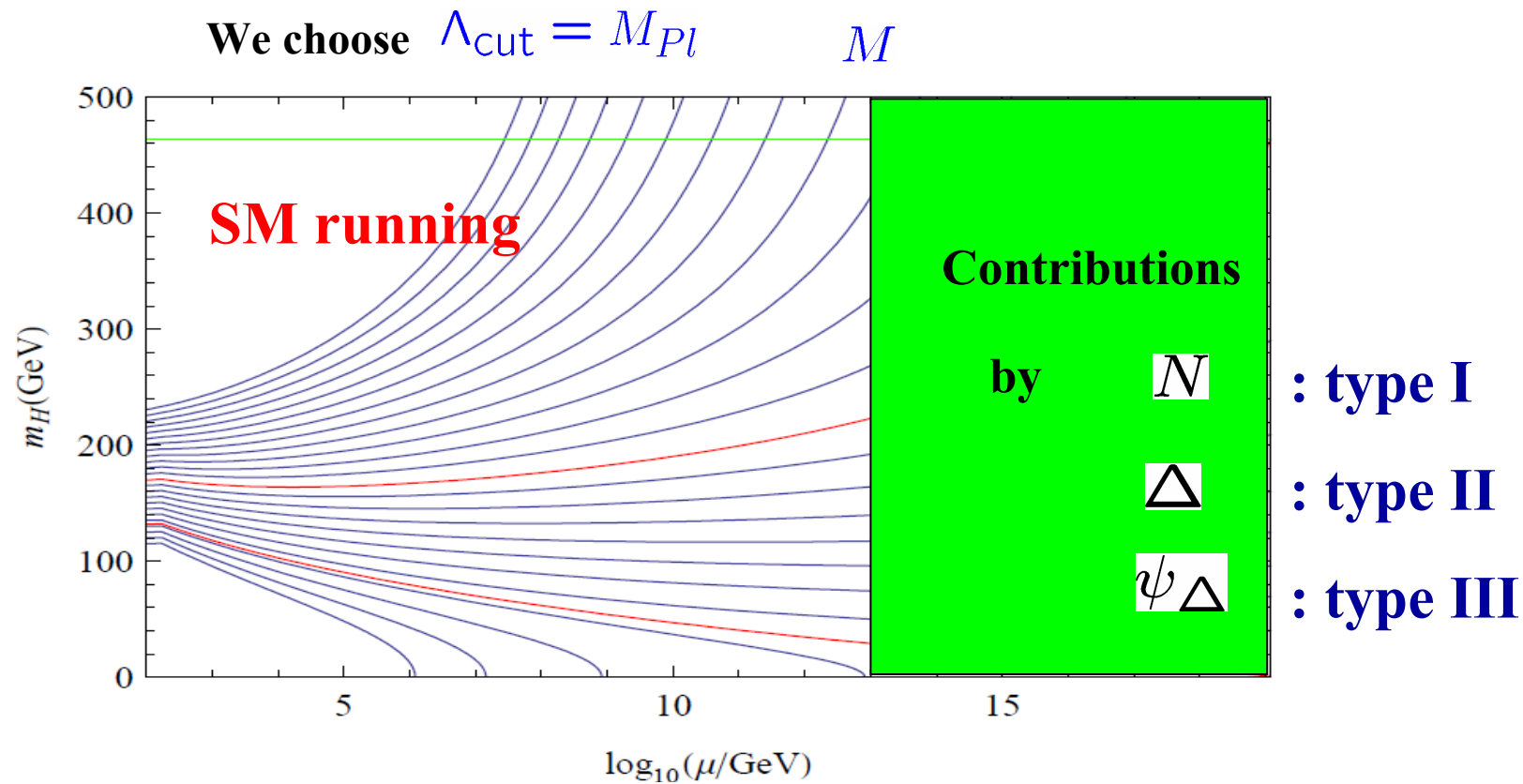
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In all seesaw scenarios, new particles couple to Higgs doublet

→ contribute to Higgs quartic RGE for $\mu > M$

Running mass: $m_H(\mu) = \sqrt{\lambda}v$



Type I seesaw

Casas, Clemente, Ibarra & Quiros,

PRD 62, 053005 (2000);

Modification of RGEs in the presence of 3 singlet fermions (Okada, Gogoladze, QS)

with $\mathcal{L}_Y = y_\nu^{ij} \bar{\ell}_i \phi N_j$

For simplicity, we assume 3 degenerate N: $M = M \times \mathbf{1}_{3 \times 3}$

Light neutrino mass matrix: $M_\nu = -\frac{1}{2}v^2 Y_\nu^T M^{-1} Y_\nu = \frac{v^2}{2M} Y_\nu^T Y_\nu$

For $\mu < M$, **SM RGEs**

$$\text{For } \mu \geq M, \quad \frac{dy_t}{d \ln \mu} = y_t \left(\frac{1}{16\pi^2} \beta_t^{(1)} + \frac{1}{(16\pi^2)^2} \beta_t^{(2)} \right) \quad \beta_t^{(1)} \rightarrow \beta_t^{(1)} + \text{tr}[\mathbf{S}_\nu]$$
$$\frac{d\lambda}{d \ln \mu} = \frac{1}{16\pi^2} \beta_\lambda^{(1)} + \frac{1}{(16\pi^2)^2} \beta_\lambda^{(2)} \quad \beta_\lambda^{(1)} \rightarrow \beta_\lambda^{(1)} + 4 \text{tr}[\mathbf{S}_\nu] \lambda - 4 \text{tr}[\mathbf{S}_\nu^2]$$

$$16\pi^2 \frac{d\mathbf{S}_\nu}{d \ln \mu} = \mathbf{S}_\nu \left[6y_t^2 + 2 \text{tr}[\mathbf{S}_\nu] - \left(\frac{9}{10}g_1^2 + \frac{9}{2}g_2^2 \right) + 3 \mathbf{S}_\nu \right] \quad \mathbf{S}_\nu = Y_\nu^\dagger Y_\nu$$

* We employ 2-loop SM RGEs + 1-loop new RGEs

Fixing the cutoff scale M_{Pl} , we investigate Higgs mass bounds

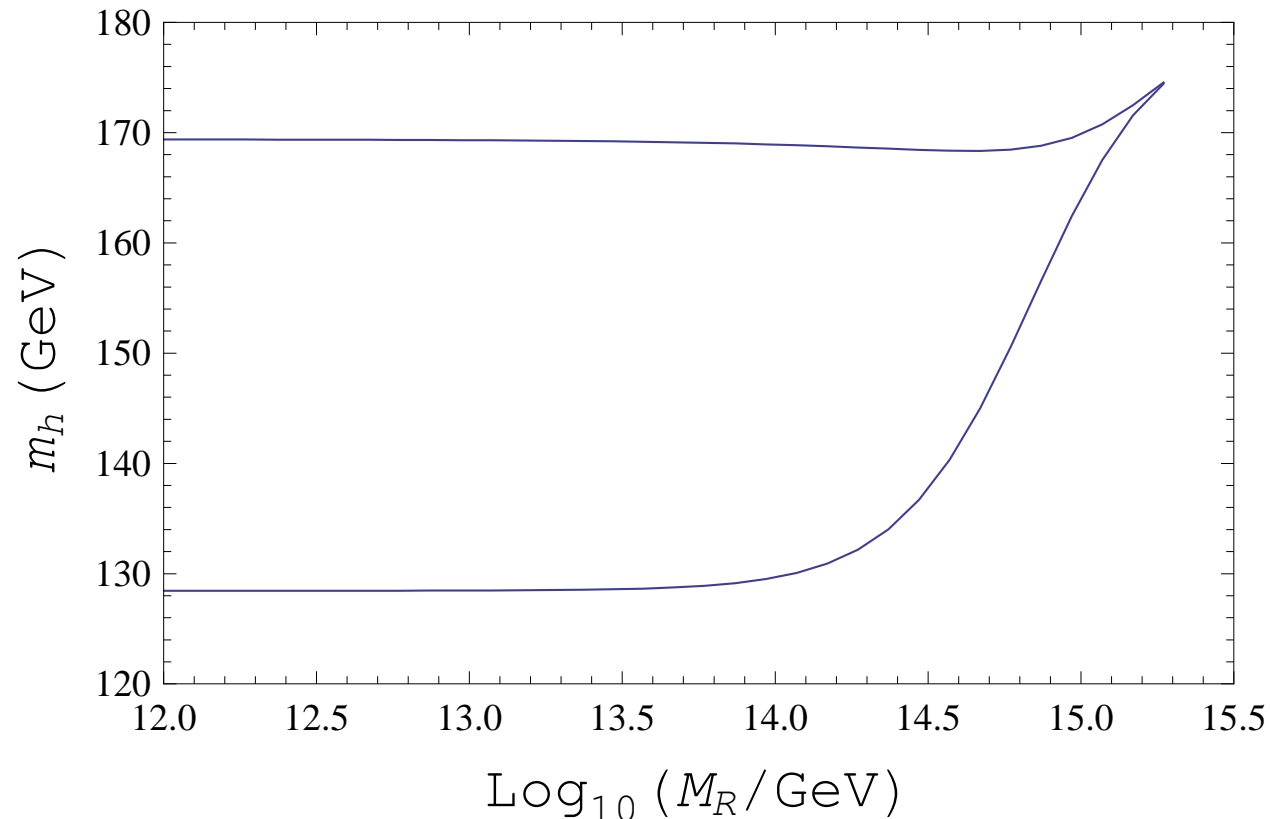
Vacuum stability bound: the lowest Higgs boson mass which satisfies

$$\lambda(\mu) \geq 0 \text{ for any scale between } M_H \leq \mu \leq M_{Pl}$$

Perturbativity bound: the highest Higgs boson mass which satisfies

$$\lambda(\mu) \leq \sqrt{4\pi} \text{ for any scale between } M_H \leq \mu \leq M_{Pl}$$

Type I Seesaw and Higgs Mass Bounds

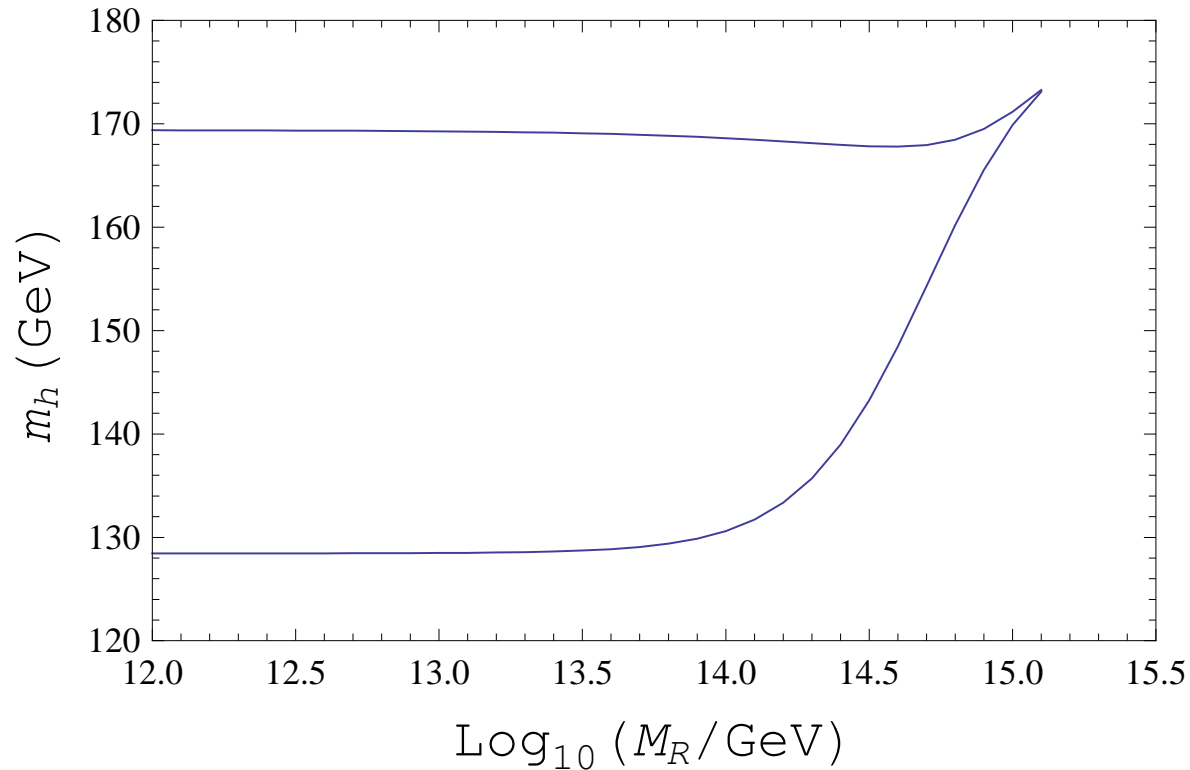


Higgs boson mass bound as a function of M_R in the case with hierarchical mass spectrum. The lower and upper lines correspond to the stability and triviality bounds, respectively. The Higgs boson mass range is closed for $M_R = 1.86 \times 10^{15}$ GeV, where $m_h = 175$ GeV.

Casas et. al. (hep-ph/9904295); Gogoladze et.al. (unpublished).



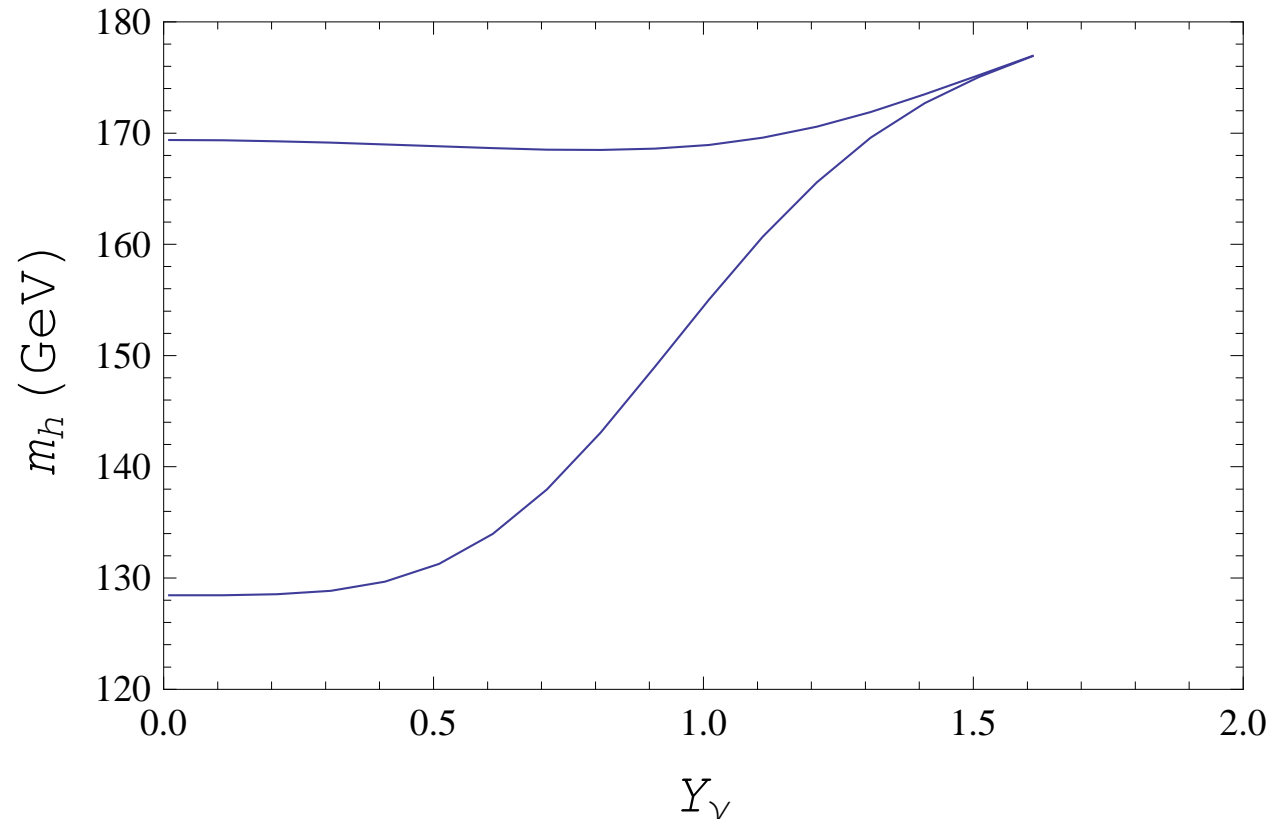
Type I Seesaw and Higgs Mass Bounds



Higgs boson mass bound as a function of M_R in the case with inverted-hierarchical mass spectrum. The lower and upper lines correspond to the stability and triviality bounds, respectively. The Higgs boson mass range is closed for $M_R = 1.26 \times 10^{15}$ GeV, where $m_h = 173$ GeV.



Type I Seesaw and Higgs Mass Bounds



Higgs boson mass bound as a function of Y_ν . The lower and upper lines correspond to the stability and triviality bounds, respectively. The limit $Y_\nu = 0$ reproduces the SM result. The Higgs boson mass range is closed for $Y_\nu \simeq 1.6$.



Type II seesaw

Gogoladze, N.O. & Shafi,

arXive: 0802.3257 [hep-ph]

We introduce a triplet scalar field

$\Delta : (\mathbf{3}, 1)$ under $SU(2)_L \times U(1)_Y$

$$\Delta = \frac{\sigma^i}{\sqrt{2}} \Delta_i = \begin{pmatrix} \Delta^+/\sqrt{2} & \Delta^{++} \\ \Delta^0 & -\Delta^+/\sqrt{2} \end{pmatrix}$$

Scalar potential

$$\begin{aligned} V(\Delta, \phi) = & -m_\phi^2 (\phi^\dagger \phi) + \frac{\lambda}{2} (\phi^\dagger \phi)^2 \\ & + M_\Delta^2 \text{tr} (\Delta^\dagger \Delta) + \frac{\lambda_1}{2} (\text{tr} \Delta^\dagger \Delta)^2 + \frac{\lambda_2}{2} \left[(\text{tr} \Delta^\dagger \Delta)^2 - \text{tr} (\Delta^\dagger \Delta \Delta^\dagger \Delta) \right] \\ & + \lambda_4 \phi^\dagger \phi \text{tr} (\Delta^\dagger \Delta) + \lambda_5 \phi^\dagger [\Delta^\dagger, \Delta] \phi + \left[\frac{\Lambda_6}{\sqrt{2}} \phi^T i \sigma_2 \Delta^\dagger \phi + \text{h.c.} \right], \end{aligned}$$

Many new couplings: $\lambda_1, \lambda_2, \lambda_4, \lambda_5$

$$\Lambda_6 = \lambda_6 M_\Delta$$

Neutrino Yukawa coupling:

$$\mathcal{L}_\Delta = -\frac{1}{\sqrt{2}} (Y_\Delta)_{ij} \ell_L^{Ti} C i \sigma_2 \Delta \ell_L^j + \text{h.c.},$$

$$\langle \phi \rangle = \frac{v}{\sqrt{2}} \rightarrow \text{Tadpole term for the triplet scalar}$$
$$\rightarrow \langle \Delta \rangle \sim \frac{\lambda_6 v^2}{M_\Delta}$$

$$\text{Neutrino mass: } M_\nu = \frac{v^2 \mathbf{Y}_\Delta \lambda_6}{2 M_\Delta}$$

After integrating out the heavy triplet, we have

$$V(\phi)_{\text{eff}} = -m_\phi^2 (\phi^\dagger \phi) + \frac{1}{2} (\lambda - \lambda_6^2) (\phi^\dagger \phi)^2$$

SM Higgs quartic is defined as

$$\lambda_{\text{SM}} = \lambda - \lambda_6^2$$

Now we solve RGEs in the presence of Type II seesaw

Many free parameters: $\lambda, \lambda_1, \lambda_2, \lambda_4, \lambda_5, \lambda_6, Y_\Delta$

For $\mu < M_\Delta$, SM RGEs

For $\mu \geq M_\Delta$,

$$\frac{dg_i}{d \ln \mu} = \frac{b_i}{16\pi^2} g_i^3 + \frac{g_i^3}{(16\pi^2)^2} \left(\sum_{j=1}^3 B_{ij} g_j^2 - C_i y_t^2 \right)$$
$$b_i = \left(\frac{41}{10}, -\frac{19}{6}, -7 \right) \rightarrow b_i = \left(\frac{47}{10}, -\frac{5}{2}, -7 \right)$$

$$\frac{d\lambda}{d \ln \mu} = \frac{1}{16\pi^2} \beta_\lambda^{(1)} + \frac{1}{(16\pi^2)^2} \beta_\lambda^{(2)} \quad \beta_\lambda^{(1)} \rightarrow \beta_\lambda^{(1)} + 6\lambda_4^2 + 4\lambda_5^2$$

with the matching condition: $\lambda_{\text{SM}} = \lambda - \lambda_6^2$

* We employ 2-loop SM RGEs + 1-loop new RGEs

$$\begin{aligned}
16\pi^2 \frac{d\lambda_1}{d\ln\mu} &= - \left(\frac{36}{5}g_1^2 + 24g_2^2 \right) \lambda_1 + \frac{108}{25}g_1^4 + 18g_2^4 + \frac{72}{5}g_1^2g_2^2 \\
&\quad + 14\lambda_1^2 + 4\lambda_1\lambda_2 + 2\lambda_2^2 + 4\lambda_4^2 + 4\lambda_5^2 + 4\text{tr}[\mathbf{S}_\Delta] \lambda_1 - 8\text{tr}[\mathbf{S}_\Delta^2], \\
16\pi^2 \frac{d\lambda_2}{d\ln\mu} &= - \left(\frac{36}{5}g_1^2 + 24g_2^2 \right) \lambda_2 + 12g_2^4 - \frac{144}{5}g_1^2g_2^2 \\
&\quad + 3\lambda_2^2 + 12\lambda_1\lambda_2 - 8\lambda_5^2 + 4\text{tr}[\mathbf{S}_\Delta] \lambda_2 + 8\text{tr}[\mathbf{S}_\Delta^2], \\
16\pi^2 \frac{d\lambda_4}{d\ln\mu} &= - \left(\frac{9}{2}g_1^2 + \frac{33}{2}g_2^2 \right) \lambda_4 + \frac{27}{25}g_1^4 + 6g_2^4 \\
&\quad + (8\lambda_1 + 2\lambda_2 + 6\lambda + 4\lambda_4 + 6y_t^2 + 2\text{tr}[\mathbf{S}_\Delta]) \lambda_4 + 8\lambda_5^2 - 4\text{tr}[\mathbf{S}_\Delta^2], \\
16\pi^2 \frac{d\lambda_5}{d\ln\mu} &= -\frac{9}{2}g_1^2\lambda_5 - \frac{33}{2}g_2^2\lambda_5 - \frac{18}{5}g_1^2g_2^2 \\
&\quad + (2\lambda_1 - 2\lambda_2 + 2\lambda + 8\lambda_4 + 6y_t^2 + 2\text{tr}[\mathbf{S}_\Delta]) \lambda_5 + 4\text{tr}[\mathbf{S}_\Delta^2]. \\
16\pi^2 \frac{d\mathbf{S}_\Delta}{d\ln\mu} &= 6 \mathbf{S}_\Delta^2 - 3 \left(\frac{3}{5}g_1^2 + 3g_2^2 \right) \mathbf{S}_\Delta + 2\text{tr}[\mathbf{S}_\Delta]\mathbf{S}_\Delta \quad \mathbf{S}_\Delta = Y_\Delta^\dagger Y_\Delta
\end{aligned}$$

RGE of λ_6 is decoupled from other RGEs, but plays an important role through the matching condition

Analysis is quite involved.....

We focus on most important parameters: $\lambda_4, \lambda_5, \lambda_6$

$\left\{ \begin{array}{l} \lambda_4, \lambda_5 \text{ appear in Higgs quartic RGE} \\ \lambda_6 \text{ shifts Higgs quartic coupling @ } M_\Delta \text{ by the matching condition} \end{array} \right.$

We analyze RGEs for various λ_6 at the cutoff with others fixed
 λ_5

Fixing the cutoff scale M_{Pl} , we investigate Higgs mass bounds

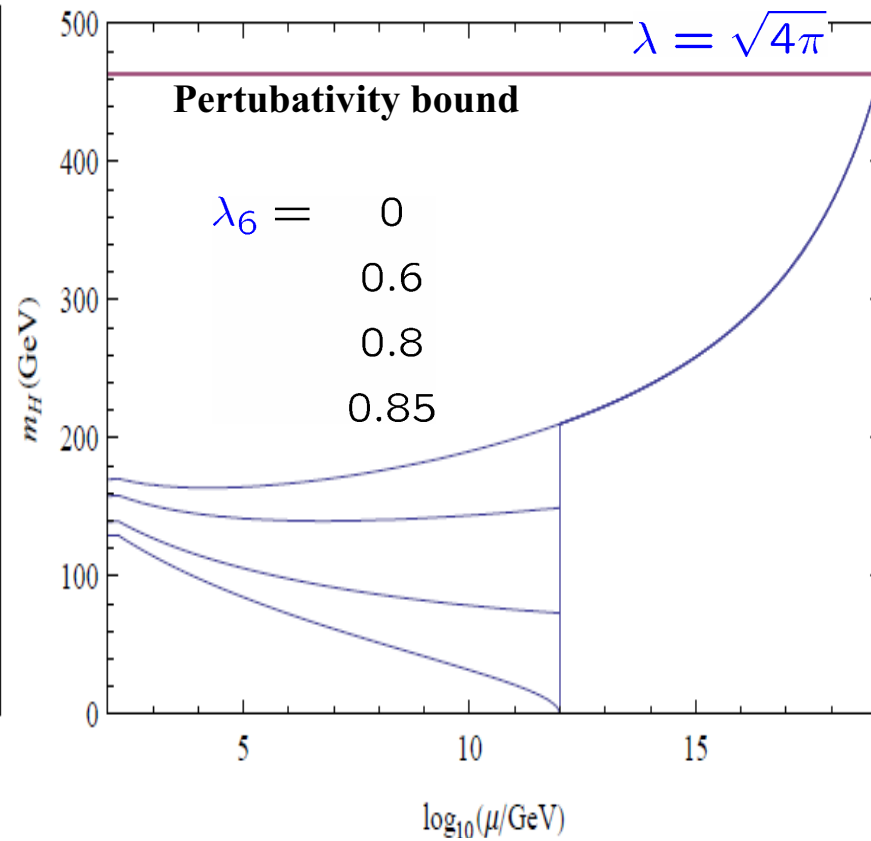
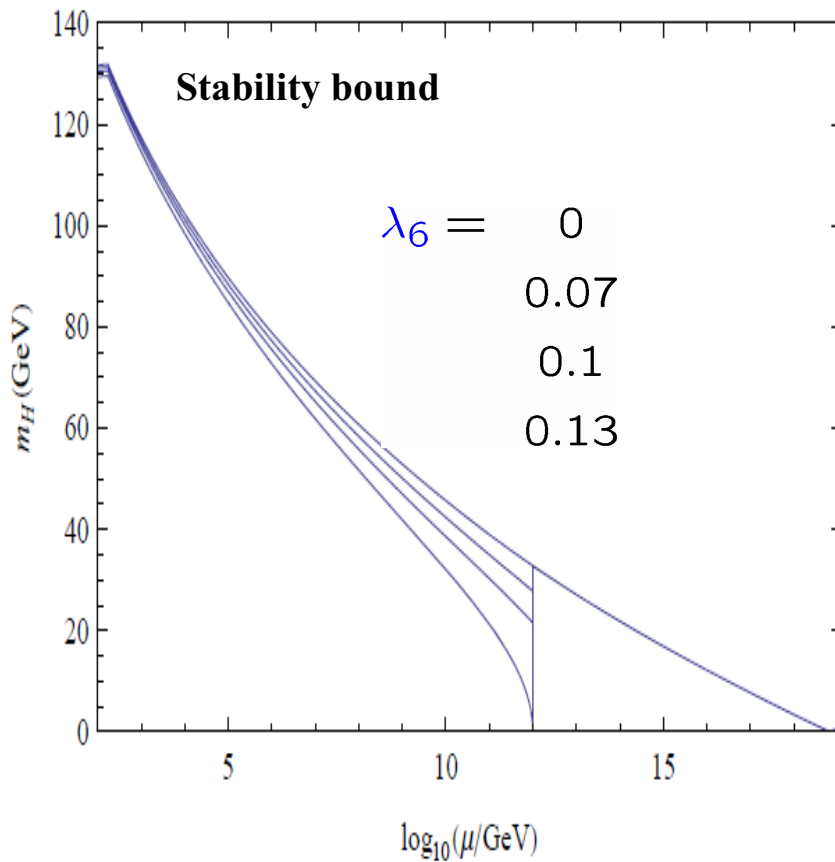
$\left\{ \begin{array}{l} \text{Vacuum stability bound: the lowest Higgs boson mass which satisfies} \\ \lambda(\mu) \geq 0 \text{ for any scale between } M_H \leq \mu \leq M_{Pl} \\ \text{Perturbativity bound: the highest Higgs boson mass which satisfies} \\ \lambda(\mu) \leq \sqrt{4\pi} \text{ for any scale between } M_H \leq \mu \leq M_{Pl} \end{array} \right.$

Sample: $\lambda_1 = \sqrt{4\pi}$, $\lambda_2 = -1$, $\lambda_4 = \lambda_5 = 0$ and $Y_\Delta = 0$.

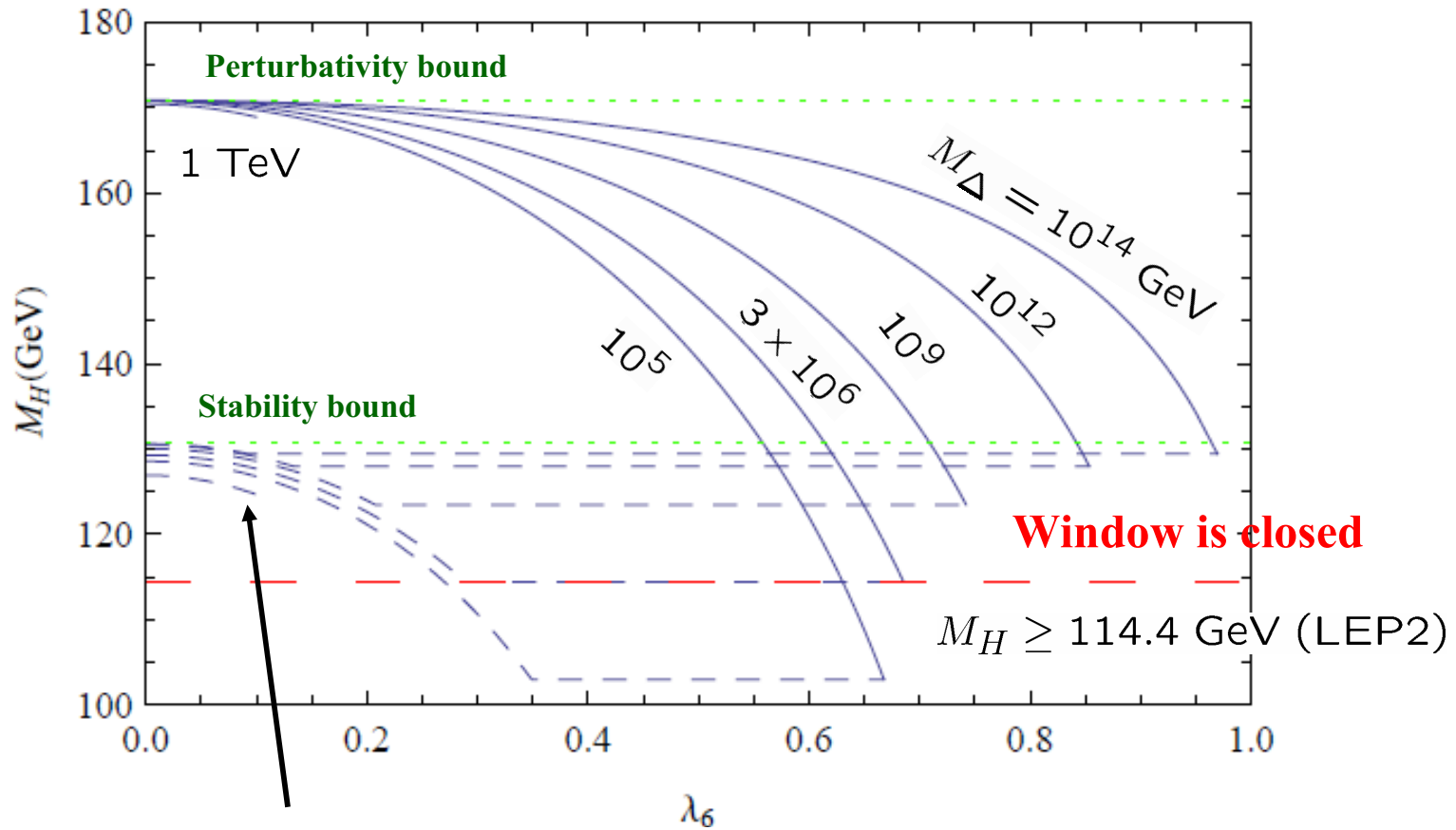
$$M_\Delta = 10^{12} \text{ GeV}$$

Input: top quark pole mass = 172.6 GeV

Running Higgs mass



Higgs mass bounds versus λ_6 for various M_Δ



ρ parameter constraint

$$\langle \Delta \rangle \sim \lambda_6 v^2 / M_\Delta \quad \rightarrow \quad \lambda_6 \lesssim 0.01 M_\Delta / v.$$

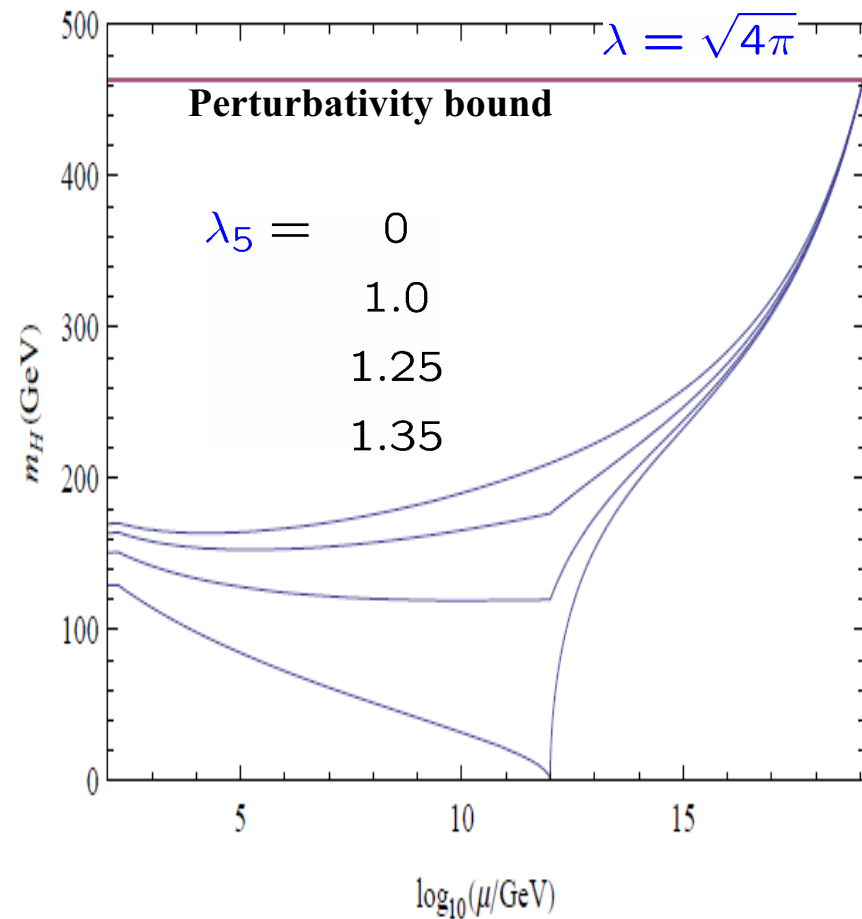
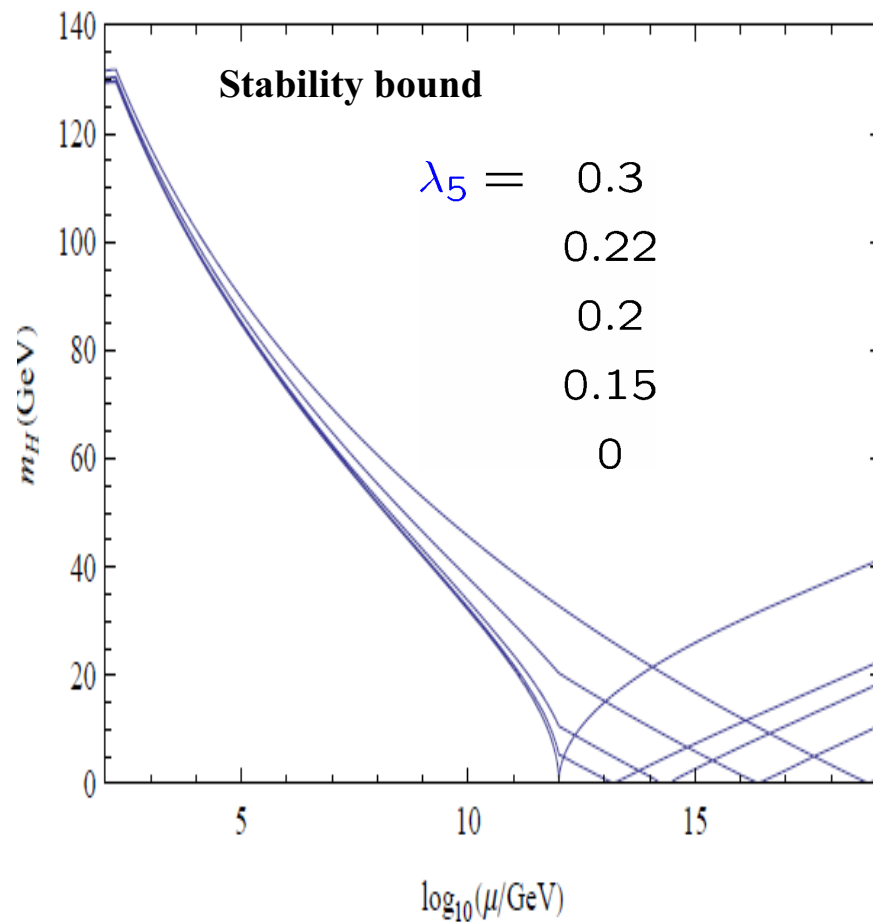
$$\Delta\rho = \rho - 1 \simeq \langle \Delta \rangle / v \lesssim 0.01$$

Sample: $\lambda_1 = \sqrt{4\pi}$, $\lambda_2 = -1$, $\lambda_4 = 0$, $\Lambda_6 = 0$ and $Y_\Delta = 0$.

$$M_\Delta = 10^{12} \text{ GeV}$$

Input: top quark pole mass = **172.6 GeV**

Running Higgs mass

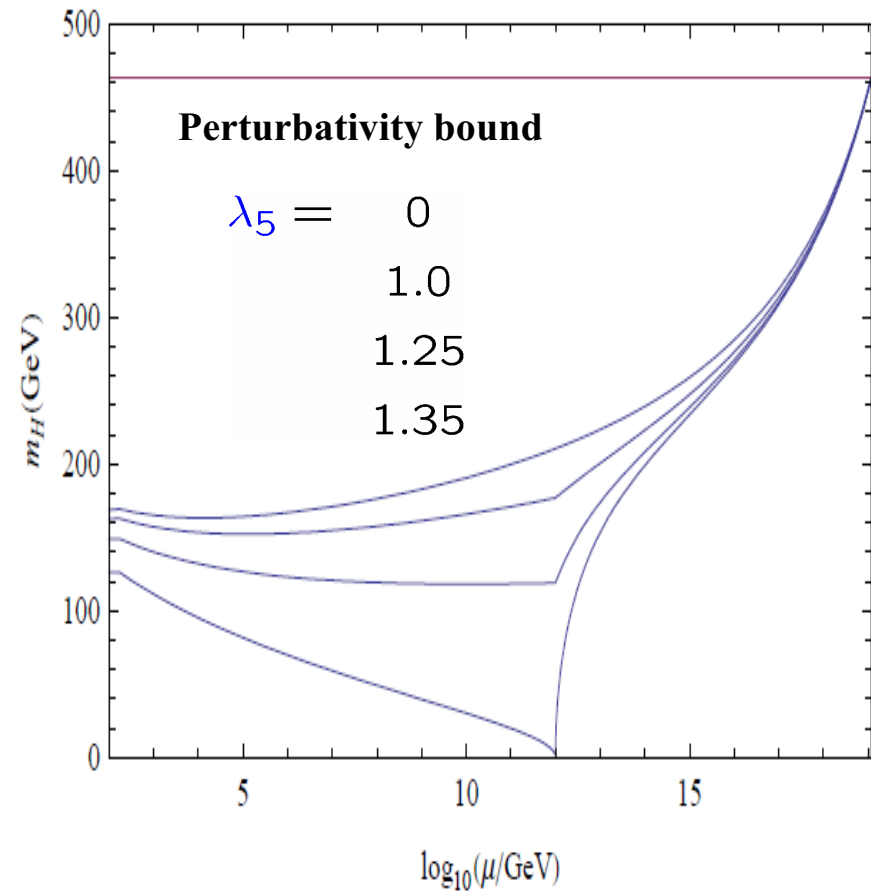
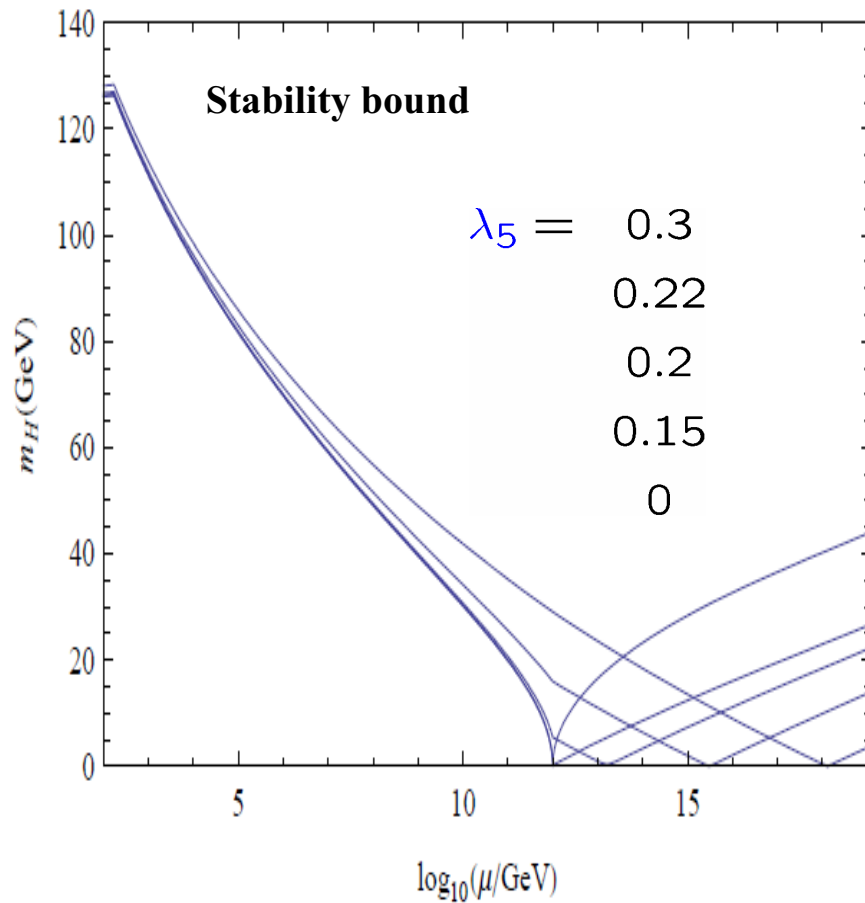


Sample: $\lambda_1 = \sqrt{4\pi}$, $\lambda_2 = -1$, $\lambda_4 = 0$, $\Lambda_6 = 0$ and $Y_\Delta = 0$.

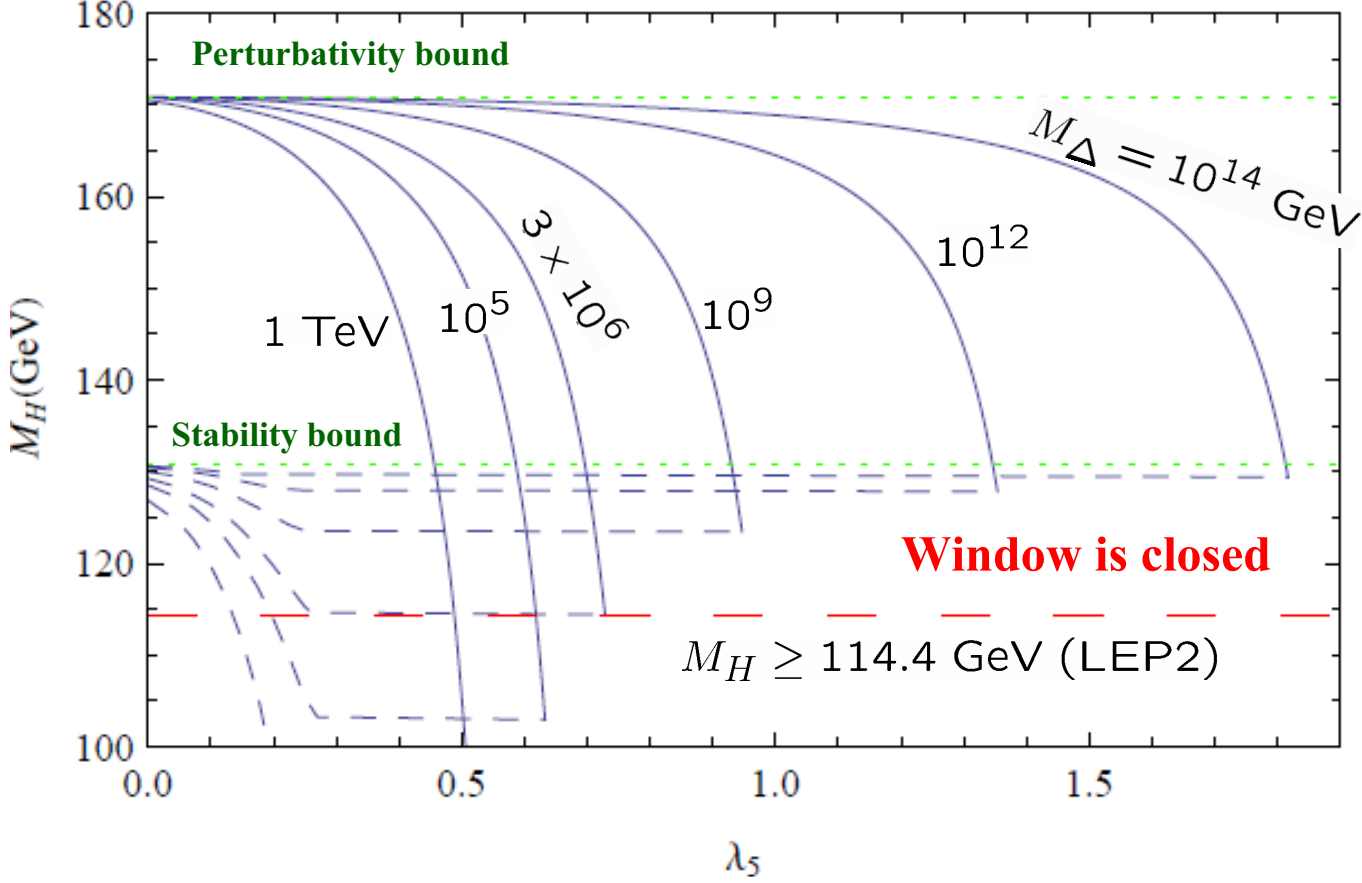
$$M_\Delta = 10^{12} \text{ GeV}$$

Input: top quark pole mass = 172.6 GeV

Running Higgs mass



Higgs mass bounds versus λ_5 for various M_Δ



Type III Seesaw and Higgs Mass

- Introduce 3 generations of fermions ψ_i ($i = 1, 2, 3$), which transform as $(\mathbf{3}, 0)$ under the electroweak gauge group $SU(2)_L \times U(1)_Y$:

$$\psi_i = \sum_a \frac{\sigma^a}{2} \psi_i^a = \frac{1}{2} \begin{pmatrix} \psi_i^0 & \sqrt{2}\psi_i^+ \\ \sqrt{2}\psi_i^- & -\psi_i^0 \end{pmatrix}.$$

- With canonically normalized kinetic terms for the triplet fermions, we introduce the Yukawa coupling

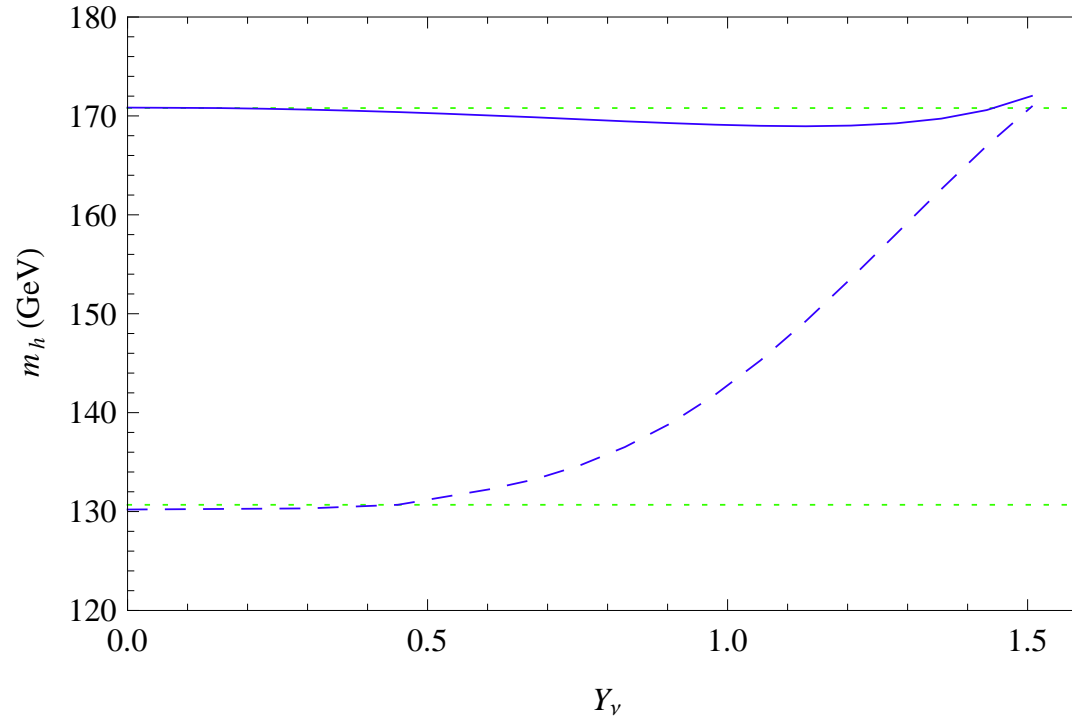
$$\mathcal{L}_Y = y_{ij} \bar{\ell}_i \psi_j \Phi,$$

where Φ is the Higgs doublet.



Type III Seesaw and Higgs Mass

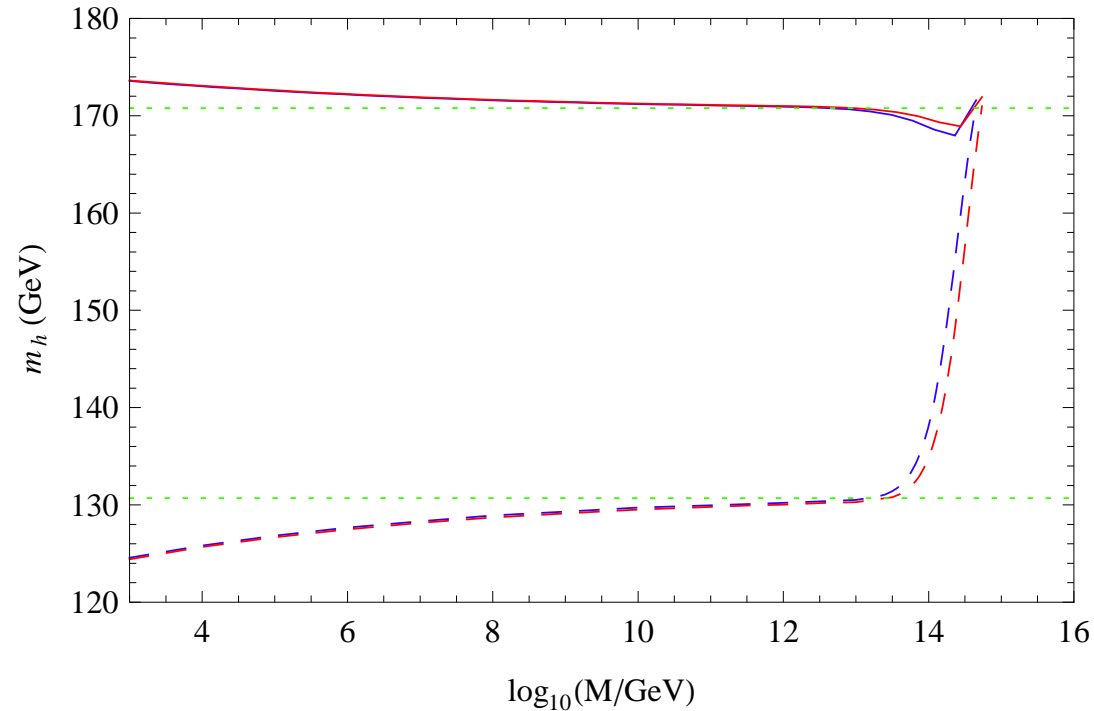
Gogoladze, Okada, Q.S



Perturbativity (solid) and vacuum stability (dashed) bounds on the Higgs boson pole mass (m_h) versus Y_ν with the seesaw scale $M = 10^{13}$ GeV. The upper and lower dotted lines respectively show the perturbativity bound ($m_h \simeq 171$ GeV) and the vacuum stability bound ($m_h \simeq 131$ GeV) in the SM case.



Type III Seesaw and Higgs Mass



Perturbativity and vacuum stability bounds versus M , with a hierarchical mass spectrum (outer region in red), and an inverted-hierarchical mass spectrum (inner region in blue).

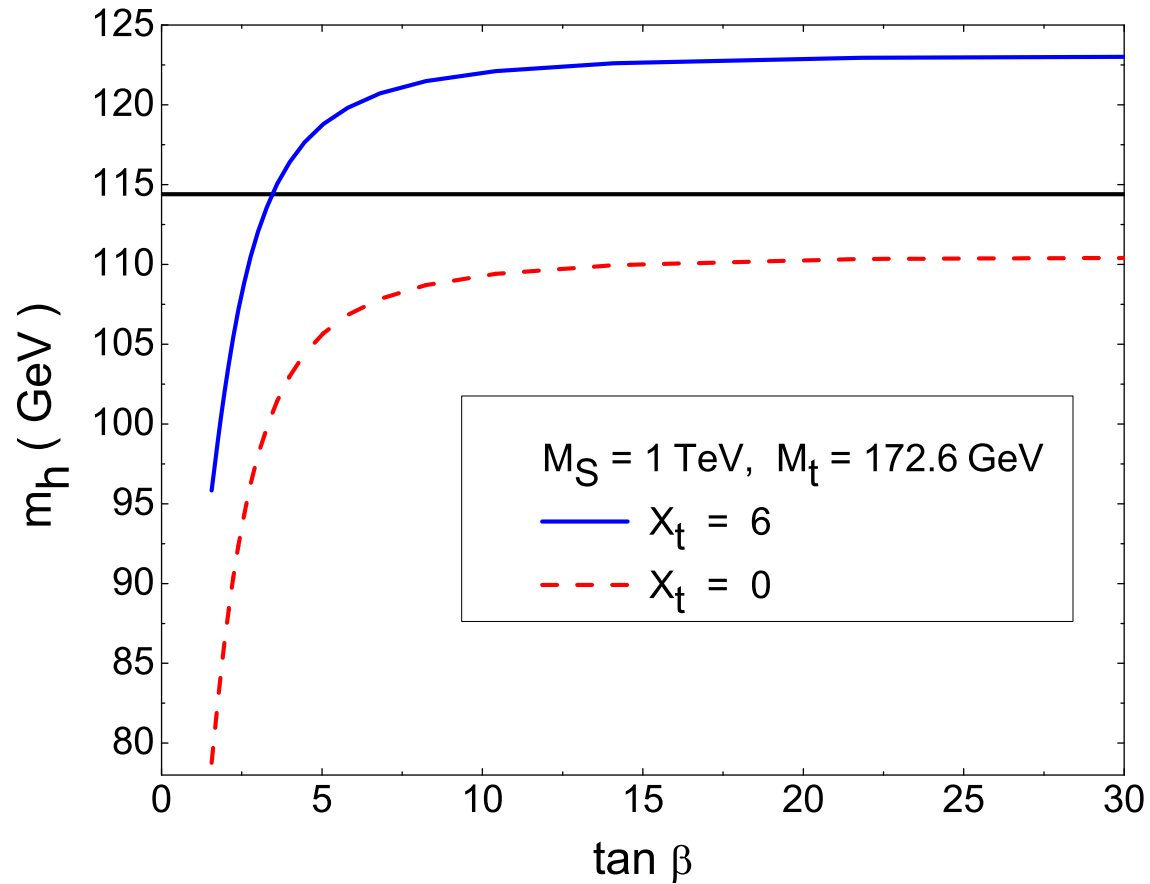


Low Energy Supersymmetry

- Resolution of the gauge hierarchy problem;
- Unification of the SM gauge couplings at $M_{GUT} \sim 2 \times 10^{16} \text{ GeV}$;
- Cold dark matter candidate (LSP);
- Predicts new particles accessible at the LHC;
Other good reasons:
- Radiative electroweak breaking;
- String theory requires susy .
Leading candidate is the MSSM (Minimal Supersymmetric Standard Model).



The MSSM Higgs Boson



WARPED EXTRA DIMENSION

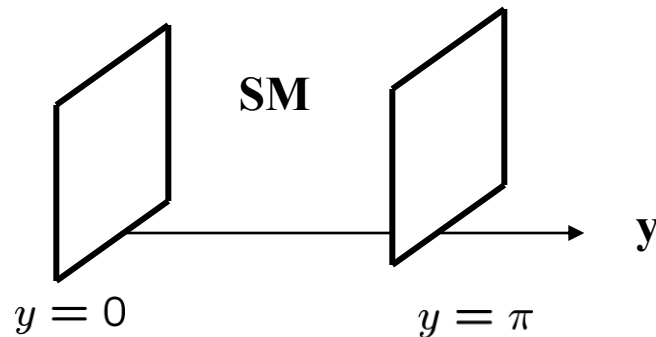
- Resolution of the gauge hierarchy problem (*without invoking susy*);
- ν Oscillations may be accommodated using **dim 5** SM operators;
- ν could be Dirac or Majorana
- may be consistent with GUTS;
- may generate even 'smaller' scales:
 $M_P \rightarrow \text{TeV}^2 / M_P$ ($\sim 10^{-3} \text{eV}$)
- KK excitations at LHC?



Gauge-Higgs Unification

Bulk Standard Model

5-dim. theory compactified on orbifold S^1/Z_2



All SM fields reside in the bulk

Higgs boson associated with 5th component of gauge fields
in higher dimension

Gauge-Higgs Unification

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- [4] N. S. Manton, Nucl. Phys. B **158**, 141 (1979); D. B. Fairlie, J. Phys. G **5**, L55 (1979); Phys. Lett. B **82**, 97 (1979).
- [5] Y. Hosotani, Phys. Lett. B **126**, 309 (1983); Phys. Lett. B **129**, 193 (1983); Phys. Rev. D **29**, 731 (1984); Annals Phys. **190**, 233 (1989).
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- [8] N. Haba, S. Matsumoto, N. Okada and T. Yamashita, JHEP **0602**, 073 (2006).
- [9] M. Sakamoto and K. Takenaga, Phys. Rev. D **75**, 045015 (2007).
- [10] N. Maru and T. Yamashita, Nucl. Phys. B **754**, 127 (2006).
- [11] For example, see G. Cacciapaglia, C. Csaki and S. C. Park, JHEP **0603**, 099 (2006), and references therein.

$$SU(3) \supset SU(2) \times U(1)$$

SU(3) gauge =

$SU(2)$	doublet
adj	
doublet	singlet

Impose non-trivial boundary conditions (parity assignment)

$$A_\mu = \left(\begin{array}{c|c} W_\mu + B_\mu & X_\mu \\ \hline X_\mu^\dagger & B_\mu \end{array} \right) \quad A_5 = \left(\begin{array}{c|c} W_5 + B_5 & H \\ \hline H^\dagger & B_5 \end{array} \right)$$

○ are Z2 even fields, others odd fields

Zero modes for odd fields are project out,

So SU(3) is broken to SU(2) X U(1) by this parity assignment

5th component of 5dim gauge field → scalar in 4D theories

We identify “H” as Higgs doublet in the SM → gauge-Higgs unification

$$\mathcal{L}_5^{gauge} = -\frac{1}{4}F^{aMN}F_{MN}^a = -\frac{1}{4}F^{a\mu\nu}F_{\mu\nu}^a + \frac{1}{2}F_5^{a\mu}F_{5\mu}^a$$

$$\frac{1}{2}F_5^{a\mu}F_{5\mu}^a \rightarrow (D_\mu H)^\dagger(D_\mu H)$$

$$H : (2, +1/2)$$

Kinetic term for Higgs is included as

5dim SU(3) gauge interaction

5 dimensional gauge symmetry → No mass and quartic coupling @ tree

Phenomenologically interesting observation “Gauge-Higgs Condition”

→ realization of gauge-Higgs unification at UV

is equivalent to imposing “vanishing quartic Higgs coupling”

at $\Lambda_{cut} = 1/(2\pi R)$

Haba, Matsumoto, N.O.&Yamashita,
JHEP 0602, 073 (2006)

Application of the gauge-Higgs condition

UV completion of the SM by (5D) gauge-Higgs unification

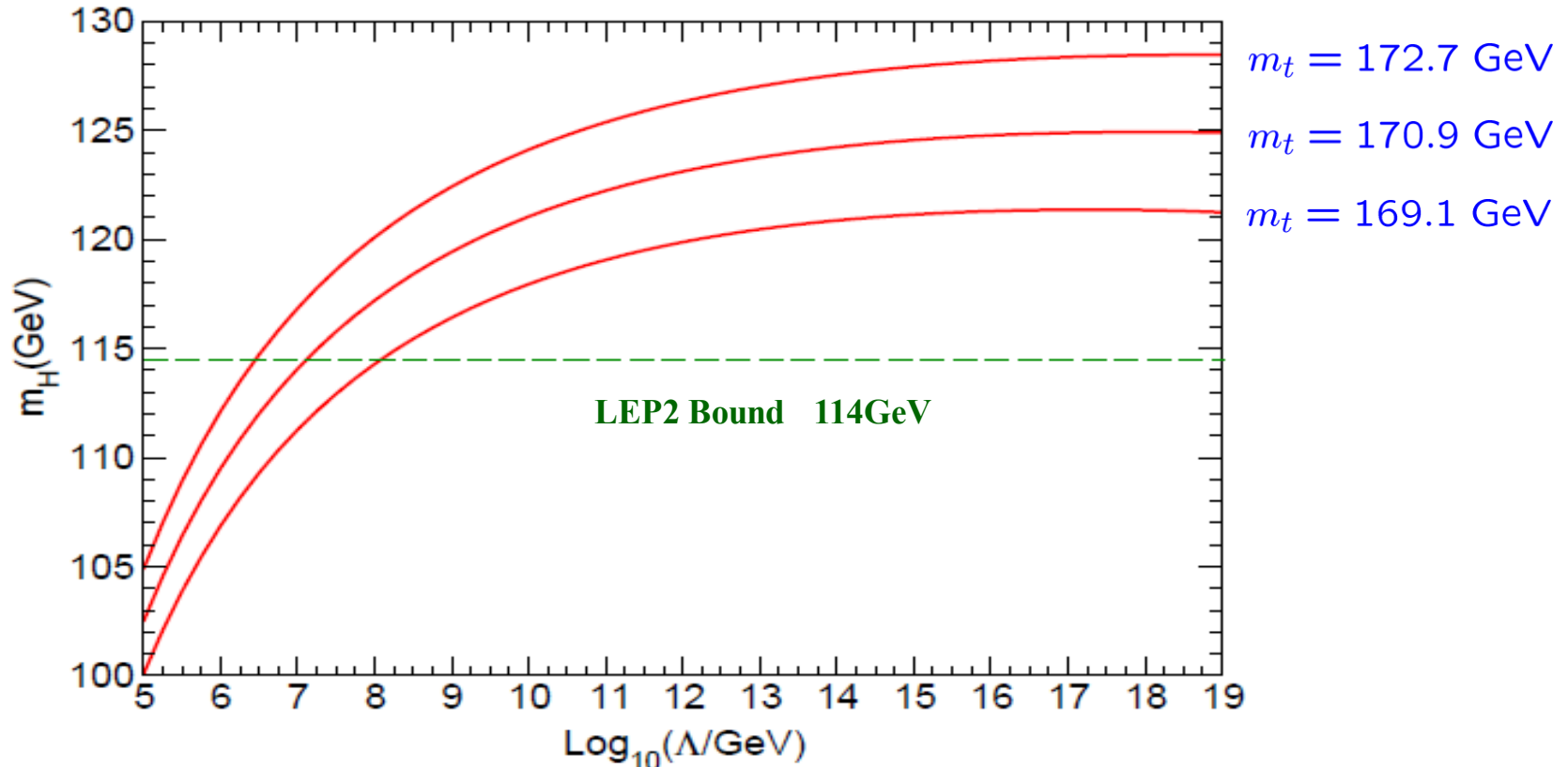
→ Higgs boson mass prediction

as a function of the compactification scale

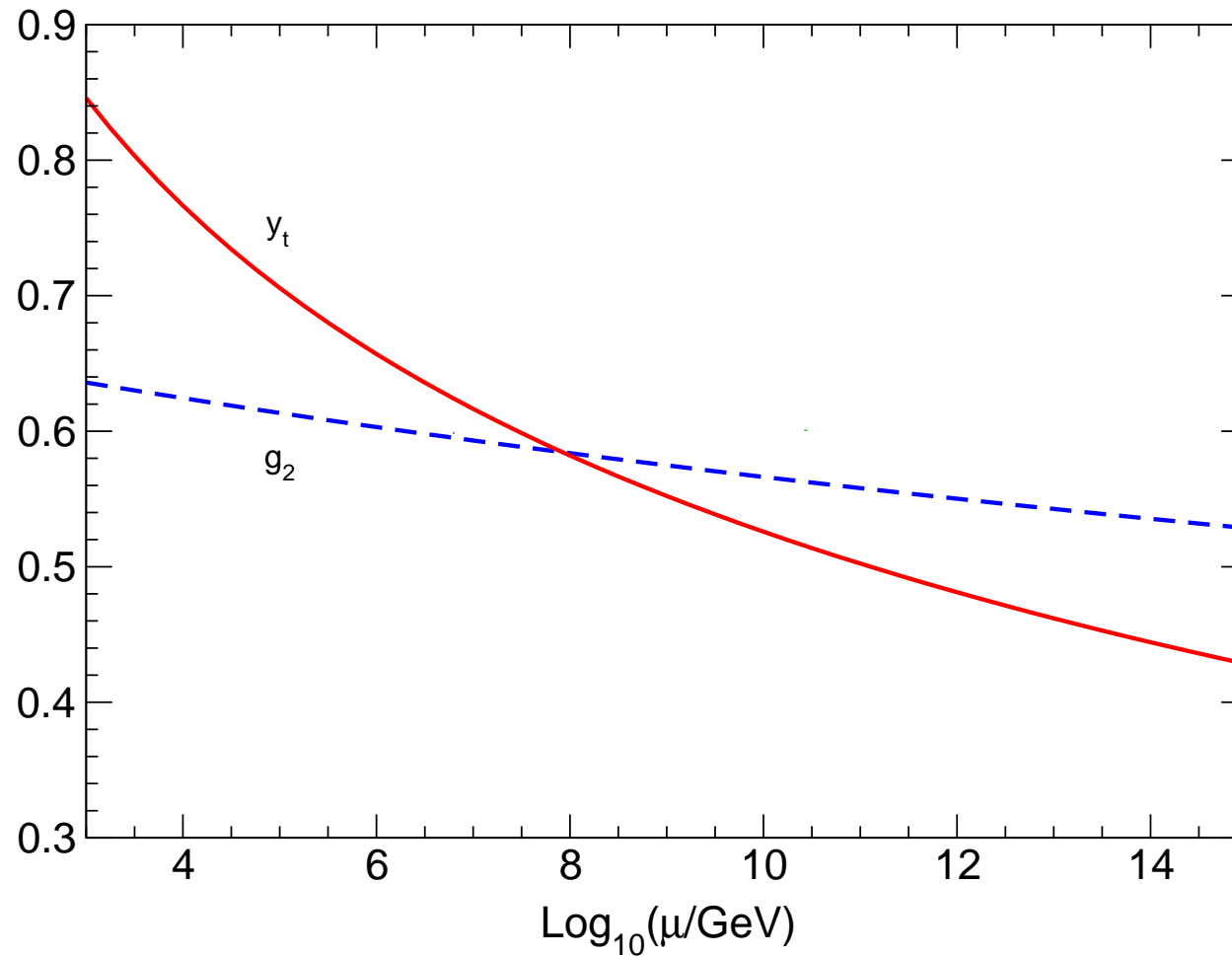
by imposing the condition $\lambda_H(\Lambda) = 0$

Gogoladze, N.O. & Shafi

Phys. Lett. B, 257 (2007)



Gauge and top Yukawa Unification



Gauge and top Yukawa Unification

	$M_t = 169.1$	$M_t = 170.9$	$M_t = 172.8$
Λ	3.26×10^7	8.41×10^7	2.34×10^8
m_h	112.9	117.0	121.1



Standard Model (SM) + Einstein' GR

⇒ Hot Big Bang Cosmology

Predictions

- Existence of CMB;
- Redshift (**Galaxies**);
- Primordial Nucleosynthesis.



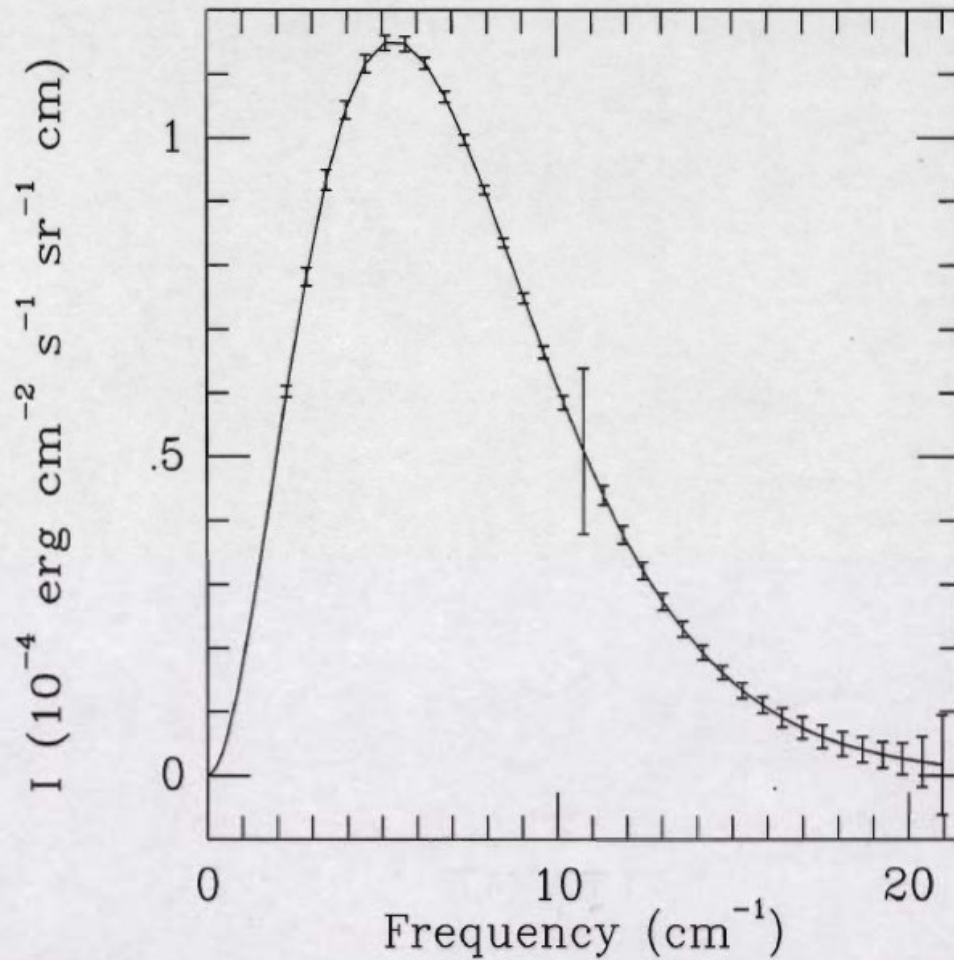
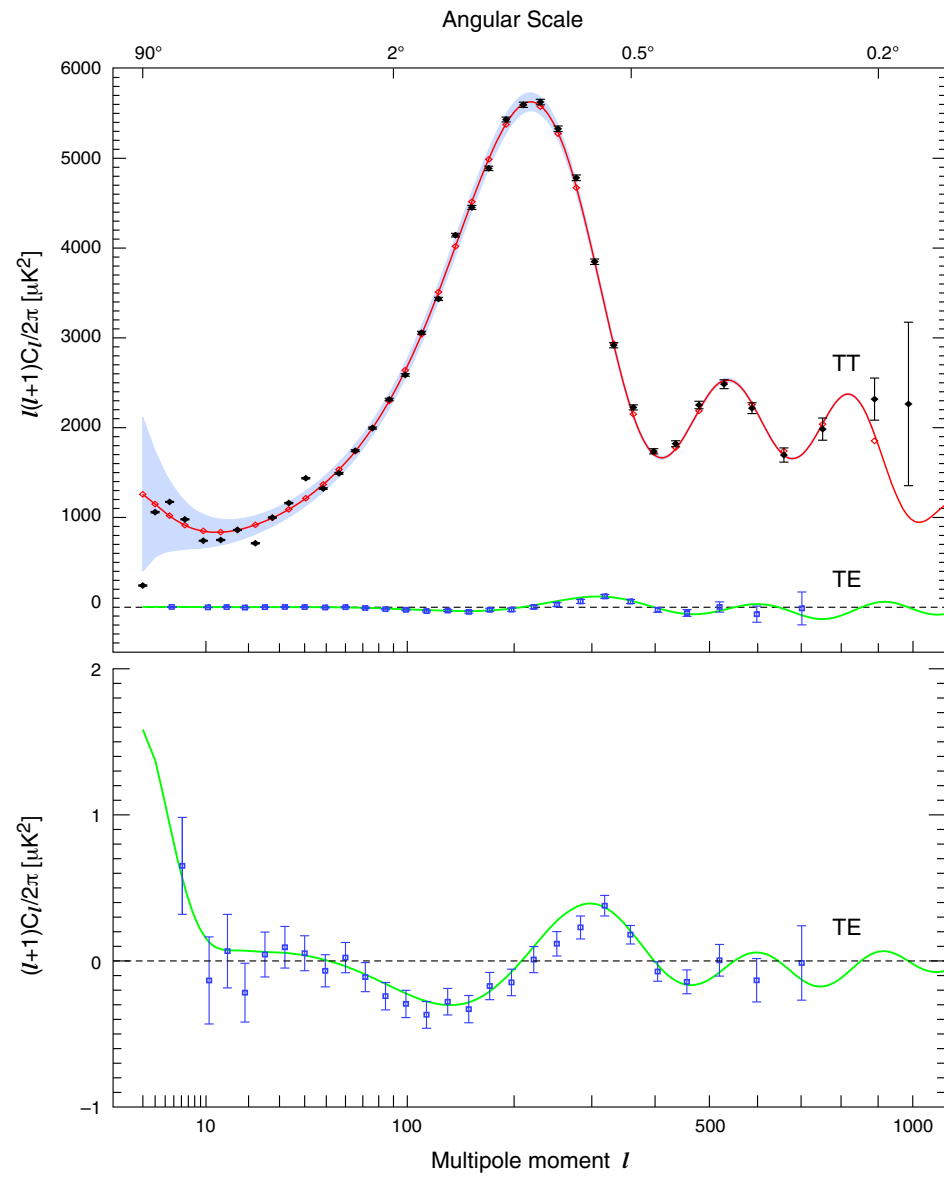


Figure 4: Spectrum of the Cosmic Microwave Background Radiation as measured by the FIRAS instrument on COBE and a black body curve for $T = 2.7277 \text{ K}$. Note, the error flags have been enlarged by a factor of 400. Any distortions from the Planck curve are less than 0.005% (see Fixsen *et al.*, 1996).



Standard Model (SM) + Einstein' GR

Hot Big Bang Cosmology fails to explain

1) Observed Isotropy of CMB(COBE)

2) Origin of $\frac{\delta T}{T}$ -COBE, ..., WMAP

3) $\Omega_{total} = 1$ (critical density)

4) $\Omega_{CDM} = 0.22$ (non-baryonic DM)

5) $n_b/n_\gamma = 10^{-10}$ (baryon asymmetry)

● If GR stays intact, an extension of the SM is needed.

(Dark Energy?)



Inflationary Cosmology

- Inflationary Cosmology can take care of (1), (2), (3) and an inflation model can be called "realistic" if it can explain (4) → CDM and (5) → n_b/n_γ . Some Models also provide a link with (6) → neutrino physics.
- Testable predictions?



Inflationary Cosmology

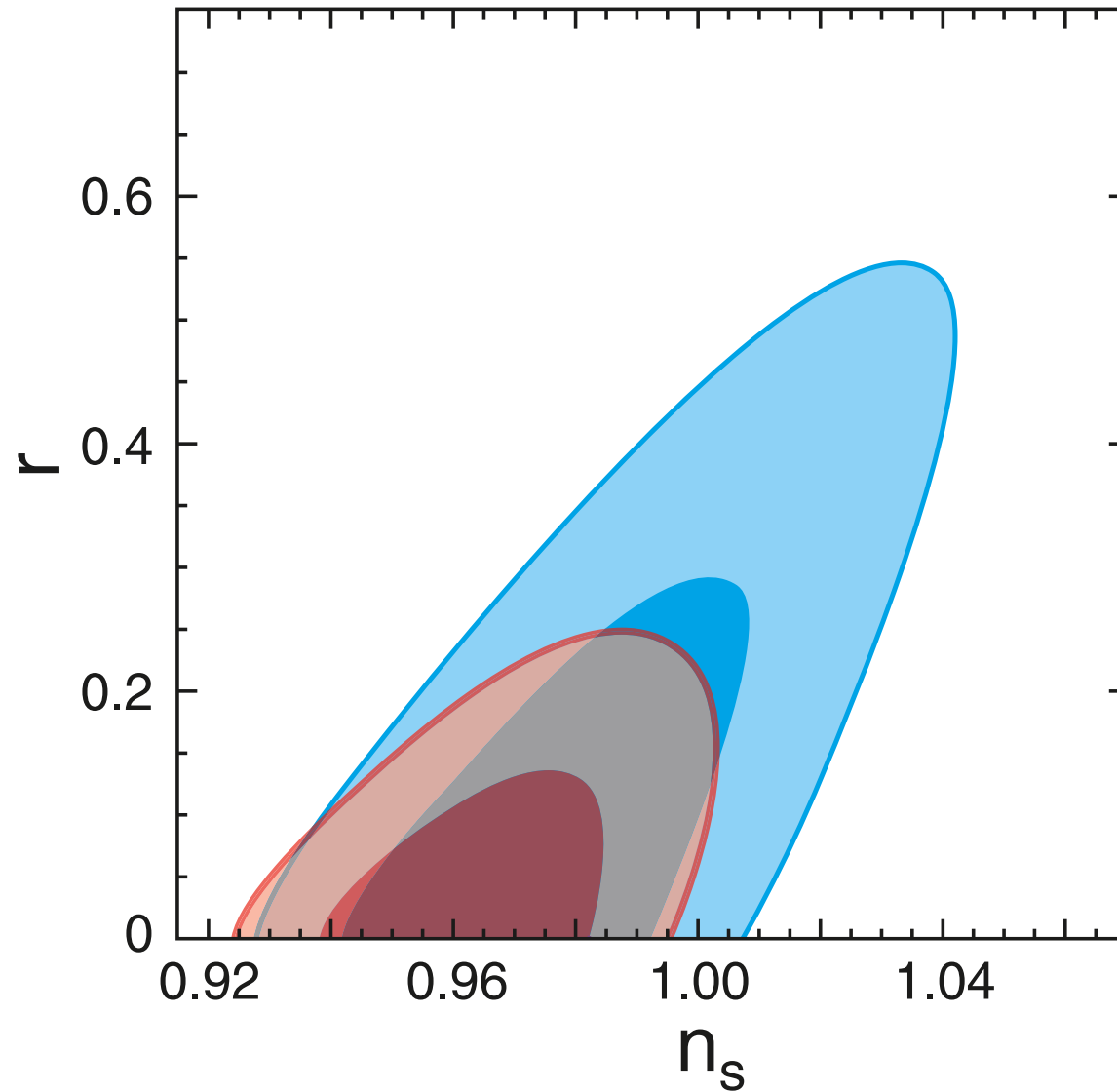
- One key parameter in cosmology is the scalar spectral index n_s . According to **Harrison and Zeldovich** (HZ), $n_s = 1$ is the most 'natural' value, referred to as the scale invariant value.
- The most recent analysis from WMAP-5 yields $n_s = 0.96 \pm 0.014$

(WMAP 1 : $n_s \approx 0.99 \pm 0.04$)

A far more precise determination of n_s is crucial for distinguishing inflation models.



Inflationary Cosmology



Inflationary Cosmology

- Inflation model come in variety of flavors. These include:
 - Chaotic Inflation (Linde,, Murayama,, Yanagida)
 - New Inflation (Linde, Albrecht, Steinhardt,, Senoguz,,)
 - Hybrid Inflation (non-susy , susy)
 - Supergravity Inflation
 - Brane Inflation (Dvali, Tye, Q.S.....)
 - Compactification (Arkani Hamed et.al,, Schmidt et.al,,)
 - Quintessence/Inflation



Quartic (CW) Potential (non-susy)

Q.S, Vilenkin; Senoguz

$$V(\phi) = A\phi^4 \left(\ln \left(\frac{\phi}{M} \right) - \frac{1}{4} \right) + \frac{AM^4}{4}$$

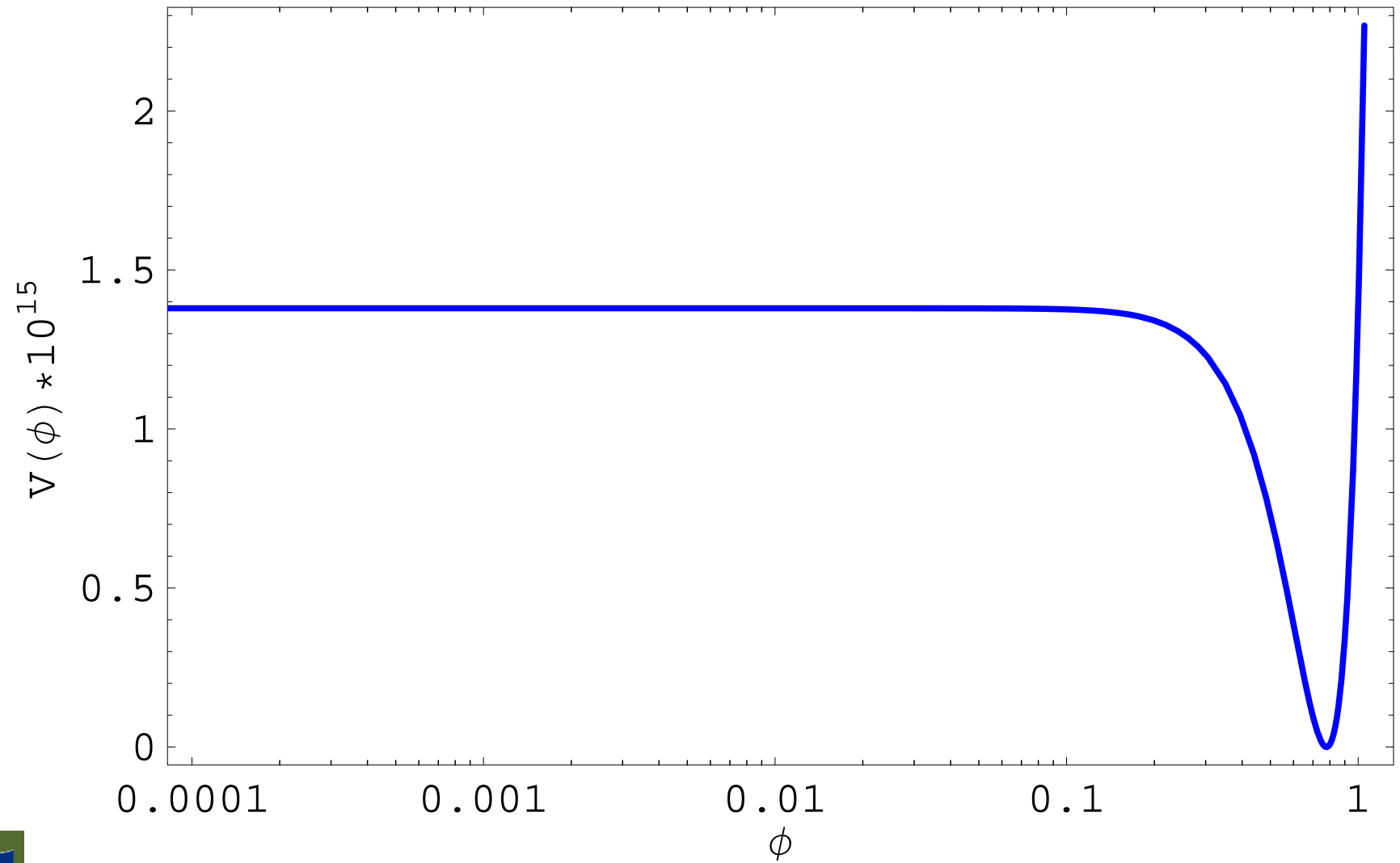
↑
 $\phi =$ gauge singlet

$$V(\phi = M) = 0; V(\phi = 0) = \frac{AM^4}{4} \equiv V_0$$

$$V(\phi \ll M) = \frac{AM^4}{4} - b\phi^4$$



Quartic (CW) Potential (non-susy)



Quartic (CW) Potential (non-susy)

• For $V_0^{1/4} < 10^{16}$ GeV,

$$\phi < m_P (\simeq 2.4 \times 10^{18} \text{ GeV})$$

$$V \simeq V_0 \left(1 - (\phi/M)^4 \right)$$

$$n_s \simeq 1 - \frac{3}{N_0}, \quad \alpha \simeq (n_s - 1) / N_0$$

↑
e-folds for $k_0 = 0.002 Mpc^{-1}$

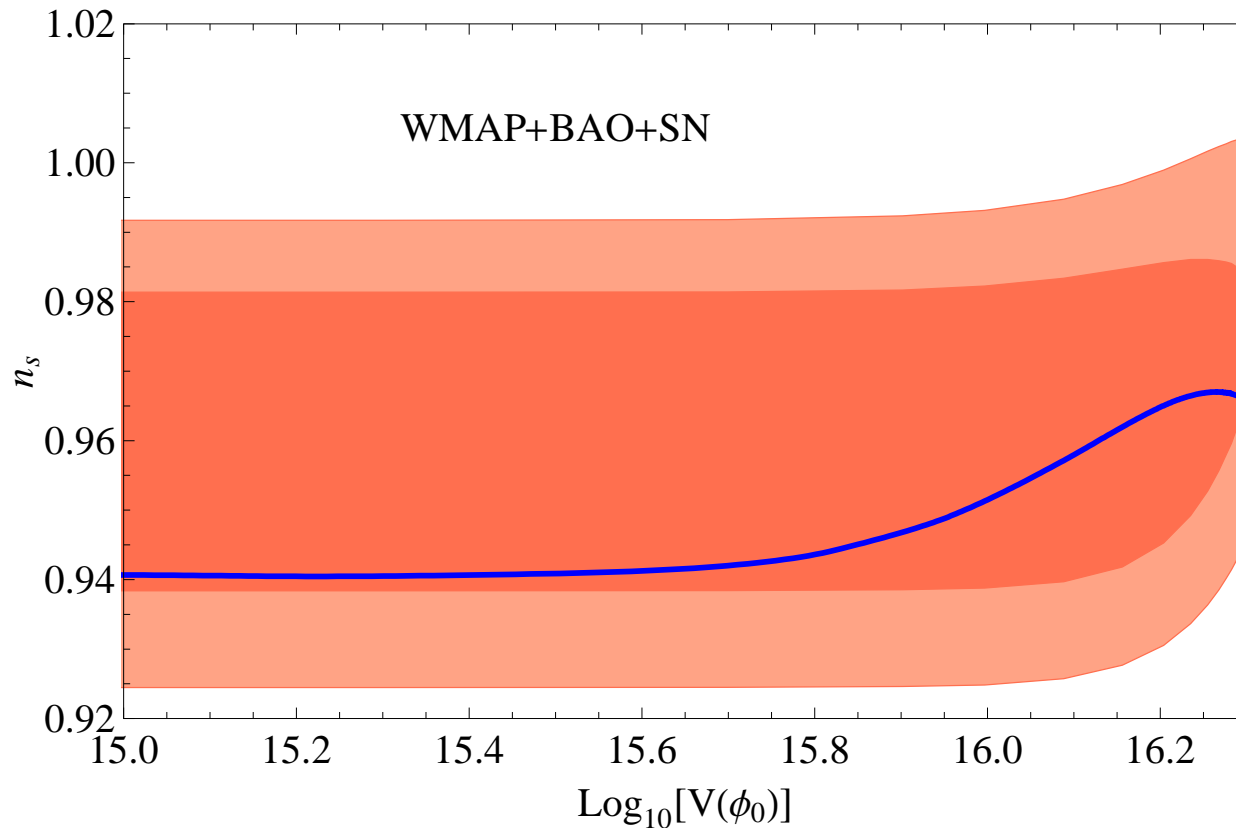
($V_0^{1/4} > 10^5$ GeV to avoid conflict with WMAP)



From New to Large Field Inflation

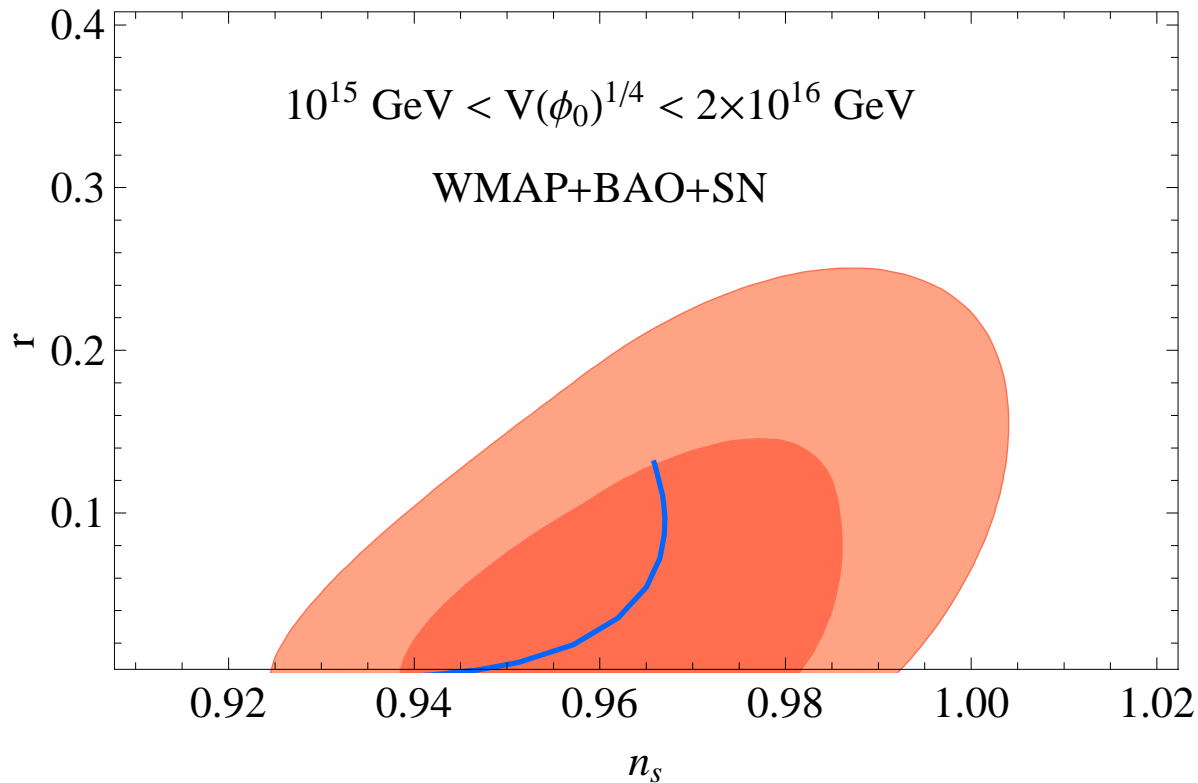
- For $V_0^{1/4} \gtrsim 10^{16}$ GeV, $\phi > m_P$ during observable inflation. Predictions approach that of ϕ^2 potential, with
$$n_s = 1 - \frac{2}{N_0} \simeq 0.96$$
$$r \simeq 0.13$$
$$\alpha \simeq -0.6 \times 10^{-3}$$





The spectral index n_s vs $\log[V(\phi_0)^{1/4}$ (GeV)] for the Coleman-Weinberg potential (green curve), compared with the WMAP+BAO+SN range for n_s (68% and 95% confidence levels, taken from Komatsu *et al.*, astro-ph/0803.0547).





The tensor to scalar ratio r vs the spectral index n_s for the Coleman-Weinberg potential (blue curve). The WMAP+BAO+SN contours (68% and 95% confidence levels) are taken from Komatsu *et al.*, astro-ph/0803.0547.



$$(m_P = 1)$$

$V_0^{1/4}(\text{GeV})$	$A(10^{-14})$	M	ϕ_e	ϕ_0	$V(\phi_0)^{1/4}(\text{GeV})$	n_s	$\alpha(-10^{-3})$	r
10^{13}	1.0	0.018	0.010	3.0×10^{-6}	$\approx V_0^{1/4}$	0.938	1.4	9×10^{-15}
5×10^{13}	1.2	0.088	0.050	7.5×10^{-5}	$\approx V_0^{1/4}$	0.940	1.3	5×10^{-12}
10^{14}	1.3	0.17	0.10	3.0×10^{-4}	$\approx V_0^{1/4}$	0.940	1.2	9×10^{-11}
5×10^{14}	1.9	0.79	0.51	7.5×10^{-3}	$\approx V_0^{1/4}$	0.941	1.2	5×10^{-8}
10^{15}	2.3	1.5	1.1	0.030	$\approx V_0^{1/4}$	0.941	1.2	9×10^{-7}
5×10^{15}	4.8	6.2	5.1	0.71	$\approx V_0^{1/4}$	0.942	1.0	5×10^{-4}
10^{16}	5.2	12	10	3.2	9.9×10^{15}	0.952	1.0	8×10^{-3}
2×10^{16}	1.1	36	35	23	1.7×10^{16}	0.966	0.6	0.07
3×10^{16}	.17	86	85	72	1.9×10^{16}	0.967	0.6	0.11
10^{17}	.001	1035	1034	1020	2.0×10^{16}	0.966	0.6	0.14



Magnetic Monopoles and Inflation

Lazarides, Q.S

- Consider the breaking

$$SO(10) \longrightarrow 4 - 2 - 2 \longrightarrow 3 - 2 - 1$$

First breaking produces superheavy monopoles carrying one unit of Dirac charge

$$\pi_2(SO(10)/4 - 2 - 2) = Z_2;$$

The second breaking at scale M_c produces monopoles which carry two units of Dirac magnetic charge. These are intermediate mass monopoles and they may survive inflation.



Magnetic Monopoles and Inflation

- Consider the quartic coupling $-c\phi^2\chi^\dagger\chi$, with $c \sim (M_c/M)^2$. Here χ vev breaks $4 - 2 - 2$ to $3 - 2 - 1$ and ϕ is the inflaton.
- Monopole formation occurs when $c\phi^2 \sim H^2$
 $\longrightarrow H(t - t_\chi) \equiv \eta \sim 3c/\lambda$.
- Initial monopole number density $\sim H^3$, which gets diluted by inflation down to $H^3 \exp(-3\eta)$; thus,
 $r_M = n_M/T_R^3 \sim (H/T_R)^3 \exp(-3\eta) \lesssim 10^{-30}$.
- Roughly 25- 30 e -folds can yield a flux close to or below the Parker bound.



Chaotic Inflation and Precision Cosmology

Scalar field ϕ with yukawa interactions $\frac{h}{2}\phi NN$.

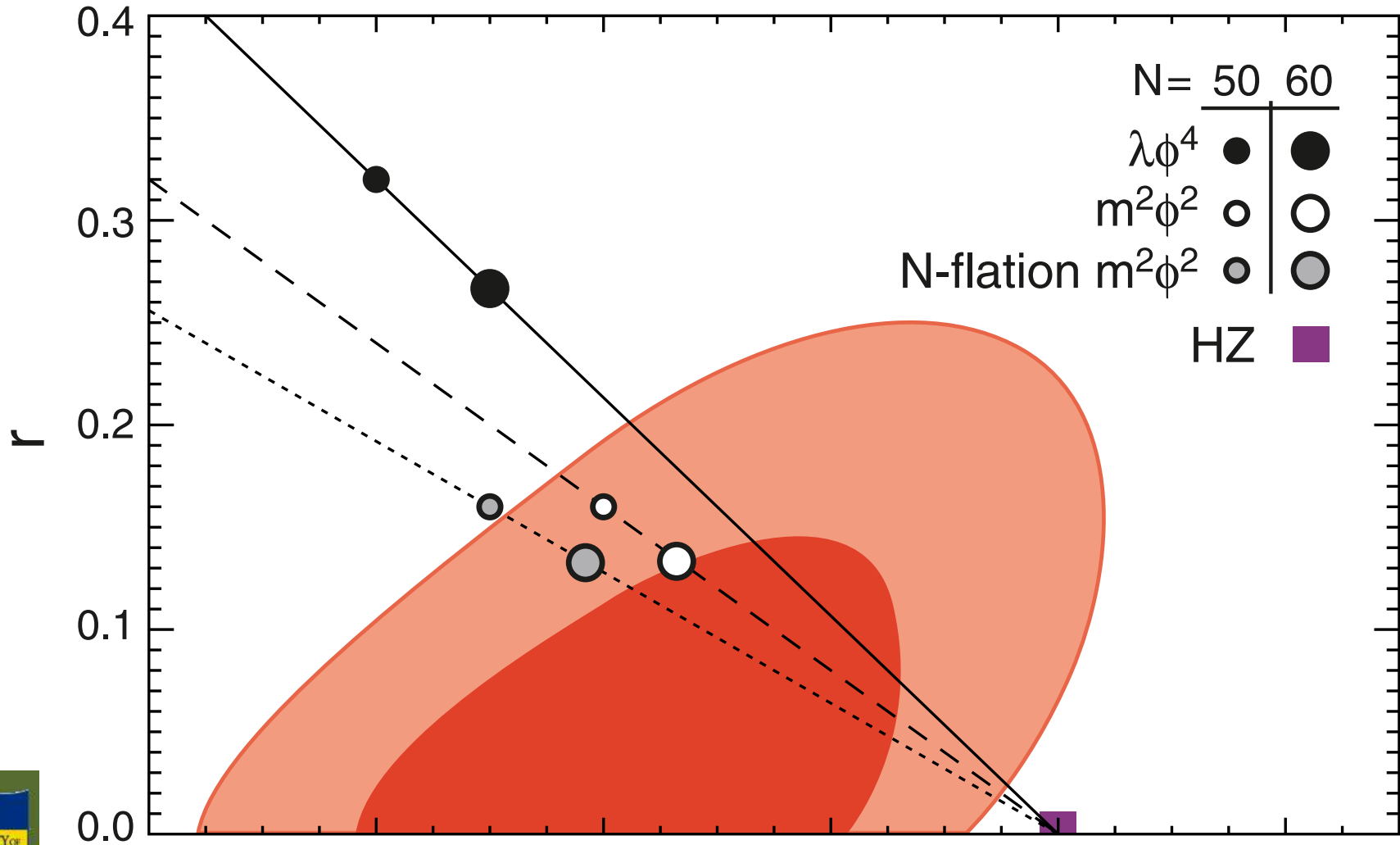
$$V = \frac{1}{2}m^2\phi^2 + \frac{\lambda}{4!}\phi^4 + V_{1-loop},$$

where

$$V_{1-loop} = \frac{1}{64\pi^2} \left[\left(m^2 + \frac{\lambda}{2}\phi^2 \right)^2 \ln \left(\frac{m^2 + (\lambda/2)\phi^2}{\mu^2} \right) - 2 (h\phi + m_N)^4 \ln \left(\frac{h\phi + m_N}{\mu} \right) \right].$$



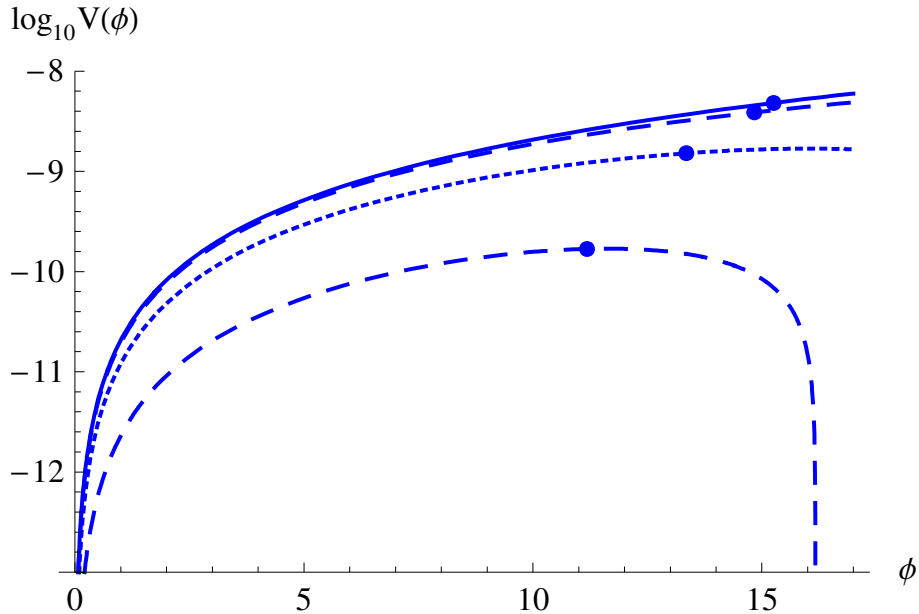
Chaotic Inflation



The inflationary parameters for the potential $V = (1/2)m^2\phi^2 - \kappa\phi^4 \ln(\phi/m_P)$

(in units $m_P = 1$)

$\log_{10}(\kappa)$	m (10^{-6})	ϕ_e	ϕ_0	$V(\phi_0)^{1/4}$	N_0	u_0	n_s	r	α (10^{-4})
$V = (1/2)m^2\phi^2$ (assuming $\rho_{\text{reh}} = 10^{-16}m^2m_P^2$)									
	6.437	1.457	15.26	0.008334	58.31		0.9657	0.1349	-5.901
ϕ^2 branch									
-16	6.434	1.457	15.25	0.008322	58.31	319.4	0.9657	0.1341	-5.901
-15	6.383	1.457	15.15	0.008204	58.47	30.2	0.9656	0.1267	-5.853
-14.5	6.245	1.457	14.83	0.007891	58.5	8.355	0.9645	0.1085	-5.647
-14.2	5.798	1.457	14.19	0.007212	58.43	3.165	0.9591	0.07567	-4.423
-14.11	4.917	1.456	13.35	0.006241	58.23	2.067	0.9459	0.04239	-1.254
Hilltop branch									
-14.11	4.917	1.456	13.35	0.006241	58.23	2.067	0.9459	0.04239	-1.254
-14.2	3.628	1.456	12.35	0.005019	57.93	0.3324	0.9219	0.01769	3.196
-14.5	2.146	1.455	11.18	0.003603	57.48	0.1447	0.8852	0.004665	6.022
-15	1.032	1.455	10.04	0.002344	56.88	0.0531	0.8424	0.000826	5.236
-16	0.268	1.453	8.617	0.001103	55.86	0.0102	0.7762	0.000039	2.254



The tree level potential (solid), the ϕ^2 and hilltop solution potentials for $\log_{10}(\kappa) = -14.5$ (dashed), and the potential for $\log_{10}(\kappa) = -14.11$ where the two solutions meet (dotted). The points on the curves denote ϕ_0 .

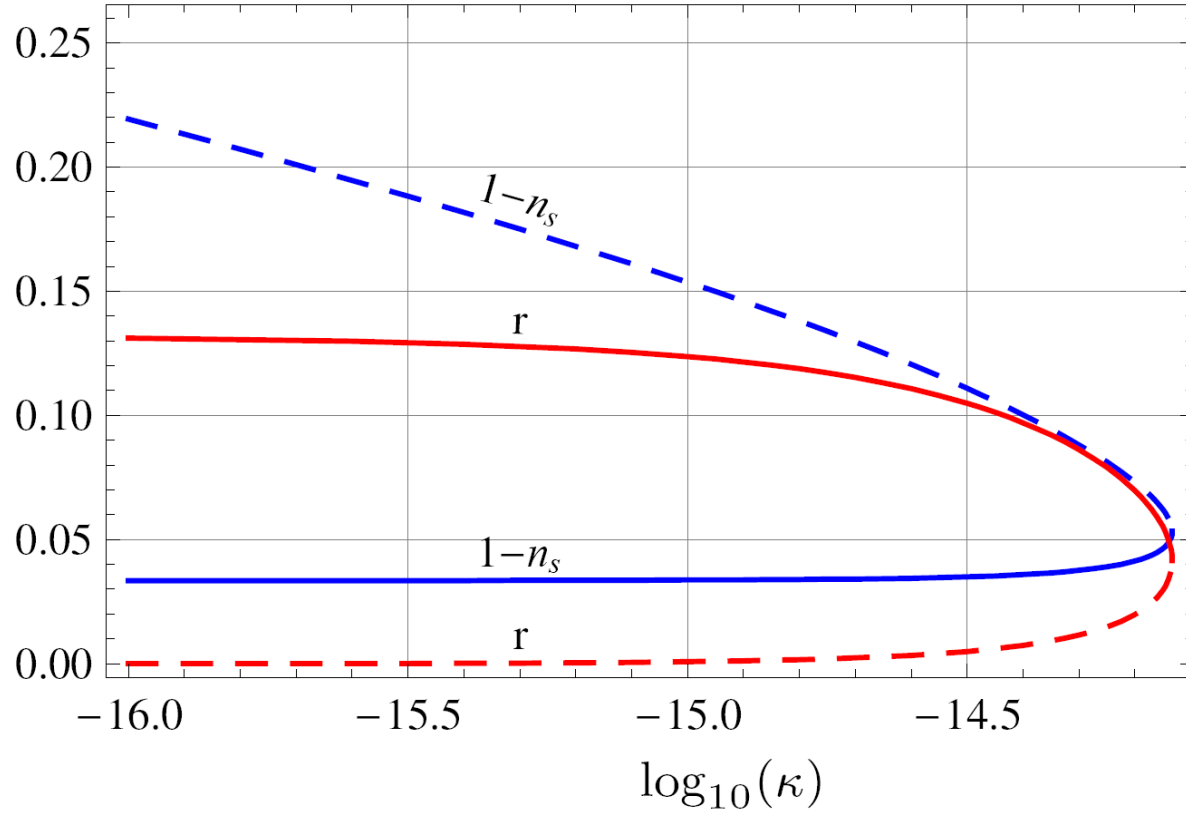
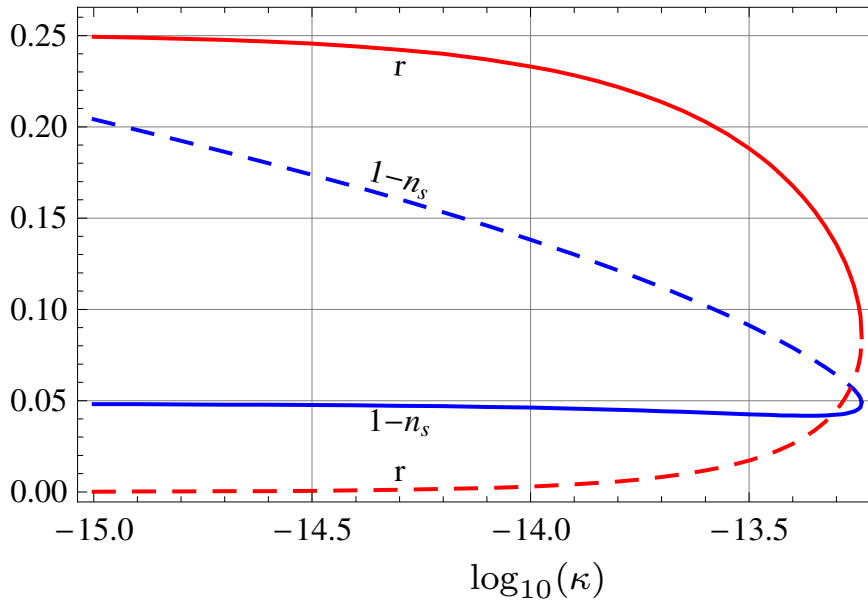


FIG. 2: $1 - n_s$ and r vs. κ for the potential $V = (1/2)m^2\phi^2 - \kappa\phi^4 \ln(\phi/m_P)$. Solid and dashed curves correspond to ϕ^4 and hilltop branches respectively. Note that the curvature perturbation amplitude and number of e-folds require $\kappa \lesssim 7.4 \times 10^{-15}$.

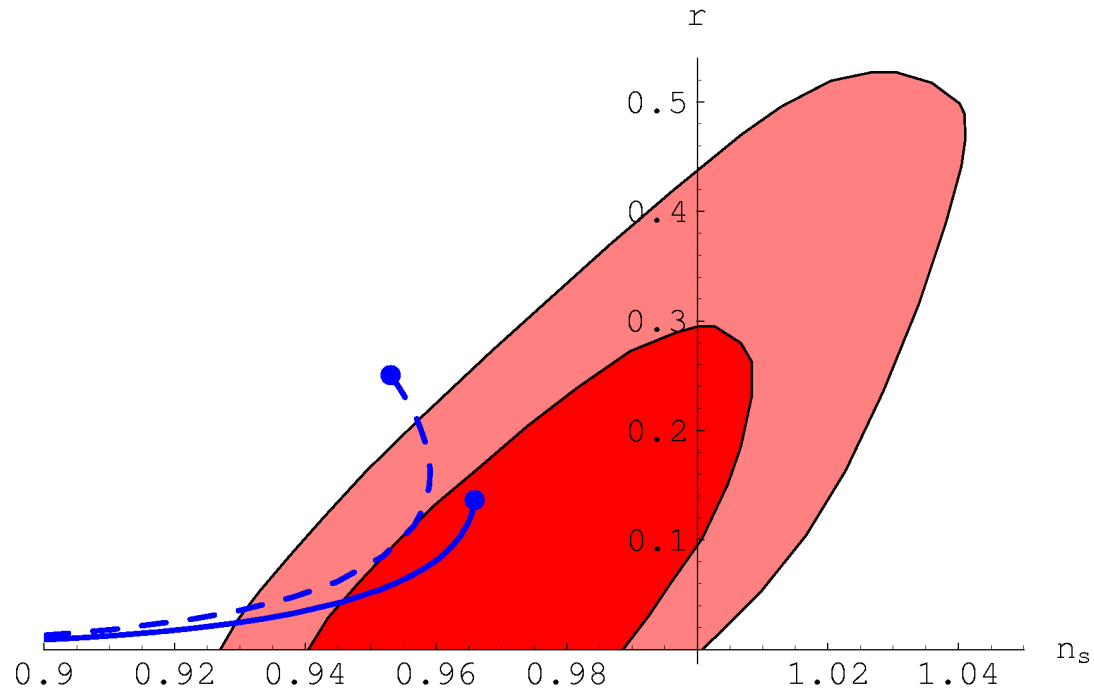
The inflationary parameters for the potential $V = (1/4!)\lambda\phi^4 - \kappa\phi^4 \ln(\phi/m_P)$

(in units $m_P = 1$)

$\log_{10}(\kappa)$	$\log_{10}(\lambda)$	ϕ_e	ϕ_0	$V(\phi_0)^{1/4}$	N_0	v_0	n_s	r	α (10^{-4})
$V = (1/4!)\lambda\phi^4$									
	-12.07	2.53	22.39	0.009737	62.55		0.9517	0.251	-7.637
ϕ^4 branch									
-15.	-12.03	2.516	22.31	0.00972	62.54	143.1	0.9519	0.2493	-7.606
-14.	-11.78	2.438	21.69	0.009558	62.43	14.08	0.9539	0.2331	-7.372
-13.5	-11.49	2.369	20.49	0.009058	62.2	3.834	0.9575	0.1881	-7.025
-13.3	-11.36	2.338	19.35	0.008344	61.97	1.762	0.9577	0.1355	-6.261
-13.24	-11.33	2.319	18.23	0.007421	61.74	0.9184	0.9512	0.08476	-3.725
Hilltop branch									
-13.24	-11.33	2.319	18.23	0.007421	61.74	0.9184	0.9512	0.08476	-3.725
-13.3	-11.41	2.305	17.11	0.006329	61.49	0.4937	0.9359	0.04481	0.9321
-13.5	-11.63	2.292	15.85	0.004985	61.14	0.2391	0.9088	0.01718	6.326
-14.	-12.15	2.276	14.15	0.003225	60.57	0.0799	0.8618	0.002978	8.232
-15.	-13.18	2.256	12.15	0.001534	59.69	0.0151	0.7959	0.000149	4.078



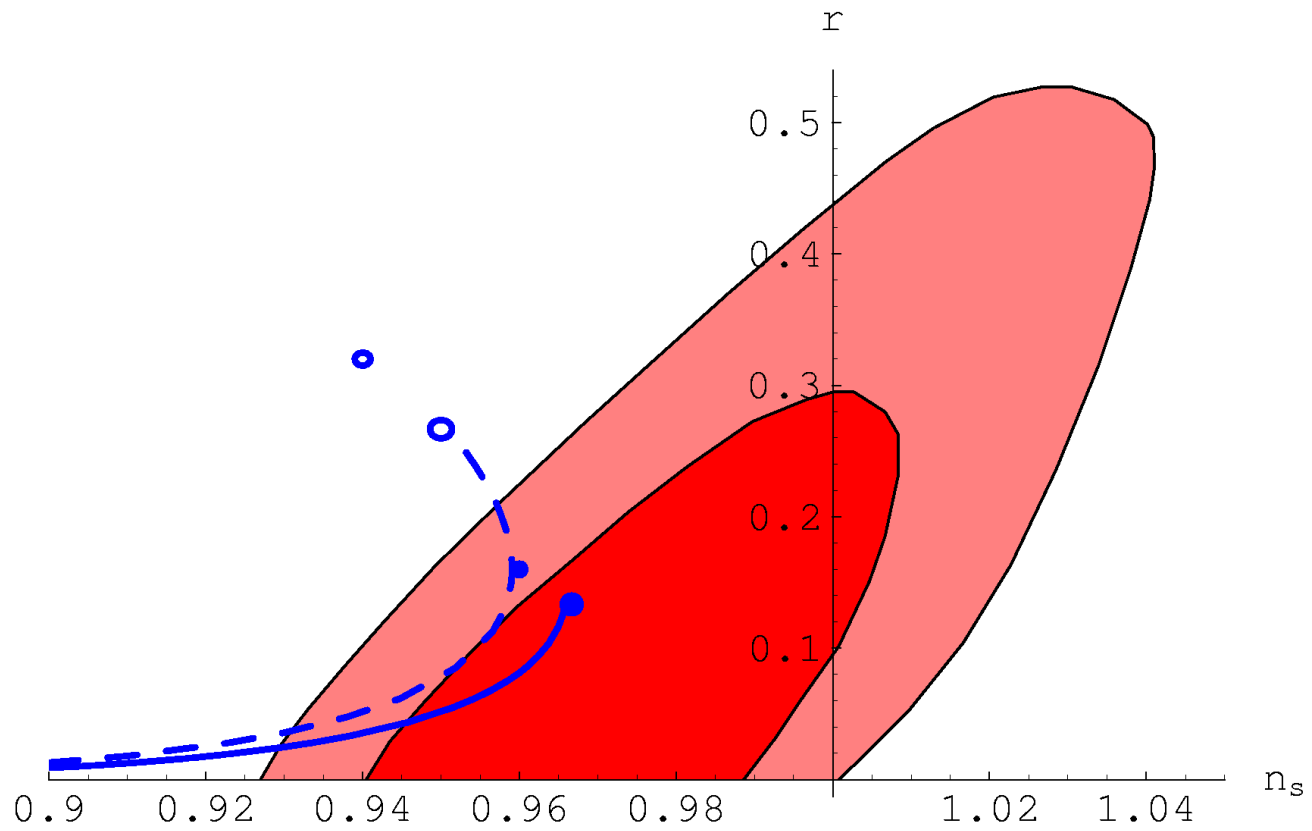
$1 - n_s$ and r vs. κ for the potential $V = (1/4!)\lambda\phi^4 - \kappa\phi^4 \ln(\phi/m_P)$. Solid and dashed curves correspond to ϕ^4 and hilltop branches respectively.



- Tree level Inflation Potential: $V = \frac{1}{2}m^2\phi^2$
 Radiative Corrections: $-\kappa\phi^4 \ln(\phi/m_P)$, $\kappa > 0$ (From Yukawa coupling $\frac{h}{2}NN\phi$)

- Tree level Inflation Potential: $V = \frac{\lambda}{4!}\phi^4$
 Radiative Corrections: $-\kappa\phi^4 \ln(\phi/m_P)$, $\kappa > 0$





Conclusion

The importance of the LHC for the future of high energy physics cannot be overemphasized. Important topics include:

- Nature of Electroweak Symmetry Breaking
- Supersymmetry
- Dark Matter (**LSP**)
- Extra Dimensions (**Kaluza Klein excitations**)
- Spontaneous Parity Violation (**New gauge bosons, other TeV scale particles**)
- TeV Scale Quantum Gravity (**Black holes,...**)
- Exotic States (**Magnetic Monopoles, Fractionally charged color singlets, Z flux tubes, Leptoquarks, diquarks, unparticle physics...**).



Conclusion

- Precision Cosmology will play an important role in the search for new physics beyond the SM.
- Challenge for PLANCK and other ongoing/future expts: Determine n_s , n_T , $dn_s/d \ln k$, r , w_{DE} , to a high degree of precision
- Find DARK MATTER
(LSP, axion, majoran, KK,...)
⇒ help discover standard model of inflation.

